A TEXT BOOK OF PRE-UNIVERSITY PHYSICS

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PREFACE

Since the beginning of the Pre-university classes, students, teachers and educationalists are worried about the falling standards. One of the main findings of the fact finding committee which went into the causes of this deterioration is *the absence of standard text-books covering the Pre-University syllabus.* The present book is an attempt in that direction and we hope, students and teachers of Physics will appreciate this The book covers fully the syllabus for the Pre-University Examination of the University of Rajasthan, and has the following special features :

1 Taking into consideration the low standard of English of the students, simple language has been used Long and complicated sentences have been avoided

2 Looking to the requirements of Biology students the mathematical steps of calculations have been explained in greater detail which will enable a student possessing elementary knowledge of mathematics to understand these steps without any difficulty

3 The topics have been discussed in such a way that even students who did not opt for optional science at their High School Examination will not find it difficult to follow them A large number of diagrams have been added to explain the construction, working and principles involved.

4. As regards the weakness of students in solving numerical problems, *a large number of solved examples have* been added at the end of concerned article The solutions have been given in such a way that they will enable the students to understand the article as well as enable him to solve successfully other unsolved problems.

5 An exhaustive list of questions is added at the end of each chapter and answers are indicated.

We are sure that in the light of above features the book will be most useful We hope like our books "A Text-book of Practical Physics" and "प्रायोगिक भौतिकी" this book will be appreciated as well

Suggestions towards the improvement of this book will be highly appreciated.

Messrs Ramesh Book Depot, our publishers and our printers deserve our special thanks because they could bring out this book even under most difficult conditions

July, 1960 AUTHORS

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CHAPTER I

UNITS

§1. **Necessity of a unit:**—In our daily life, there is a great importance of weighing and measuring various quantities We want to know the distance between Jaipur and Jodhpur How much time would it take to reach from Jaipur to Jodhpur? When we give cloth to a tailor we measure it in yards and inches When we purchase vegetables in the market we want to know how many seers of potatoes are being weighed? In this way at every step, we want to assess various quantities Similarly the study of any physical event or the formulation of any law governing a phenomenon requires the exact measurement of various quantities involved

§2. **Measurement of a quantity:**—When we say that the length of a particular rod is 200 centimetres, the weight of a body is 200 grams or the time taken by a person in going from *A* to *B* is 200 seconds every statement carries two important things Firstly it is the unit in which the quantity has been measured It is centimetre, gram and second respectively in the above statement Secondly the number 200 which states how many times the unit is contained in that quantity Without any of these two things the statement will have no meaning If we say that the length of a rod is 200 mass of a body in grams or time is 200, the meaning is not clear A body of many persons by joint agreement or the government arbitrarily fixes a certain quantity which we call a unit To measure any thing we compare its quantity with that unit *i e* we decide how many times the unit is contained in the given quantity

§3. **Fundamental quantities:**—In our study we would come across a large number of quantities Neither it is necessary nor convenient to select an independent and arbitrary unit for each of them In physical science three quantities are fundamental The Unit of (*i*) *length* (*ii*) *mass* and (*iii*) *time* They are independent of each other All other quantities can be expressed in terms of these three quantities For example the speed or velocity of a body can be determined if we know the distance travelled (*i e* length) by the body in a certain time If we know the length of a box in three perpendicular directions we can find out its volume Such units which can be expressed in terms of the fundamental units are known as derived units

Practical units:—Sometimes the derived units are either too small or too large to express any physical quantity In such cases a convenient unit is selected which bears some relation to the derived unit This is known as practical unit, while the derived units as explained above are known as absolute units

§4. **Kinds of systems:**—Two kinds of systems are prevalent for measuring the various quantities (i) *Metric System* or Decimal System (ii) *British System* Our Government has completely decided to introduce the Decimal System.

(i) *Metric system* :—In Metric System the fundamental units are (i) centimetre for length, (ii) gram for mass and (iii) second for time Therefore this system is also known as centimetre gram second system or C G S system

(ii) *British system* :—In this system the fundamental units are (i) foot (ii) pound and (iii) second. This system is therefore known as (F P S) foot-pound-second system

§5. **The unit of length:**—According to decimal system, the unit of length is centimetre This is 1/100th part of a metre According to international agreement a rod made of an alloy of 90% platinum and 10% iridium is placed in a laboratory in Paris (France) Two marks have been put on this rod at a certain distance The distance between these two marks at 0°C is known as the metre (fig 1) This kind of standard rod is only one in the world and by some accident this may be destroyed Scientists have, therefore, calibrated this distance in terms of the wave-length of cadmium red According to it 1 metre contains 1553163 5 wave-lengths of cadmium red light

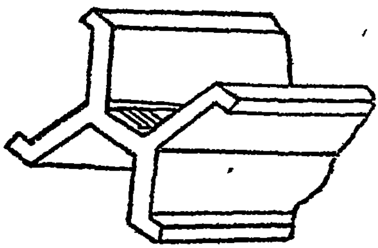


Fig 1

As you have read in your VIII class the following tables give the relation between various divisions and subdivisions of different units of length

DECIMAL SYSTEM

10 Millimetres	=1 Centimetre
10 Centimetres	=1 Decimetre
10 Decimetres	=1 Metre.
10 Metres	=1 Decametre
10 Decametres	=1 Hectometre
10 Hectometres	=1 Kilometre
1 Micron	$=10^{-3}$ Millimetre
	$=10^{-6}$ Metre
1 Angstrom	$=10^{-8}$ Centimetre

BRITISH SYSTEM

12 Inches	=1 Foot.
3 Feet	=1 Yard
220 Yards	=1 Furlong.
8 Furlongs	=1 Mile

1760 Yards	=1 Mile.
1 Light Year	=Distance travelled by light in one year
	$=186000\times60\times60\times24\times365$ miles
	$=5\ 865\times10^{12}$ miles
2 54 Centimetres	=1 Inch
30 5 Centimetres	=1 Foot

§6. **Unit of mass:**—Like the metre, a lump of a similar alloy has been placed in the same laboratory The mass of this lump at 0°C is 1 kilogram This is also equal to the mass of one litre of water at 0°C 1000 grams=1 kilogram Therefore mass of 1 c c of water is 1 gram

Table of divisions and subdivisions of unit of mass

METRIC SYSTEM

10 Milligrams	=1 Centigram
10 Centigrams	=1 Decigram
10 Decigrams	=1 Gram
10 Grams	=1 Decagram
10 Decagrams	=1 Hectogram
10 Hectograms	=1 Kilogram

BRITISH SYSTEM

16 Drams	=1 Ounce (óz)
16 Ounces	=1 Pound (lb)
28 Pounds	=1 Quarter (Qr)
4 Quarters	=1 Hunderweight (Cwt)
20 Hunderweights	=1 Ton
1 Pound	=453 6 Grams

At 0°C the mass of 1 cu foot of water is 1000 oz or 62 5 pounds

§7. **Unit of time:**—In both the systems the unit of time is second Earth rotates about its axes and also revolves round the sun in a fixed orbit The time taken by the earth in completing one rotation is known as one day while that for completing one revolution is one year The time interval of each day is different for different positions of the earth in its orbit The average time of rotation for the whole year is known as mean solar day One second is 1/86400 part of a mean solar day

60 Seconds	=1 Minute
60 Minutes	=1 Hour
24 Hours	=1 Day
365 Days	=1 Year

§8. **A few derived units and their definitions:**—All quantities like velocity volume, area, etc except fundamental quantities of length, mass and time are known as derived quantities Their corresponding units are known as derived units

Velocity :—Velocity is defined as the rate of change of distance in a given direction.

Therefore, $\text{Velocity} = \dfrac{\text{Length}}{\text{Time}} = \dfrac{\text{Centimetre}}{\text{Second}}$

$= \text{Centimetres per second.}$

$= \text{Cm/Sec}$

The unit of velocity is centimetres per second. In British system the unit is feet per sec

Acceleration :—Rate of change of velocity is known as acceleration in a given direction

Therefore, $\text{Acceleration} = \dfrac{\text{Velocity}}{\text{Time}} = \dfrac{\text{Centimetre per sec}}{\text{Second}}$

$= \text{Centimeters per second per second}$

$= \text{Cms./Sec}^2$

In British system the unit is foot per second per second.

Force :—It is that thing which produces acceleration in a body. That force which produces unit acceleration in unit mass is known as unit force

Force $= \text{Mass} \times \text{Acceleration}$

$= \text{Gram} \times \text{centimetre per second per second}$

$= \text{Dyne}$

The unit of force is known as a dyne 1 dyne force is that force which will produce an acceleration of 1 centimetre per second per second in a mass of 1 gram in a given direction The unit of force in British system is poundal. One poundal force is that force which will produce an acceleration of 1 foot per second per second in a mass of 1 pound in a given direction

Work :—When the point of application of a force moves through a certain distance in the direction of force, work is done

Work $= \text{Force} \times \text{distance.}$

$= \text{Dyne} \times \text{centimetre.}$

$= \text{Ergs.}$

When a force of one dyne is displaced through a distance of 1 cm. the amount of work done is one erg. Similarly when a force of 1 poundal is moved through a distance of 1 foot, the work done is one foot-poundal. The practical unit of work is joule 1 joule$=10^7$ ergs. Similarly 1 foot-pound$=32$ foot-poundal

Power :—The rate of doing work is known as power.

$$\text{Power} = \frac{\text{Work}}{\text{Time}} = \frac{\text{Ergs}}{\text{Seconds}} = \text{ergs per sec}$$

$$\text{Here } \frac{\text{Joule}}{\text{Seconds}} = \text{Watts}$$

1 Kilowatt=1000 Watts

The unit of power in British system is known as Horse Power. If an engine can move a force of 550 pounds through a distance of 1 foot in 1 second its power is 1 horse power

$$1 \text{ Horse Power} = 746 \text{ Watts}$$

In this way the unit of any other derived quantity can be fixed

§9. Inter-relation between a few derived units in both the systems :—

$$\begin{aligned}
1 \text{ Poundal} &= 1 \text{ pound} \times 1 \text{ foot per sec}^2 \\
&= 453\ 6 \text{ gram } \times 12 \times 2\ 54 \text{ cm per sec}^2 \\
&= 453\ 6 \times 12 \times 2\ 54 \times \text{gram} \times \text{cm per sec}^2 \\
&= 453\ 6 \times 12 \times 2\ 54 \text{ dynes} \\
&= 13834\ 8 \text{ dynes} \\
1 \text{ foot pound} &= 1 \text{ pound} \times 1 \text{ foot} \\
&= 1 \times 32 \text{ poundal} \times 12 \times 2\ 54 \text{ cm} \\
&= 32 \times 12 \times 2\ 54 \text{ poundal} \times \text{cm} \\
&= 32 \times 12 \times 2\ 54 \times 13834\ 8 \text{ dynes} \times \text{cm} \\
&= 1365\ 28 \times 10^4 \text{ ergs} \\
&= 1\ 36 \times 10^7 \text{ ergs}
\end{aligned}$$

QUESTIONS

1 What is a unit? What is its importance? Define fundamental, and derived units Give examples (*See* §§1, 3, and 8)

2 Define velocity, acceleration, force, work and power Mention their units in both the systems and give their inter-relation (*See* §8 and §9).

CHAPTER II

MEASUREMENTS OF LENGTH

§1. Measurement of Length:—You already know from your school time that the main instrument for measuring length is either a metre scale or a foot rule. You also know how to find out the length of a straight line which can be measured with the help of a thread and a scale While using a scale the following precautions are to be observed

(*i*) Do not measure from the beginning of the scale because its end may be worn out (*ii*) Whenever you are taking the reading of the scale keep your eye vertically above the division

§2. Principle of a vernier:—While using a scale many times we come across a situation as shown in fig 2 Here point B lies between 2 1 cm and 2 2 cm Therefore the length of the line *AB* is more than 2 1 cm and less than 2 2 cm In this way we can find out the length of a line correct up to the first place of decimal

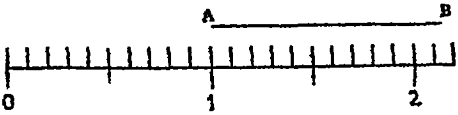


Fig 2

In order to find out length more accurately we make use of a second short scale known as vernier scale This vernier scale is fixed on the main scale and can slide alongside it There are generally 10, 20 or 25 divisions on the vernier scale Suppose there are 10 divisions on the vernier scale These ten divisions of the vernier scale are made equal to 9 divisions of main scale Therefore 1 division of the vernier scale is equal to 9/10 division of main scale If the main scale is divided in millimetres, then 1 division of vernier scale will be 9/10 mm Therefore, the difference between 1 division of the vernier scale and 1 division of the main scale will be 1−9/10 or 1/10 mm Suppose for a certain position of vernier the zero of the vernier scale is in line with 2 cm on the main scale then as shown in fig 4 the 10th division of the vernier scale will coincide with 2 9 cm No other division in between will coincide anywhere

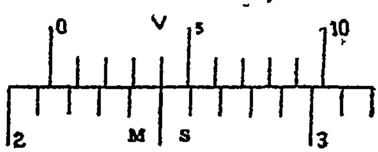


Fig. 3

The first division of the vernier scale will be shorter than the first division of main scale by 1/10 mm. Similarly 2nd division will be shorter by 2×1/10 mm and so on Now if the vernier scale is moved forward so that the first division of the vernier scale coincides with

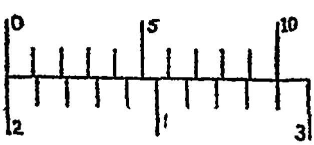


Fig 4

2.1 cm then the zero of the vernier scale will move forward by 1/10 mm. Similarly when the second division coincides, the distance advanced by the zero of the vernier scale is $2\times 2/10$ and so on. In this way the distance moved forward by the zero of the vernier scale in fig. 5 is $6\times 1/10$ mm. In this case the position of the zero of the vernier scale is 2.06 cm.

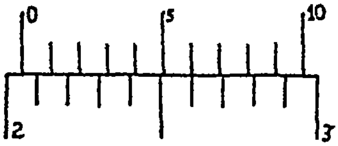


Fig 5

Consider the same case as shown in fig 2 when point *B* does not coincide with the main scale division. In such a case move the vernier scale so that its zero coincides with point *B* (fig 3). Take the reading of the main scale on the left of the zero of the vernier. This gives the main scale reading. Now find out which division of the vernier scale is coinciding with main scale division. Suppose it is the 4th division. It means the zero of the vernier scale has moved forward from 2.1 by $4\times 1/10$ mm or 4 mm or .04 cm. Therefore the position of the zero of the vernier scale is $2.1+.04$ cm that is 2.14 cm. In this way we can measure length accurately to the second place of decimal. The smallest length which can be measured with this vernier is .01 cm and is known as vernier constant. In any case, vernier constant can be determined by the formula given below :—

$$\text{Vernier constant}=\frac{\text{Smallest division on M.S.}}{\text{No. of divisions on V.S.}}$$

§3. Vernier Callipers :—This is a very useful instrument. We can find out the internal or external diameter or measure length, provided it is small, with the help of a vernier callipers.

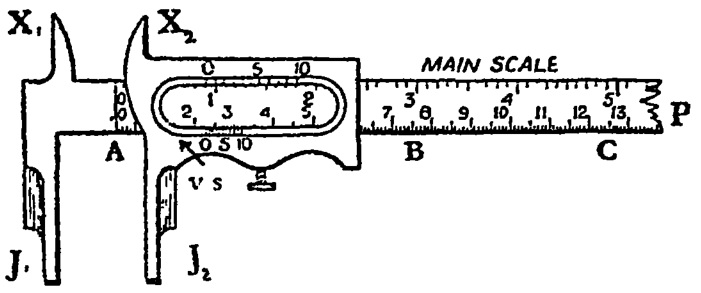


Fig 6

Construction :—We have already discussed the construction and working of a simple vernier. Vernier callipers is shown in fig 6. *P* is a thin strip of metal carrying a fixed jaw *J*, at one end. Main scale (M.S.) is graduated in centimetres and millimetres on *P*. J_2 is another movable jaw parallel to J_1. J_2 is attached to such a strip which can be moved along the main scale. V.S. is the vernier scale marked on the movable strip. Generally there are 10 divisions on the vernier scale. Therefore its least count (L.C.) is 1/10 mm or .01 cm. When J_2 touches J_1, the zero of the vernier scale coincides with zero of the main scale.

Working:—(*See* Text Book of Practical Physics by the authors for greater detail) Suppose we want to find out the external diameter of a calorimeter Place it between the two jaws in such a way that neither it is too tight nor too loose Now find the position of the zero of the vernier scale because the diameter of the calorimeter is equal to the distance between the two jaws which is equal to the distance between zero of the vernier scale and zero of the main scale

Take the reading of the main scale up to the left of the zero of vernier scale Find out the division of the vernier scale coinciding with a main scale division Multiply this by vernier constant and add it to the main scale reading. This will give the diameter of the calorimeter

To measure internal diameter:—We can find out the internal diameter with the help of jaws λ_1 and λ_2 Put these jaws inside the body and move the vernier till the jaws touch both the sides Take the reading of the vernier as shown above This gives the internal diameter

To measure depth:—A thin strip is also attached to the vernier strip When the jaws are in contact, the strip is just equal to strip *P* On moving the vernier scale the strip comes out Put this inside the vessel till it touches its bottom Take the reading on the vernier scale as explained above This will give the depth of the body

Zero-correction:—If the zero of the vernier scale does not coincide with the zero of the main scale when the two jaws are in contact then there is a zero error in the instrument In order to find out the necessary correction, put the two jaws in contact and take the reading of the zero of the vernier scale as explained above If the zero of V S lies to the left of M S Zero the correction is to be added if otherwise it is to be subtracted

§4. **Screw Gauge**—Generally we can measure length correct up to the second place of decimal with the help of a vernier callipers Therefore it is useful for measuring diameters of thicker bodies For measuring diameters of wires etc we make use of a screw gauge

Construction.—A screw gauge is shown in fig 7 *A* is a rectangular or *U* shaped frame of metal A steel plug (*B*) with carefully planed face is fixed at one end of the frame A nut is attached to the other end of the frame A hollow cylinder *M* is attached to the nut Main scale in millimetre or 1/2 millimetre is marked on this cylinder along a reference line *R* A screw *D* with flat end moves inside the nut. To the other end of the screw is attached a cap *E* which moves on the main scale The circular scale of the cap is divided in 100 or 50 equal parts. The scale can be rotated with the help of a milled head

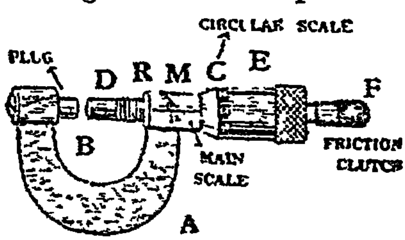


Fig. 7

Principle:—When a screw is given one complete rotation, generally it moves forward by 1 mm and the zero of the scale rotates by 100 divisions That is when the circular head rotates by 100 divisions the screw moves by 1 mm and therefore when the screw rotates by 1 div it will move by 1/100 mm If it rotates by n divisions it will advance by $n \times 01$ mm

When B and D touch each other, zero of the circular scale should coincide with zero of the main scale Therefore the distance between B and D will be equal to the distance moved by the zero of the circular scale

Definitions.—1 *The distance travelled by the screw in one complete rotation of the circular scale is known as the* **pitch** *of the screw*

2 *The distance travelled by the screw when the circular scale is rotated by 1 division is known as* **least count** *of the instrument*

$$\text{Least count} = \frac{\text{Pitch}}{\text{No of divisions on circular scale}}$$

$$= \frac{1 \text{ mm}}{100} = 01 \text{ mm} = 001 \text{ cm (Generally)}$$

Working:—Find out the pitch and least count as shown above, Place the wire between B and D and move the head till the wire is neither too loose nor too tight Take the main scale reading and note the division of circular scale in line with the reference line Find the total reading by the formula, total reading=M S reading+C S reading×least count This will give the diameter of the wire

Zero-correction·—If the zero of the circular scale does not coincide with the zero of the main scale when B and D are touching each other there is zero error in the instrument To determine this find out as to how many divisions the zero of the circular scale is above or below the main scale line Multiply this by least count This will give the zero error If the zero is above the line, correction should be added to the total reading, if otherwise it should be subtracted

Note —Sometimes a ratchet head F is attached at the top of the milled head In such a case when we rotate the ratchet head the screw will not move forward after it has touched the wire or end B Thus it will be saved from damage

§5. **Spherometer:**—This instrument is also based on the principle of the screw gauge This is used for finding out the radius of curvature of spherical surfaces and therefore it is known as a spherometer

Construction:—(*See* fig 8) It consists of a metallic frame standing on three legs A, B and C which form the vertices of an equilateral triangle A screw D passes through the centre of the frame such that it will pass through the centre of the triangle A circular disc E is fixed at the top of the screw This disc is divided into 50 or 100 divisions and can be rotated by means of a head F The

screw will move up and down when so rotated by F. Along one of the legs is attached a vertical strip G which touches the circumference of the disc. Main scale is graduated on this strip in millimetres. Generally when the disc is given one complete rotation it moves on the main scale by 1 division *i.e.* by 1 millimetre This is known as the pitch of the screw. When this is divided by the number of divisions on the circular scale we get the least count. When the spherometer is placed on a glass plate and the screw is rotated so that it also touches it, then the three legs will form an equilateral triangle and the screw will touch at the centre of the triangle (*See* fig. 10)

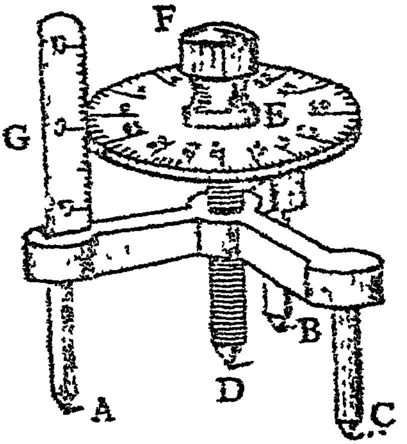


Fig. 8.

Working:—(*See* A Text Book of Practical Physics by the authors). To find out the thickness of a plate:—

First find out the pitch of the screw and the least count. Now place the spherometer on a plane glass plate and rotate the screw

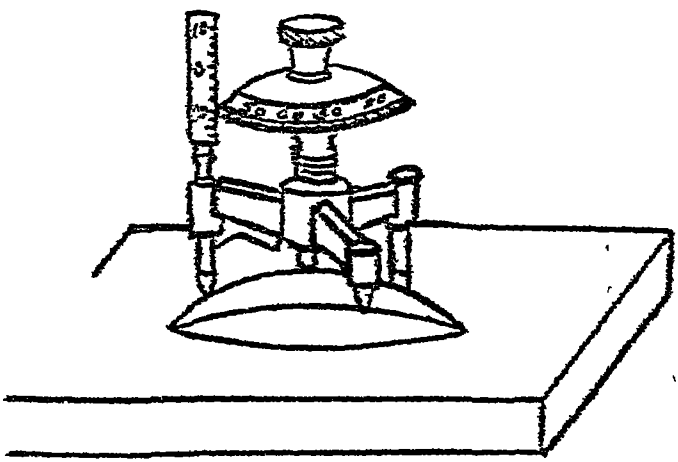


Fig. 9.

till it touches the glass plate In this position the tip of the screw D and its image in the glass plate will appear to touch each other. Now take the reading of the upper edge of the disc. If there is no zero error this reading will be zero otherwise note the reading. Move the screw upwards and place the glass piece below the screw and move the screw such that it touches the glass piece. Again take the reading of the main scale and note the division of the circular scale conciding with the main scale. Multiply this by the least count and add it to the main scale reading. This reading $\pm$ zero error (*i.e.* initial reading) is the thickness of the plate.

To find the radius of curvature of a spherical surface:—*See fig.* 9 and 9(a)

Place the spherometer on a plane glass plate and rotate the screw till it touches the glass plate. In this position the screw and all the legs will be in the same plane. Now place the convex mirror or convex lens whose radius of curvature is to be determined on the glass plate. In fig. 9 XBAYZ is the portion of a spherical surface whose centre is at *O*. XBY is curved surface and XYZ is plane. Move the screw upwards and place the spherometer on the spherical surface. Rotate the screw till it touches the top of the curved surface as shown at *D*. In this case *ABC* will be in one plane and *D* will be above this plane. Take the reading of the disc in this position. The difference between the two readings gives *DD'* or height (*h*) of the spherical surface. Press the spherometer on a piece of paper and measure the distances *AB*, *BC* and *AC* and find out mean side (*a*). The radius of curvature *r* is given by the following formula

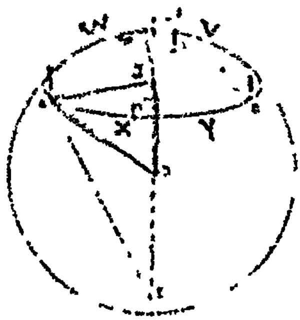


Fig. 9 (a)

$$r=\frac{a^2}{6h}+\frac{h}{2}$$

Proof of the formula:—Join *AD*, *Do* and *Ao* and produce it to meet *BC* in *F*. In the $\triangle ABC$ (fig. 10) since *AF* is perpendicular to *BC* and *BF*=*FC*,

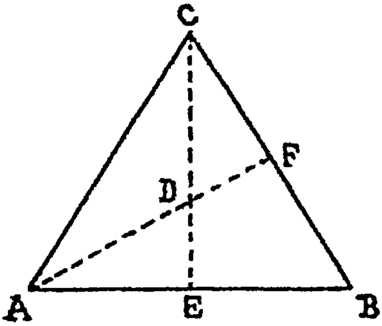


Fig. 10

$$AB^2=AF^2+FB^2$$

or
$$AF^2=AB^2-FB^2$$
$$=a^2-(a/2)^2$$

Because $AB^2=a^2$, $FB=\frac{1}{2}BC=a/2$

$$\therefore\quad AF^2=a^2-\frac{a^2}{4}=\frac{3a^2}{4}$$

or
$$AF=\sqrt{\frac{3a^2}{4}}$$

Since *AF* is also median of the triangle *ABC*, $AD=2/3\ AF$

$$\therefore\quad AD=2/3\sqrt{\frac{3a^2}{4}}=\frac{2}{3}\times\frac{a}{2}\sqrt{3}$$

$$=\frac{a\sqrt{3}}{3} \qquad (i)$$

In the triangle *AOD* Fig. (9)

Since angle *ADO* is a right angle, therefore, we have

$$Ao^2=AD^2+Do^2$$
$$=AD^2+(OO'-DO')^2$$
$$r^2=AD^2+(r-h)^2 \qquad \therefore\quad Ao=O'o=$$
$$\therefore\qquad r^2=AD^2+r^2-2rh+h^2$$

or $$r=\frac{AD^2}{2h}+h/2 \qquad ...(ii)$$

From (i) $$AD^2=\left(\frac{a\sqrt{3}}{3}\right)^2=\frac{a^2\times 3}{9}=\frac{a^2}{3}$$

$$\therefore \quad r=\frac{a^2}{6h}+\frac{h^2}{2h}$$

$$=\frac{a^2}{6h}+h/2$$

Since h is small it may be neglected and we get

$$r=\frac{a^2}{6h} \text{ approx} \qquad .. (iii)$$

§6. **Measurement of time:**—Time is measured by a stop-clock or stop watch. The least count is generally 2 second. On first press the watch starts, on second press it stops when reading is taken and on third press there is a fly black action when all the hands come to initial position and the stop-watch is ready for use again A stop watch is shown in fig 11. The small circle near the key reads minutes and the large central circle reads seconds and sub-divisions

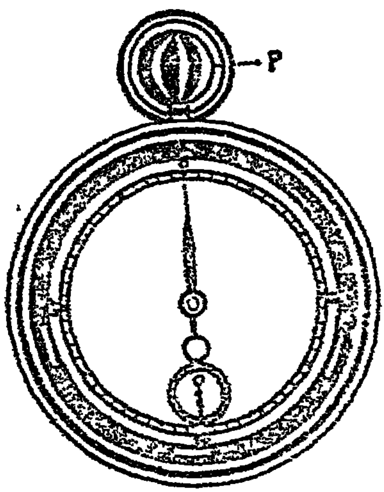


Fig 11.

In the old days people used to judge time by the position of sun in daytime and by star positions during night Then were developed the sand clocks With the advent of modern era came the days of spring watches and pendulum clocks. The accuracy of measurement of time increased Now in this atomic era are developed some atomic clocks which can measure time to the accuracy of a second in thousands of years

QUESTIONS

1. Discuss the principle of vernier scale and explain how vernier constant is calculated. (*See* § 2)

2. Describe the construction and working of a screw gauge. Describe its use in determining the radius of a wire (*See* § 4).

3 What is a spherometer? Describe its construction and working. How is it used to determine radius of curvature of a spherical body? Deduce the formula you use. (*See* § 5).

CHAPTER III

VOLUME

§1. **Volume :**—The space occupied by a body is known as its volume You have read in your general science that the volume of a book is less than that of a box Every body occupies some space according to its size Greater is the length or breadth of a body greater will be its volume If two bodies are equal in length and breadth their volumes will depend upon the height Thus we see that volume of a body depends upon length, breadth and height,

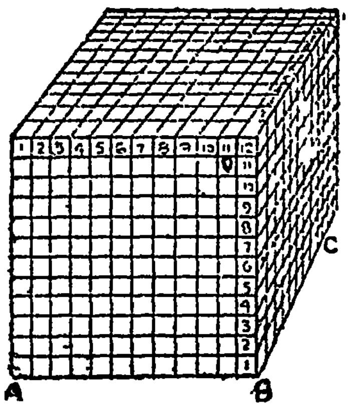


Fig 12

In case of a uniform body as shown in fig 12, *AB* is length, *BC* breadth and *BD* is height

$$\text{Volume}=L\times B\times H$$

§2. **Unit of volume :**—Unit of volume in metric system is cubic centimetre It is the volume of a cubic whose side is 1 centimetre

Fig 13

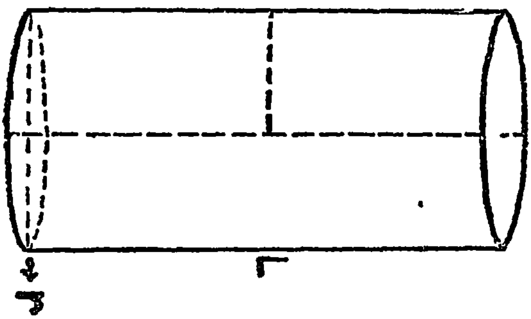


Fig. 14.

Similarly the volume of a cubical body whose side is 1 metre is one cubic metre Its side can be expressed in centimetre also It will be 100 cms therefore its volume will be $100\times100\times100$ cubic centimetres Thus we see that 1 cubic metre is equal to $100\times100\times100$ cubic centimetres In this way we can form the whole conversion table as on page 2, unit of volume in British System is cubic feet It is the volume of a cube whose side is 1 foot When we say that the volume of a solid is 1,000 c c it means that its volume is equal to a cube whose side is 10 cms or its volume is 1,000 times the volume of a unit cube

§3. **Volume of regular solids :**—Regular bodies are those bodies whose sides follow certain regular pattern like book, box, sphere, cylinder, cone etc Irregular bodies have got no definite shape like a piece of stone

Volume of regular bodies can be calculated with the help of formulae given below

(i) *Volume of a rectangular parallelopiped*

$=$ length $\times$ breadth $\times$ height

(ii) *Volume of a cube* $=$ side $\times$ side $\times$ side $=$ (side)^3

(iii) *Volume of sphere* $= 4/3\pi$ (radius)$^3 = 4/3\pi r^3$

Here r = radius of the sphere and $\pi = 3.14$

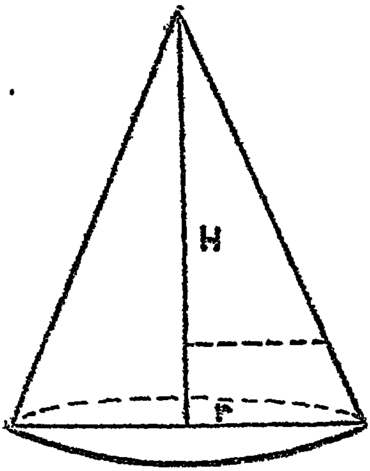


Fig 15.

(iv) *Volume of a cylinder* $= \pi r^2 h$, or $\pi r^2 l$

Here r is the radius and h or l is height or length of the cylinder.

(v) *Volume of a cone* $= \frac{1}{3}\pi r^2 h$

Here r is radius of the base and h is height In order to find out the volume of above solids measure their sides or radius with the help of a scale, vernier callipers or screw gauge depending upon their size

§4. To find the volume of irregular bodies:—For this purpose we use measuring glass (fig 15) burette or pipette as shown in the figure

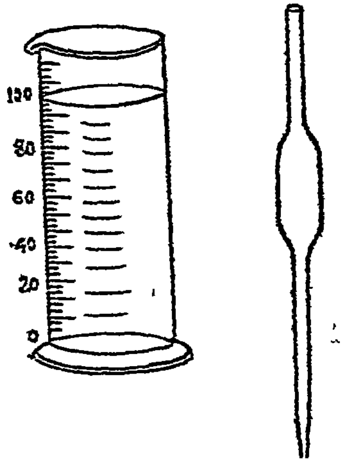


Fig 16.

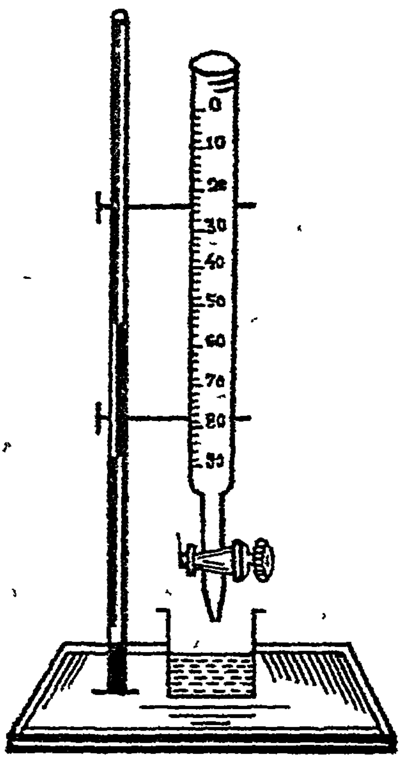


Fig. 17.

§5. **To find the volume of a piece of stone:**—We know that whenever a body is dipped in water it will displace water equal to its own volume Put some water in a graduated jar Take the reading of the lower meniscus water level While reading these instruments eye should be placed in the same level The surface of the liquid will be curved Take that reading which is tangential to this surface at the lowest point Now put the body gently inside the cylinder Water will rise again and take the reading The difference of the two readings gives the volume of the body In order to find out volume of bodies which cannot be placed in graduated jar we can use burette or pipette

QUESTIONS

1 What do you understand by volume of a body ? How is it measured for regular and irregular bodies ? (*See* §1, §2 and §3)

CHAPTER IV

MEASUREMENT OF MASS*

§1. **Mass:**—The amount of substance contained in a body is known as mass The amount of wood required in the construction of chair is less than that in the construction of table Therefore mass of chair is less than that of table

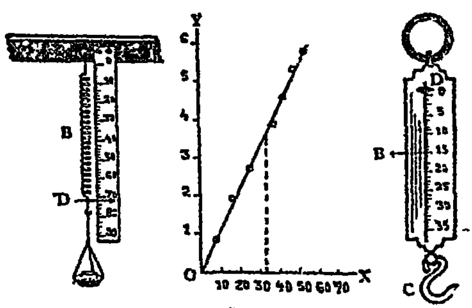
The unit of mass in metric system is gram and in British system is pound.

§2. **Variation of mass:**—The mass of a body is not altered by moving it from one place to another Unless the body is divided or something is added to it its mass remains constant

§3. **Weight:**—According to law of gravitation everybody is attracted towards the centre of the earth with a certain force This force is proportional to the mass of the body and is known as the weight of the body As you will read later on this force varies from place to place on the surface of the earth or with height from the surface of the earth and with depth below the surface of the earth Thus weight of a body is not constant

§4. **Methods for measuring mass:**—The instrument which is used for measuring mass is known as balance They are of two types (*a*) spring balance, (*b*) physical balance In fact we measure the weight of a body and not the mass, with the help of a spring balance.

§5. **Spring balance:**—[*See* Fig 18, (*i*) and (*iii*)] This consists of a hollow flat tube in which a spring *B* is attached from the top of the tube The lower end of the spring carries a hook in which a body whose weight is to be determined is suspended A pointer *D* is attached to the spring which moves over the scale When there is no load on the spring the pointer reads zero When the load is suspended from the spring, it extends, the extension is proportional to the load placed [fig 18 (*ii*)] and the pointer gives the weight of the body If a body is weighed with the help of a spring balance at different places on the surface of the earth, its weight will be different It will be more on the poles, less on the equator, less at a certain height and also less inside the surface of the earth



(*i*) (*ii*) (*iii*)
Fig 18

*This chapter is not included in course but it has been given for the sake of continuity

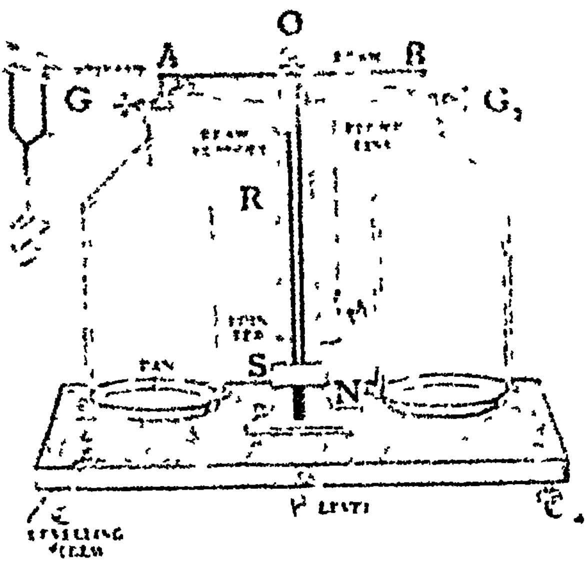


Fig 20

rod carries a platform of agate at the upper end and can be raised or lowered by means of lever H at the lower end AB is a girder shaped beam carrying knife edges at the centre and at the two ends A and B Ordinarily the beam rests on a beam support When the rod is raised the beam rests on its knife edges on the agate platform The knife edges at the ends support two stirrups in which pan holders are suspended These pan holders carry the pans G_1 and G_2 are two nuts which can be moved in and out A pointer S is attached to the beam at O which moves on the scale N A plump line M is suspended over a metal point

Working:—Adjust the levelling screws till the plumb line is vertically above the metallic point

(*ii*) Gently raise the beam and see that the pointer moves equally in both the directions, if not adjust it by moving G_1 and G_2 till it deflects equally in both the sides

(*iii*) Place the body in right hand side and place approximately equal weight in left hand side Raise the lever and note the deflection Adjust the weights till equal deflection is obtained on both the sides Note the weights This will give the weight of the body

Precautions :—1. Place the body in left hand side and weights in right hand side

2 Do not place any weights when the beam is raised

3 Close the doors of the box while taking reading

4 Do not touch weights by your hand

5 Do not weigh a hot or damp thing

§7. **Requisites of a good balance :**—A good balance should be

(*i*) True

(*ii*) Sensitive and

(*iii*) Stable

(*i*) **True :**—A balance will be true if the weight of a body comes out to be same in both the pans For this, its arms must be equal in length and weight The pans must also be equal in weight Otherwise the balance will be false

(*ii*) **Sensitive :**—If a balance gives a large deflection for a small difference in weight in the two pans it is known as sensitive For this the beam of the balance should be long, it should be light and the distance between fulcrum and centre of gravity should be small

(*iii*) **Stable :**—A balance is stable if it comes to rest very soon. This will be possible when the beam is short, heavy and the distance between the fulcrum and centre of gravity is large

Thus we see that sensitivity and stability are opposite to each other and the same balance cannot satisfy both the conditions We select a sensitive or a stable balance according to its use For weighing costly things or scientific measurement we use sensitive balance and for weighing heavy things we use stable balance.

§8. **To find a correct weight with the help of a false balance :—** There are two methods for finding correct weight :—

(*i*) **Gauss's method :**—First place the body in one pan and find out its weight (W_1). Then place the body in another pan and again find out its weight (W_2) The true weight W is given by

$$W=\sqrt{W_1 \ W_2}$$

(*ii*) **Boarda's method :**—Place the body in right hand side and place something like sand or stone pieces in left hand pan. Remove the body and place weights in right hand side pan This will give the true weight of the body

QUESTIONS

1 Distinguish between mass and weight (*See* §§1, 2, 3)

2. How will you measure the mass of a body ? (*See* §6)

CHAPTER V

DENSITY AND RELATIVE DENSITY

§1. **Density:**—Take two pieces of iron and wood having equal volume Try to lift them, you will find that the iron piece is heavier than the wooden one That is mass of iron is more than that of equal volume of wood In this way if we find out the masses of different substances having equal volumes we can compare their relative heaviness This idea is contained in density which is defined <u>*as mass of the substance per unit volume*</u> Density of iron is 7 6 grams per c c It means the mass of 1 c c of iron will be 7 6 grams Density is given by the following relation,

$$D=\frac{M}{V} \text{ grams per c c}$$

where M is mass in grams and V is volume in c c and D is density The unit of density in British system is pounds per cubic foot

§2. **Density of water:**—Mass of 1,000 c c of water is 1 kilogram or 1,000 grams Thus density of water is 1000/1000 *i e*, 1 gram per c c Generally this is true at 4° C Density of water in British system is 62 5 pounds or 1,000 oz per cubic foot That is one cubic foot of water will weigh 62 5 pounds

§3. **To determine density:**—In order to find out density of a body we should know its mass and volume Mass can be determined with the help of a physical balance In case of regular bodies volume can be found out by measuring its dimensions In case of irregular bodies volume can be determined by graduated jar or any other method, given in the chapter of volume Density can be calculated by the above formula $\left(D=\frac{M}{V}\right)$

Numerical Problems.—1 *Find the density of a cylindrical body whose length is 15 cms and radius is 2 cms Its mass is 113 4 grams*

We know that the volume of a cylinder V is given by $V=\pi r^2 l$

here $r=2$ cms and $l=15$ cms

$\therefore$ $V=3\ 14\times2\times2\times15$

$=188\ 4$ c c

$M=113\ 4$ grams

$\therefore$ Density $D=\frac{M}{V}$

$=\frac{113\ 4}{188\ 4}$

$=\ 6$ gram per c c

Calculations :—

$$\log 113.4 = 2.0547$$
$$\log 188.4 = 2.2751$$

$$\bar{1}.7796$$

$$\text{Anti log } \bar{1}.7796 = .6020$$

2. *How much water should be added to 125 c c of copper sulphate solution of density 1.5 grams per c c so as to reduce it to 1·25 grams per c c ?*

Suppose we add x c c , of water

Mass of copper sulphate solution before adding water

$$= V d = 125 \times 1.5 \text{ grams}$$

Mass of water added $= x$ grams (1 c c = 1 gram)

$\therefore$ Total mass $= (125 \times 1.5 + x)$ grams

$= (187.5 + x)$ grams

Total volume $= (125 + x)$ c c

$\therefore$ Density of mixture $= \dfrac{\text{Total mass}}{\text{Total volume}}$

$$1.25 = \frac{187.5 + x}{125 + x}$$

Cross multiplying

$$1.25\ (125 + x) = (187.5 + x)$$

$$\therefore \quad x = \frac{187.5 - 1.125 \times 125}{1.25 - 1} = 125 \text{ c c}$$

§4. Relative density :—Often we want to know the relative densities of different substances Since water is a more common substance and is easily available we compare the density of every other substance with respect to water *The ratio of the density of the substance to the density of water is known as* **Relative Density** or **Specific Gravity**. Since it is the ratio of two similar quantities it has no unit When the density of any substance is compared with any other substance (including water) we call it relative density but when it is compared with density of water it is known as specific gravity.

$$\text{Specific gravity} = \frac{\text{Density of the substance}}{\text{Density of water at } 4°C}$$

$$= \frac{\text{Wt of the substance.}}{\text{Wt of an equal vol of water at } 4°C.}$$

Suppose the density of the substance $= 7.6$ grams per c.c

$$\therefore \text{ Specific gravity} = \frac{7.6}{1} = 7{\cdot}6$$

Since the density of water is one in metric system therefore density is numerically equal to specific gravity But in British system density of water is 62.5, therefore density of the substance is

equal to 62.5 times the specific gravity. In the above example, specific gravity is 7.6. It will be the same in both the systems. But density in metric system will be 7.6 grams per c.c. while density in British System will be 62.5 × 7.6 = 475 pounds per cu foot. Therefore

In metric system Density = specific gravity

In British system Density = 62.5 × specific gravity

§5. **To determinee specific gravity:—**

$$\text{Specific gravity} = \frac{\text{Density of the substance}}{\text{Density of water}}$$

$$= \frac{\text{Mass of the substance} / \text{volume of substance}}{\text{Mass of water} / \text{volume of water}}$$

$$= \frac{\text{Mass of the substance}}{\text{Mass of equal volume of water}}$$

$$= \frac{\text{Mass of the substance}}{\text{Mass of water displaced by the substance}}$$

$$= \frac{\text{Mass of the substance}}{\text{Volume of water displaced by the substance}}$$

Therefore in order to find out specific gravity first find out its mass. Then find out the volume of water displaced by it since mass of water is numerically equal to its volume specific gravity can be obtained by the above formula

§6. **Specific gravity bottle:**—This bottle is shown in the figure (*i*) Its volume is generally 25 c c or 50 c c It carries a glass stopper with a fine hole in it This is to ensure that the bottle is completely full with liquid The bottle is completely filled with the given liquid and then stopper is gently pressed so that some of the liquid comes out

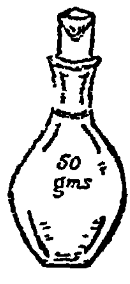


Fig 21

§7. **To find out the specific gravity of a liquid with the help of specific gravity bottle:**—Take the specific gravity bottle, clean it and dry it Find out its mass with the help of a physical balance Let this be W_1 gram Now fill up the bottle completely with water and put the stopper and after drying it from outside find out its mass again let this be W_2 gram Throw away the water and after drying it fill it up with the liquid and find out its mass again Let this be W_3 gram Record your observations as shown below :

1 Mass of specific gravity bottle $= W_1$ gram

2 Mass of specific gravity bottle + Water $= W_2$ gram

3 Mass of sp gravity bottle + liquid $= W_3$ gram

Mass of liquid $= W_3 - W_1$ gram

Mass of water $= W_2 - W_1$ gram

Since volume of water = volume of liquid

Therefore specific gravity of liquid $= \dfrac{W_3 - W_1}{W_2 - W_1}$

In this way we can find the specific gravity of any liquid

§8 **To find the specific gravity of lead shots with specific gravity bottle:**—Find out the mass of sp gravity bottle as explained above Put a few lead shots in it and again find out its mass Fill up the bottle with water and again find out its mass Now remove the lead shots from the bottle and fill it up with water completely and find out its mass Find out the specific gravity as shown below:—

1 Mass of specific gravity bottle $= W_1$ gram

2 Mass of sp gravity bottle + lead shots $= W'$ gram

3 Mass of sp gravity bottle + lead shots in it + water $= W_2$ gram

4 Mass of sp gr bottle + water $= W_3$ gram

∴ Mass of lead shots $W = W' - W_1$ gram.

Mass of equal volume of water $= W_3 + W - W_2$ gram

Because in the second case since lead shots are inside bottle, therefore mass of water contained in it will be less by the amount of water whose volume in equal to the volume occupied by the shots

∴ Specific gravity of shots $= \dfrac{\text{Mass of shots}}{\text{Mass of equal volume of water}}$

$$= \frac{W}{W_3 + W - W_2}$$

Numerical Problem:—*The mass of specific gravity bottle is 27 52 grams when empty and 51 25 grams when filled with some lead shots. When it is further filled with water its mass is 74 15 grams When it is completely full of water alone its mass is 52 52 grams Find the specific gravity of lead shots*

Mass of lead shots $= 51\ 25 - 27\ 52$

$= 23\ 73$ grams

Mass of water displaced, $= 52\ 52 + 23\ 73 - 74\ 15$

$= 2\ 1$ grams

∴ Specific gravity $= \dfrac{23\ 73}{2\ 1}$

$= 11\ 3$ *Ans*

§9. **To find the specific gravity of a soluble substance like sugar or common salt:**—Find out the relative density of sugar with respect to some other liquid in which it is insoluble like kerosene as explained above. Take kerosene instead of water Again find out the relative

density of kerosene with respect to water as explained in §7 The specific gravity of solid will be given by the following formula Specific gravity = relative density of the substance with respect to kerosene × relative density of kerosene with respect to water

Proof of the above formula :—Specific gravity of the substance

$$=\frac{\text{Density of the substance}}{\text{Density of water}}$$

$$=\frac{\text{Density of the substance}}{\text{Density of the liquid}}\times\frac{\text{Density of liquid}}{\text{Density of water}}$$

Relative density of the substance with respect to liquid × Relative density of liquid

Numerical Problem. *The following observations were taken during an experiment :—*

(i) *Mass of specific gravity bottle = 15 72 grams*

(ii) *Mass of sp gr bottle + salt = 20 52 grams*

(iii) *Mass of sp gr bottle + salt + spirit = 39 1 grams.*

(iv) *Mass of sp gr bottle + spirit = 36 22 grams.*

(v) *Mass of sp gr bottle + water = 40 72 grams*

Find out the specific gravity of spirit and salt

Mass of spirit $= 36\ 22 - 15\ 72$

$= 20\ 50$ grams

Mass of water $= 40\ 72 - 15\ 72$

$= 25$ grams

∴ Sp gr of spirit $= \frac{20\ 50}{25} = \ 82$

Mass of salt $= 20\ 52 - 15\ 72$

$= 4\ 80$ grams

Mass of spirit displaced $= 36\ 22 + 4\ 80 - 39\ 1$

$= 1\ 92$

Relative density of salt with respect to spirit

$$=\frac{4\ 80}{1\ 92}$$

Specific gravity of salt with respect to water

Calculations :—

log 4 8 = 6812

log 1 92 = 2833

log 82 = $\bar{1}$ 9138

5950

2833

3117

Ant log 3117 = 2 050

$= \frac{4\ 80 \times \ 82}{1\ 92}$ grams

$= 2\ 05$

§10. To determine the specific gravity of a liquid with the help of a U-tube.—Take a *U* tube of glass fixed up on a wooden stand in a vertical position (fig 22). Two scales are fixed on the stand by the side of the two limits Put some mercury in the tube The level of mercury in both columns will be the same In one of the columns put some amount of liquid. Pore water in the second limb till the level of mercury in both the limbs is the same Note the length of liquid column and water column The R.D. can be calculated according to the formula

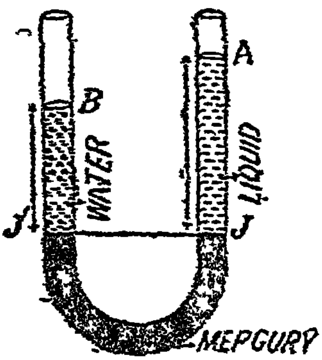


Fig 22.

Specific gravity of liquid

$$=\frac{\text{Length of water column}}{\text{Length of liquid column}}$$

$$=\frac{H_1}{H_2}$$

where H_1 is the height of water column and H_2 the height of liquid column.

Proof of the formula:—Since the level of mercury column is the same in both the limbs, therefore pressure of liquid on mercury must be equal to pressure of water on mercury. Now pressure of a liquid column is equal to HDg dynes per sq centimetre, where H is height, D is density of the liquid and g is acceleration due to gravity.

Pressure at J, P_A = Pressure at J, P_B.

P_A = Atmospheric pressure + pressure of the water column.

$= P + H_1 . D_1 . g$. (Density of water is D_1)

Similarly $P_B = P + H_2 D_2 g$

$\therefore \quad P + H_1 D_1 g = P + H_2 D_2 g$

or $$\frac{D_2}{D_1}=\frac{H_1}{H_2}$$

Specific gravity $=\frac{D_2}{D_1}=\frac{H_1}{H_2}$

Numerical Problem:—*After some mercury is put in a U-tube, water is poured in one column and glycerine in another column such that the level of mercury is the same in both the tubes. The length of water column is 40 cms and the length of glycerine is 32 cms Find the Sp. gravity of glycerine.*

Specific gravity of glycerine $=\frac{\text{Height of water column}}{\text{Height of liquid column}}$

$$=\frac{40}{32}=\frac{5}{4}=1.25$$

QUESTIONS

1 Define density and relative density How are they interrelated? Is the relation the same in both the systems of units? (*See* §1 and §4)

2 How will you determine density of a substance? (*See* §3)

3 Describe how you will use R D bottle to determine

(*i*) R D of a liquid, (*See* §7)

(*ii*) R D of lead shots, (*See* §8)

(*iii*) R D of sugar (*See* §9)

4 How will you determine R D of a liquid by a U-tube? (*See* §10)

5 For numerical problems (*See* Chapter VI)

CHAPTER VI

ARCHIMEDES' PRINCIPLE

§1. **Archimedes :**—He was born in Cisili in 287 B C He spent his whole life in the study of Science and Mathematics Once he was asked to test the purity of the material of the crown of Emperor, Hero. The crown was made of gold. One day, when he was taking his bath Archimedes found himself lighter in water, he cried out 'Ureka Ureka' i e, found out Generally we all know that a bucket of water is lighter so long as it is inside water and becomes heavier when it is pulled out of water

Experiment :—Take a spring balance and suspend a weight from it Take its reading Now hold the spring balance in such a manner that the weight remains in water fig 23. Again take the reading It would be less than the previous one Take out the weight again and read the balance it will give some weight It shows that *whenever a body is immersed either wholly or partially in a liquid it loses weight This loss in weight is equal to the weight of the liquid displaced by the body.* This is known as **Archimedes' Principle.**

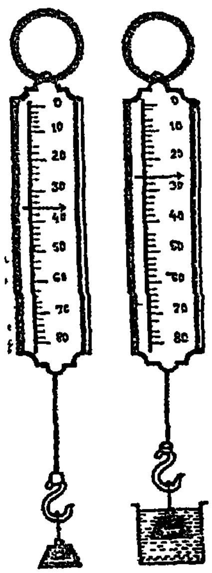


Fig 23.

Suppose the volume of a body is 100 c c and its weight is 500 grams in vacuum When it is immersed in any liquid it will displace 100 c c of liquid The loss in weight of the body will be equal to the weight of 100 c c of the liquid In case of water 100 c c of water weighs 100 grams therefore loss in weight will be 100 grams *i e*, the weight of the body in water will be 500—100=400 grams In any other liquid of density *d*, the loss in weight will be 100×*d* grams

§2. **To verify Archimedes' Principle experimentally :**—Take a hydrostatic balance as shown in fig 24 A bucket (*B*) and a solid cylinder (*C*) is suspended from one of its arms The size of the cylinder is equal to that of the bucket Place weights in the right hand pan so that the pointer reads zero Place a beaker containing water below the cylinder in such a way that the whole of it dips inside water This will disturb the equilibrium of the balance The right-hand pan will go up showing that the cylinder has lost in weight Now pore water in the

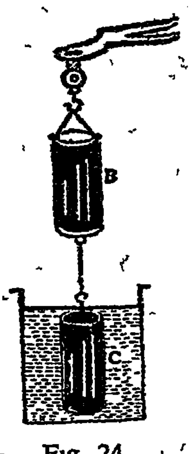


Fig 24.

bucket till it is completely full It will be found that again the pointer will read zero That is again the equilibrium is restored It proves that the loss in weight is equal to the weight of water contained in the bucket whose volume is equal to the volume of the cylinder Therefore loss in weight is equal to weight of equal volume of water In this way the Archimedes' Principle can be verified

§3. Explanation of Archimedes' Principle —Why does a body lose weight when immersed in a liquid? If we press a log of wood in water and leave it, it will come up again with a force It shows that water exerts force on the body in upward direction This force is known as upthrust Weight is the force with which earth attracts a body towards its centre It acts vertically in the downward direction When a body is dipped in a liquid two forces will act on it, one is its weight W, acting in the downward direction while the other is upthrust (T) acting in upward direction Therefore resultant force in the downward direction is (W−T) Hence there is loss in weight (*See* fig 25) In fig 26, a rectangular body *ABCD* is dipped in water *h cm* below the surface The pressure exerted by water on the two sides *AD* and *BC* is equal and opposite and therefore balance each other The pressure on the top *AB* of the body is due to atmospheric pressure plus pressure due to liquid column *h cm* This acts in downward direction while the pressure on the lower surface *DC* is more due to extra water column of length *l cm* Therefore pressure in upward direction is more than that in downward direction This difference gives rise to upthrust force T

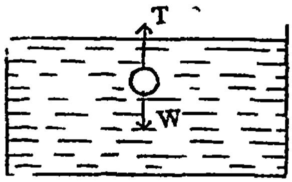


Fig. 25

Remember mass of the body remains the same

§4. Actual weight of a body:—Generally we weigh a body in air which will also apply upthrust and therefore its weight will be less than its actual weight by the weight of air displaced by the body Since the density of air is small this loss in weight is negligible and hence we neglect this loss in weight To find accurate weight we must weigh the body in vacuum

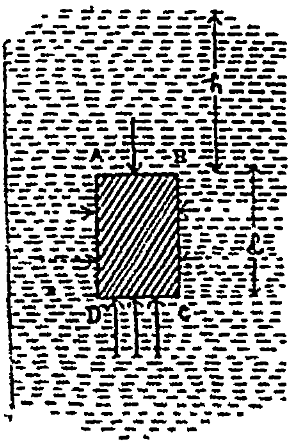


Fig 26

§5. To find the specific gravity of a body with the help of Archimedes' Principle.—We know that,

Specific gravity of a body

$$= \frac{\text{Mass of the body}}{\text{Mass of equal volume of water}}.$$

Since the value of *g* is the same for both, we can put weight instead of mass in the above formula

$$\therefore \quad \text{Sp gravity} = \frac{\text{Weight of the body}}{\text{Weight of equal volume of water}}$$

As shown above,

Loss of weight in water = Weight of water displaced

= Weight of equal volume of water.

$$\therefore \quad \text{Sp gravity} = \frac{\text{Weight of the body in air}}{\text{Loss of weight in water}}.$$

§6. To find the sp. gravity of a solid by Archimedes' Principle —Find out the weight of a body in air using hydrostatic balance Let this be W_1 Suspend the body in a beaker containing water and again find out its weight Let this be W_2 Then loss of weight in water $= W_1 - W_2$ gram

$$\therefore \quad \text{Sp gravity} = \frac{\text{Weight in air}}{\text{Loss of weight in water}}$$

$$= \frac{W_1}{W_1 - W_2}$$

If the body is soluble in water then find out its R D. with respect to some other liquid as explained above and multiply it with the R D of the liquid

Numerical Problem —*A body weighs 60 03 grams in air and 42 grams in water Find its sp gravity.*

$$\text{Sp gravity of a body} = \frac{\text{Weight in air}}{\text{Loss in wt in water}} = \frac{W_1}{W_1 - W_2}$$

Loss in wt in water $= W_1 - W_2 = 60\ 03 - 42$

$$\therefore \quad \textit{Sp. gravity of the body} = \frac{60\ 03}{60\ 03 - 42}$$

Calculations.—

Log 60 03 = 1 7803	$= \frac{60\ 03}{18\ 03}$
Log 18 03 = 1 2625	= 3 3
5178	
Anti log 5178 = 3 295	

§7. To find the specific gravity of a liquid by Archimedes' Principle —Take a solid which is insoluble in water as well as in the given liquid Find out its weight in air Also find out its weight when it is suspended in the liquid and then in water Let these weights be W_1, W_2 and W_3 respectively Then, we have,

Loss of weight in liquid $= W_1 - W_2$

Loss of weight in water $= W_1 - W_3$

Since we have found the loss of weight of the same solid in both liquid and water, the volume of liquid and water displaced is

the same Hence W_1-W_2 and W_1-W_3 are the weights of equal volume of liquid and water

Therefore sp gravity of liquid

$$=\frac{\text{Weight of liquid}}{\text{Weight of equal volume of water}}$$
$$=\frac{W_1-W_2}{W_1-W_3}$$

Numerical Problem:—*A body weighing 50 grams in air weighs 45 grams in water and 45 6 grams in alcohol Find the sp gravity of alcohol*

Weight of the body in air $(W_1)=50$ grams
Weight of the body in water $(W_3)=45$ grams
Weight of the body in alcohol $(W_2)=45\ 6$ grams
Loss of weight in water $W_1-W_3=50-45$
$= 5$ grams
Loss of weight in alcohol $W_1-W_2=50-45\ 6$
$=4\ 4$ grams

$$\therefore \text{ Sp gravity}=\frac{\text{Loss of wt in liquid}}{\text{Loss of wt in water}}$$
$$=\frac{4\ 4}{5}=\ 88$$

§8. **To find the sp. gravity of a light body:**—Suppose we want to find out the sp gravity of cork or wax Find out the weight of the body in air (W_1) Take a sinker and suspend it in water keeping cork in air and find out its weight (W_2) Tie both cork and sinker inside water and find out its weight (W_3) Then, we have,

Weight of the cork in air $=W_1$ gram
Loss of weight of the cork in water $=W_2-W_3$ gram

$$\text{Sp gravity of cork} =\frac{W_1}{W_2-W_3}$$

Numerical Problem:—*The weight of a piece wax is 18 03 grams in air. A piece of metal weighs 17 03 grams in water When the piece of the metal is tied to the wax and weighed in water, its weight is 15 23 grams Find the sp gravity of wax*

Weight of wax in air $=18\ 03$ grams
Weight of sinker in water $=17\ 03$ grams
Weight of wax in air and sinker in water $=18\ 03+17\ 03$
$=35\ 06$ grams
Weight of sinker and wax in water $=15\ 23$
Loss in wt of wax in water $=35\ 06-15\ 23$
$=19\ 83$ grams

$$\therefore \text{ Sp gravity of wax} =\frac{18\ 03}{19\ 83}=\ 9$$

§9. **Principles of floating:**—We have seen that when a body is immersed in a liquid two forces act on it. One is the weight of the body which acts vertically downward and another is the upthrust which acts in the upward direction This upthrust is equal to the weight of the displaced liquid As more and more of the body sinks inside liquid more and more liquid is displaced which increases the upthrust till the whole of the body is immersed

First Law:—*If in any case upthrust is equal to the weight of the whole body the resultant force on the body will be zero and it will begin to float* (i) If the density of the liquid is less than the density of the solid, upthrust will be less than weight of the body and body will sink.

(ii) If the density of the liquid is equal to the density of the solid, the body will float just submerged.

(iii) If the density of the liquid is more than the density of the solid, only part of the body will dip inside liquid In this case the weight of displaced liquid is equal to the weight of the whole body Let V be the volume of whole body and D its density Let v be the volume of the body inside liquid and d be the density of the liquid Then according to the above principle,

Weight of the body=Weight of displaced liquid

$$V D.=v d$$

This is the first law of floating bodies

Second Law:—*The weight of the body and upthrust should act along the same line* Weight of the body acts vertically through a point known as centre of gravity, upthrust acts vertically through a point which is the centre of gravity of the displaced liquid This point is known as centre of buoyancy According to this law the centre of buoyancy and centre of gravity should be along the same vertical line This is known as centric line

Third Law.—*For stable equilibrium.*—When a floating *body is slightly* disturbed, its new buoyancy centre shifts towards the leaning side The new centre line cuts the old one at a point known as meta centre *If the meta centre lies above the centre of gravity, the body will come back to its original position while of the meta centre lies below centre of gravity the body will topple down i e the equilibrium will be unstable*

Numerical Problems:—1 *The specific gravity of a body is 11 4 Its weight in air is 57 2 Find its weight in water. What will be its weight in alcohol if sp gr =0 8)*

Suppose its weight in water is W_2 gram

$$\therefore \qquad \text{Specific gravity}=\frac{W_1}{W_1-W_2}.$$

Substituting the values of the various quantities we get,

$$11\ 4=\frac{57\ 2}{57\ 2-W_2}$$

$$\text{or } (57{\cdot}2 - W_2)\,11{\cdot}4 = 57{\cdot}2$$

$$57{\cdot}2 \times 11{\cdot}4 - W_2 \times 11{\cdot}4 = 57{\cdot}2$$

$$11{\cdot}4\, W_2 = 57{\cdot}2 \times 11{\cdot}4 - 57{\cdot}2$$

$$= 57{\cdot}2 \times (11{\cdot}4 - 1)$$

$$= 57{\cdot}2 \times 10{\cdot}4$$

$$W_2 = \frac{57{\cdot}2 \times 10{\cdot}4}{11{\cdot}4} = 52{\cdot}18$$

Calculations.—

log of numerator	log of denominator
1·7574	
1·0170	1·0569
2·7744	
2·7744	
−1·0569	
1·7175	

Ant log 1·7175 = 52·18

Loss in weight in water	= 57·2 − 52·18
	= 5·02 grams
Weight of displaced water	= 5·02 grams
Volume of displaced water	= 5·02 c c
Volume of displaced alcohol	= 5·02 c c
Weight of displaced alcohol	= 5·02 × ·8 grams
	= 4·016 grams
Loss in weight in alcohol	= 4·016 grams
	= 4·02 grams
∴ Weight in alcohol	= 57·2 − 4·02
	= 53·18 grams

2. *A piece of a cork tied to a piece of metal just floats in alcohol Find the ratio of their masses (Given sp. gravity of alcohol = ·8, cork = ·25 and metal = 8).*

Suppose the weight of cork	$= M_1$ gram
and the weight of metal	$= M_2$ gram
∴ Volume of cork (V_1)	$= \dfrac{M_1}{\cdot 25}$ c c
Volume of metal V_2	$= \dfrac{M_2}{8}$ c c
∴ Total weight of combination	$= M_1 + M_2$
and total volume	$= \left(\dfrac{M_1}{\cdot 25} + \dfrac{M_2}{8}\right)$ c c.
Volume of alcohol displaced	$= \left(\dfrac{M_1}{\cdot 25} + \dfrac{M_2}{8}\right)$ c c.

Weight of alcohol displaced $=\left(\frac{M_1}{25}+\frac{M_2}{8}\right)\times{\cdot}8$ gram.

This must be equal to total weight

$$\therefore \quad M_1+M_2=\left(\frac{M_1}{{\cdot}25}+\frac{M_2}{8}\right)\times{\cdot}8$$

$$=\frac{80M_1}{25}+\frac{8\,M_2}{8}$$

$$=3\,2\,M_1+{\cdot}1\,M_2$$

or $$3{\cdot}2\,M_1-M_1=M_2-{\cdot}1\,M_2$$

or $$2\,2\,M_1=9\,M_2$$

$$\frac{M_2}{M_1}=\frac{2\,2}{9}=\frac{22}{9}$$

3 *The specific gravity of ice is 918 and that of sea water is 1 03 An ice berg floats on water with 224 c c. outside water Find the total volume of ice berg*

We know that for a floating body,

$$V\,D=v\,d$$

Here V=Volume of whole body $=V$ c c

D=Density of ice $=918$

v=Volume of water displaced $=V-224$

d=Density of water $=1\,03$

Substituting these values in above relation,

We get,

$$V\times 918=(V-224)\,1\,03$$

$$V\times 918=1\,03\,V-224\times 1{\cdot}03$$

$$\text{or}\quad 1\,03\,V-918\,V=224\times 1\,03$$

$$V\,(1\,03-918)=224\times 1\,03$$

Calculations —

Log Nr	Log Dr
2 3502	$\bar{1}$ 0492
0128	
2 3630	2 3630
	$-\bar{1}$ 0492
	3 3138
Ant log.	3 3138=2060

or $V=2060$ c c

$$V=\frac{224\times 1\,03}{112}$$

$$=2060\text{ c c}$$

§10. **Nicholson's Hydrometer :—**This principle of floating bodies has been utilised for measuring the specific gravity of a solid or liquid by means of Nicholson's Hydrometer

It consists of a hollow cylindrical barrel (A) of brass carrying a narrow stem at one end and a conical bucket (B) at the other The bottom of the bucket is heavily loaded with lead shots or

mercury The upper end of the stem carries a circular pan D When placed in a jar containing water or liquid the hydrometer will float vertically A fixed mark M is put on the stem (Fig 27)

Fig 27

§11. To find the sp. gravity of a solid:—*Principle and working* (Fig 28) —1 Float the hydrometer in water and place some weights on the upper pan till the hydrometer sinks up to M Let these weights be W_1 In this position the weight of the displaced water (v d) is equal to weight of hydrometer $Wh+W_1$, where v is the volume of hydrometer inside water

2 Now remove the weights W_1 and place the body on the upper pan and again place weights W_2 on the pan to sink it up to the same mark M In this case the weight of the hydrometer $+W_2+$weight of the body$=v$ d

$$\therefore \quad W_h+W_2+B=v\,d=W_h+W_1$$

$$\therefore \quad B=W_1-W_2$$

This gives the weight of the body in air (B)

3 Place the body in lower pan and again place weights W_3 on the upper pan to sink it up to M

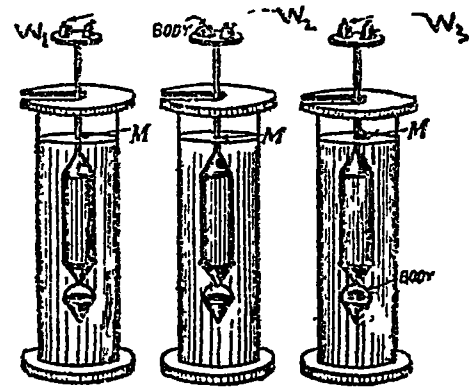


Fig 28

This time W_h+W_3+weight of body in water$=W_h+W_1$

$\therefore$ Weight of body in water$=W_1-W_3$

$$\therefore \quad \text{Loss in weight in water}=(W_1-W_2)-(W_1-W_3)$$
$$=W_3-W_2$$

$$\therefore \quad \text{Sp gravity}=\frac{\text{Weight of the body in air}}{\text{Loss in weight in water}}=\frac{W_1-W_2}{W_3-W_2}$$

Having known W_1, W_2 and W_3 sp gravity can be calculated

Numerical problem:—*In an experiment, we have the following observations, Wt required to sink the hydrometer up to the fixed mark* (W_1)*=16 84 grams*

Wt required to sink when a piece of glass is placed on the upper pan (W_2)*=4 96 grams*

Wt required to sink when the glass piece is placed in the lower pan $=9{\cdot}71$ *grams*

Find out the sp gravity of glass

Weight of glass piece in air	$=16\ 84-4\ 96$
	$=11\ 88$ grams
Weight of glass piece in water	$=16\ 84-9\ 71$
	$=7\ 13$
Loss in weight	$=11\ 88-7\ 13$
	$=4\ 75$
∴ Sp gravity	$=\dfrac{11\ 88}{4\ 75}=2\ 5$

Calculations :—

1 0748 Anti log=2·501
6767
3981

§12. Specific gravity of a liquid :—

$$\text{Sp gr of liquid} = \frac{\text{Weight of liquid}}{\text{Weight of equal vol of water}}$$
$$= \frac{\text{Weight of equal vol of liquid}}{\text{Weight of equal vol of water}}$$
$$= \frac{\text{Loss in weight of a body in liquid}}{\text{Loss in weight of the same body in water}}$$

Find out this loss in weight in water ($W_3 - W_2$) of a body as explained above Similarly find out the loss in weight of the same body in liquid Let this be $W'_3 - W'_2$

$$\therefore \quad \text{Specific gravity of liquid} = \frac{W'_3 - W'_2}{W_3 - W_2}$$

Numerical problem :—*A Nicholson's hydrometer is floated in a liquid and a piece of solid is placed on its upper pan It requires 6 5 grams to sink it up to the fixed mark. When the piece of metal is placed in the lower pan it requires 10 7 grams when the experiment is repeated with water the corresponding wts are 8 5 and 14 8 grams. Find the sp. gravity of liquid.*

$$\text{Sp gravity of liquid} = \frac{\text{Wt of liquid displaced}}{\text{Wt of water displaced}}$$
$$= \frac{\text{Loss in wt in liquid}}{\text{Loss in wt water}}$$
$$= \frac{10\ 7-6\ 5}{14\ 8-8\ 5}$$
$$= \frac{4\ 2}{6\ 3}$$
$$= {\cdot}66$$

§13. **To find the sp. gravity of a liquid by weighing the Hydrometer** :—First find out the weight of the hydrometer with the help of a physical balance. Let this be W_h. Float the hydrometer in a jar containing water and place weights (W_1) on it till it sinks up to the mark. Now float the hydrometer in liquid and find weight (W_2) required to sink it up to the same mark. Then according to Archimedes' principle,

$$W_h + W_1 = \text{Weight of displaced water}$$

and $W_h + W_2 =$ weight of displaced liquid since the same volume of hydrometer is inside in both the cases, the volume of water displaced is equal to volume of liquid displaced.

$$\therefore \text{ Sp. gravity of liquid} = \frac{\text{Weight of liquid}}{\text{Weight of equal vol. of water}}$$

$$= \frac{W_h + W_2}{W_h + W_1}$$

Numerical problem —*The weight required to sink a Nicholson's hydrometer up to fixed mark in water is 3.32 grams and in liquid is 9.41 grams. If the sp. gravity of liquid is 1.02. Find the weight of hydrometer.*

$$\text{Sp. gravity of liquid} = \frac{W + W_1}{W + W_2}$$

Substituting the value we get

$$1.02 = \frac{W + 9.41}{W + 3.32}$$

or $$1.02\,(W + 3.32) = (W + 9.41)$$

$$W(1.02 - 1) = 9.41 - 3.32 \times 1.02$$

or $$.02\,W = 9.41 - 3.39$$

$$= 6.02$$

$$W = \frac{6.02}{.02} = 301 \text{ grams}$$

Note.—If the body is lighter than water, then also perform the experiment in the same manner. But when the cork is placed in the lower pan it has to be tied to the bucket.

Numerical problem :—*The sp. gravity of gold is 19.3 and that of silver is 10.4. What is the proportion of gold and silver in an alloy of sp. gravity 17.6?*

Suppose the volume of gold is V_1 c.c.
and the volume of silver is V_2 c.c.

Mass of gold $= V_1 \times 19.3$

Mass of silver $= V_2 \times 10.4$

$$\text{Sp. gravity of alloy} = \frac{M_1 + M_2}{V_1 + V_2}$$

$$\therefore\ 17.6 = \frac{19.3\,V_1 + 10.4\,V_2}{V_1 + V_2}$$

or $$17{\cdot}6\,V_1+17{\cdot}6\,V_2=19{\cdot}3\,V_1+10{\cdot}4\,V_2$$

or $$(19{\cdot}3-17{\cdot}6)\,V_1=(17{\cdot}6-10{\cdot}4)\,V_2$$

$\therefore$ $$\frac{V_1}{V_2}=\frac{7{\cdot}2}{1{\cdot}7}=4{\cdot}23.$$

§14. A Few Applications of Archimedes' Principle:—

(a) To find out the radius of a wire.—Take a long wire and find out its weight in air (W_1) and in water (W_2). Therefore loss of weight in water $=(W_1-W_2)$ gram Volume of water displaced is equal to (W_1-W_2) c.c Therefore volume of wire $=(W_1-W_2)$ c.c since the wire is in cylindrical form its volume is $\pi r^2 l$ c.c

$$\pi r^2 l=W_1-W_2$$

or $$r^2=\frac{W_1-W_2}{\pi l}\ \text{cm}$$

or $$r=\sqrt{W_1-W_2/\pi l}\ \text{cm}$$

In this way we can find its radius

(b) To find out the diameter of a capillary tube:—Find out the weight of the capillary tube (W_1). Fill it with a column of mercury and

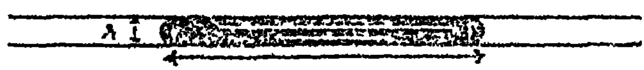


Fig 29

again find out its weight W_2 The weight of mercury is equal to (W_2-W_1) gram Measure the length of mercury column (l) cm. The radius of mercury column is r where r is the radius of the tube. Again volume of mercury is also equal to $\frac{W_2-W_1}{13{\cdot}6}$ where 13·6 gms. c.c is the density of mercury

$$\pi r^2 l=\frac{W_2-W_1}{13{\cdot}6}$$

or $$r=\sqrt{\frac{W_2-W_1}{13{\cdot}6\times\pi l}}\ \text{cm}$$

Numerical problem:—*The weight of a capillary tube is 15·05 grams when it is filled with mercury column of length 10·6 cm its weight is 19·13 grams If the density of mercury is 13·6, find the radius of the tube.*

Mass of mercury contained in the tube

$$=19{\cdot}13-15{\cdot}05$$
$$=4{\cdot}08\ \text{grams}$$

Volume of mercury $$=\frac{4{\cdot}08}{13{\cdot}6}\left(v=\frac{m}{d}\right)$$
$$=3\ \text{c.c}$$

If the radius of the tube is r, the internal volume of 10·6 cm., of the tube=volume of mercury contained in it

$$=3\ \text{c.c}$$

We get, $\pi r^2 l = 3$

or $$r^2=\frac{3}{\pi l}=\frac{3}{3.14\times 10.6}$$

$$r=\sqrt{\frac{3}{3.14\times 10.6}}=.092\text{ cm}$$

Calculations:—

Log of numerator	Log of Denominator
$\bar{1}.4771$	.4969
1.5522	1.0253
$\bar{3}.2249$	1.5222

$$\tfrac{1}{2}(\bar{3}.9249)=\tfrac{1}{2}(-4+1.9249)$$
$$=-2+.9624$$
$$=\bar{2}.9624$$

Anti log $\bar{2}.9624$ $= .09168$

$r = .092$ cm

(*c*) **Variable immersion type hydrometer (Lactometer).**—Hydrometers are of two types (*i*) constant immersion type like Nicholson's hydrometer In this case the hydrometer always sinks up to the same mark and we vary the weights placed on it (*ii*) Variable immersion type In this case the weight of the hydrometer is constant but it dips to different heights in different liquids It will dip more in lighter liquid than in heavier one The stem (*S*) of the hydrometer is graduated directly in density It is shown in fig 30 Mercury is filled in lower bulb (*B*) to make it float vertically lactometer is also of this type

Fig 30

(*d*) **Ice berg.**—*See* fig 30(*a*) This is known as ice berg It floats on water in oceans Generally we find these ice bergs in cold currents coming from North and South regions We know that density of ice is .907 gm/c c while that of sea water is 1.026 gm c c therefore, approximately 1/9 part of the ice berg remains outside water and 8/9 part inside water. Since only a small part is visible outside therefore in bad weather sometimes a ship is unable to see it and crashes against it resulting into accident

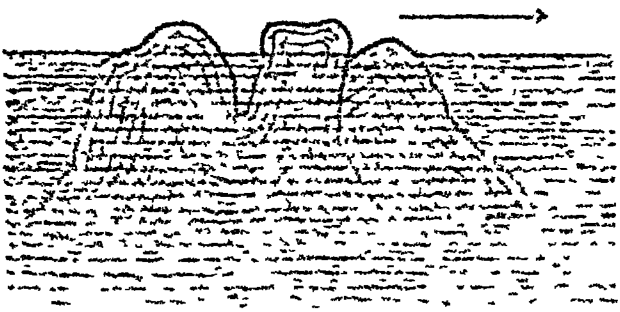


Fig 30(a)

(*e*) **Floating of a ship:**—You already know that the sp gravity of iron is 7 8 How it is possible that a ship made of such a heavy metal floats on water? You must have seen a vessel floating on water. This is possible because the ship or the vessel is hollow from inside The extent of their surface is large, when a small height of it goes inside water the amount of water displaced is so large that the weight of displaced water is equal to the weight of whole ship If we start filling the vessel with water it will sink down.

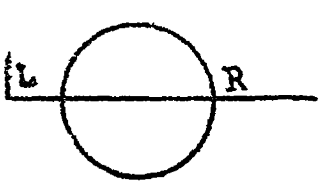


Fig 31

As shown in fig 31 a circle is drawn on the vertical side of the vessel and a line is drawn across this circle. This line is known as plimsoll line after the scientist Plimsoll The ship is loaded to such an extent that it does not sink in below this line *L and R* is written on this line The meaning of this is that according to the shape and size of the ship, this line limit has been decided by Lloyds Registrar of shipping

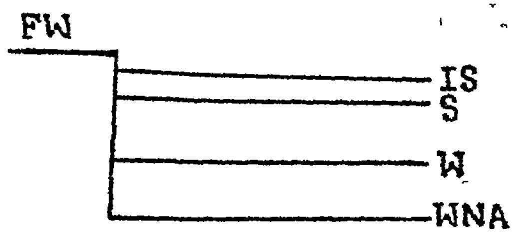


Fig 31(*a*).

The ship has to sail in river water in cold water in northern seas where density of water is different and accordingly different lines are drawn keeping in view all these factors fig 31(*a*)

(*f*) **Balloon:**—(Fig. 32). Just as lighter bodies float in water in the same way if bodies are lighter than air, they will *float* in air The laws of floating bodies will also apply to their cases hydrogen and helium are lighter than air If balloons are filled with hydrogen or helium, they will float in air The density of air is not uniform throughout As we go higher and higher it goes on decreasing The balloon being lighter will rise up and will go on ascending till the weight of whole balloon is equal to the weight of air displaced. Under this condition it will float at that height

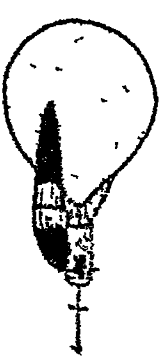


Fig 32

If on reaching this height sand bags stored in it are thrown out it will become lighter and will ascend farther. If on the other hand some of the hydrogen or helium is forced out its volume will decrease and so it will become heavier and sink down

Balloons are filled with different scientific instruments for automatic recording of temperature, pressure, or presence of charged particles. They also carry films for photographing various phenomena After some time by automatic device the balloon bursts and the section containing all these instruments comes down on earth with the help of a parachute like that of a man jumping from aeroplane.

(g) Submarine :—This is a special kind of ship which can float on the surface of water as well as at any desired depth inside water

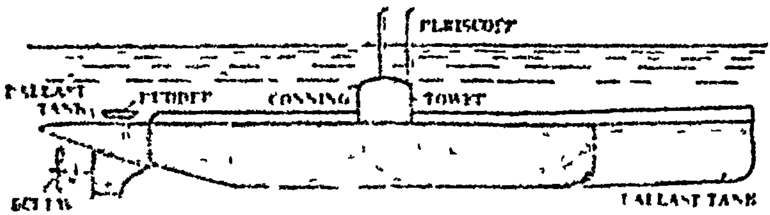


Fig 33

(Fig 33) It carries different storage tanks which when filled with water make it heavier and it will sink down when it wants to come out water will be forced out In this way it can swim at any depth

In times of war it is used for crashing a ship In peace-time it is used for scientific discoveries They carry an instrument known as periscope with which a person sitting in it can see any object on the surface of water when the submarine is inside water

Recently we have been successful in sending atomic submarines under frozen seas of north They have also been able to complete the global tour under water

QUESTIONS

1 State Archimedes' principle How is it verified ? How is it used to determine R D of a substance ? (*See* §§ 1. 2 and 5)

2 State the laws of floating body How are these laws satisfied in the construction and working of Nicholson's hydrometer (*See* §9 and §10)

3. How is Nicholson's hydrometer used to determine

(*i*) R D of a solid

(*ii*) R D of a liquid ? (*See* §11 and §12)

4 How will you determine radius of a capillary tube ? (*See* 14*b*)

5 Distinguish between constant and variable immersion types of hydrometer (*See* §14*c*)

Numerical questions —

1 A balloon has a volume of 1000 cubic metres How much weight it will lift when filled with (*a*) Hydrogen (*b*) with Helium Take density of Hydrogen 09 gram per litre. That of Helium twice that of Hydrogen and that of air 14 time that of Hydrogen [*Ans* 1170 K gm 1080 K gm]

2 A Nicholson's hydrometer sinks up to a fixed mark in liquid of sp gr. 0 6 but it takes 120 grams to sink up to the same mark in water What is the weight of Hydrometer ? [*Ans* 180 gms]

3 A piece of wax weighs 18 03 grams in air A piece of metal is found to weigh 17 03 grams in water It is tied to wax and both together weighs 15 23 grams in water What is sp gr of wax ? [*Ans* 91]

4 The densities of three liquids are in the ratio of 1 2 3 what will be the relative density of the mixture by combining (*a*) equal volumes (*b*) equal weights of the liquid

[*Ans* (*a*) $2s_1$ (*b*) $\frac{18}{11}s_1$ where s_1 is the sp gr of lighter liquid]

5. The density of sea water is 1·025 grams per c.c. and the density of ice is ·917 gram per c.c. Find what portion of ice berg is visible above the water surface ? When it is in sea water and when it is in fresh water ? [*Ans.* ·105 ·083]

6. Show that a hollow sphere of radius R made of metal of sp. gr. S will float on water if the thickness of its wall is $\frac{R}{3S}$

7. The crown of Hero weighed 20 lbs. Archimedes found that immersed in water it lost 1·25 lb. The crown was made of gold and silver. Find the weight of these metals (Sp. gr. of gold 19·3 and Sp. gr. of silver 10·5,
[*Ans.* 15·078 and 4·922 lbs.]

8. A ship with a cargo sinks by 14 feet on entering a river from sea. On unloading it rises by 10 feet and when it goes to sea it further rises by 12 feet. Find the sp. gr. of sea water. Take the sides of the ship as vertical. [*Ans.* 1·25]

9. 52 grams of an alloy of two metals of sp. gravity 8 and 12 respectively is found to weigh 46 grams in water. Find the mass of metal in the alloy.
[*Ans.* 40 gms., 12 gms.]

10. A block of ice weighing 1000 grams is thrown in the sea. Find the volume of ice submerged. The density of ice is ·917 and of sea water is 1·03
[*Ans.* 970873·79 c.c.]

11. A piece of lead weighing 17 grams and a piece of sulphur have equal apparent weights when suspended from the arms of a balance and immersed in water. When water is replaced by alcohol sp. gr. ·9, 1·4 gram must be added to the pan from which the lead is suspended to restore equilibrium. Determine the weight of sulphur. Density of lead is 11·333 grams per c.c.
[*Ans.* 31 gms.]

12. A bent tube contains paraffin oil on one side and water on other side, the water rising a little way up in the other tube. If the reading of the top of paraffin column is 17·4 cms. and the bottom of paraffin is 5·4 cms. Top of water column 15·6 cms. Find the sp. gr. of paraffin. [*Ans.* ·85]

CHAPTER VII

COMPOSITION AND RESOLUTION OF FORCES

§1. **Scalar and Vector:**—Quantities which we come across are in general of two kinds—Scalars and Vectors

Those quantities which have only magnitude and no direction are known as scalars Like mass volume area time etc Their meaning is complete as soon as their magnitude is stated For example when we say mass of a book is 1000 grams, volume of a body is 1000 c c the meaning is clear

Those quantities which have magnitude as well as direction are known as vectors like velocity, force, acceleration, etc When we say the velocity of a body is 15 cms per sec, it has no meaning unless we state the direction in which the velocity is acting say in east or west Similarly we have to state that a force of 5 lbs acts on a body towards north Vector can be represented by a straight line The length of the line is proportional to the magnitude of the vector and the line is drawn in the direction of vector and the sense is shown by an arrow

§2 **Force.**—It is that agent which produces acceleration in any mass This acceleration is produced in the direction of the force

Force is a vector quantity and therefore can be represented by a straight line passing through the point at which the force acts The length of the line is taken to be proportional to the force

§3 ***Resultant of two or more than two forces:***—When two forces act on a point in the same direction the resultant force will be equal

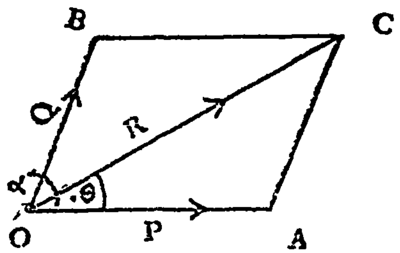


Fig 34

to the sum of the two forces and will act in the same direction while if they act in opposite direction the resultant force will be equal to the difference of the two forces and will act in the direction of greater force If the two forces are equal the resultant will be zero and the point will be at rest When the two forces are inclined at a certain angle the resultant can be found by applying the law of parallelogram of forces

Law of parallelogram of forces :—*If two forces acting at a point be represented in magnitude and direction by the adjacent sides of a parallelogram, the resultant will be represented in magnitude and direction by the diagonal of the parallelogram passing through that point.*

If two forces acting at the point O are represented in magnitude and direction by *OA* and *OB* their resultant will be represented by *OC* the diagonal in magnitude and direction (fig 34)

Proof of the law .—In order to prove this law we take an apparatus as shown in the figure 35(*A*). A wooden board is fixed vertically

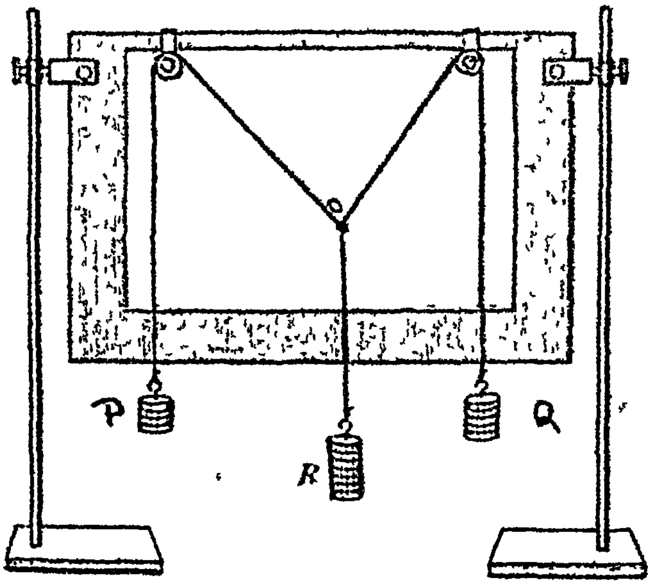


Fig. 35 (A).

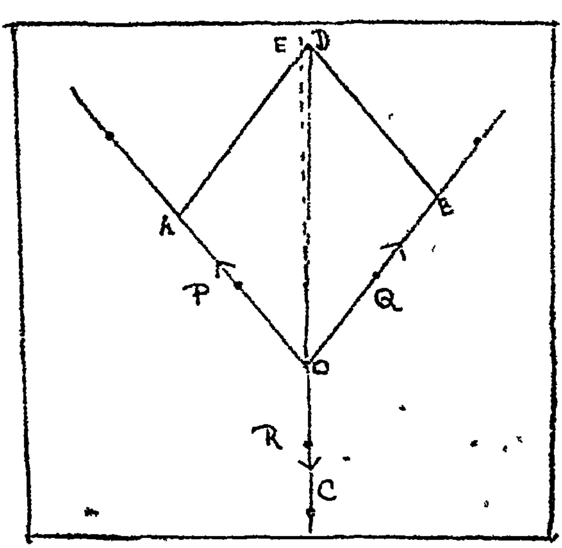


Fig 35 (B).

on two stands There are two frictionless pulleys at the top of the board Fix a white sheet of paper on the board Suspend two weights P and Q by means of a long thread and place the thread on the pulleys Suspend another weight R from a second thread and tie it to the previous one at any point between the pulleys See that the point O where all the threads meet remains in the middle of the board and none of the weights touches the board The values of P, Q and R are so adjusted that O remains in equilibrium *i e*, at rest Observe the shadows of the threads on the paper and put pencil dots on each of the shadows Remove the paper and draw lines joining these points All these lines will meet at O Cut off OA and OB proportional to P and Q Complete the parallelogram $OADB$ Join OD Now the length of OD will come out to be proportional to R and OD will be in the same straight line as R That is R is equal and opposite to OD Since point O is in equilibrium therefore the resultant of P and Q must be equal and opposite to R That is the resultant is represented by the diagonal

§4. **Geometrical calculation of the diagonal:**—Let P and Q be the two forces acting at O inclined at an angle α Let them be represented by OA & OB Complete the parallelogram $OADB$ Then OD will represent R

To calculate R in terms of P Q and α

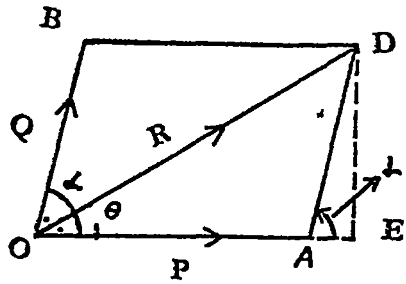


Fig 36

Draw $DE \perp$ to OA

In the triangle ODE

$$OD^2=OE^2+DE^2$$
$$=(OA+AE)^2+DE^2$$
$$=OA^2+AE^2+2OA\ AE+DE^2$$
$$=OA^2+(AE^2+DE^2)+2OA\ AE \qquad (i)$$

In $\triangle ADE$ $AD^2=AE^2+DE^2 \qquad (ii)$

Substituting this value of AE^2+DE^2 in (i) we get,

$$OD^2=OA^2+AD^2+2OA\ AE$$

Angle $BOA=\alpha$

$\therefore$ $DAE=\alpha$

Now $\dfrac{AE}{AD}=\cos\alpha$ and $\dfrac{DE}{AD}=\sin\alpha$

$\therefore$ $AE=AD\cos\alpha$ and $DE=AD\sin\alpha$

$\therefore$ $OD^2=OA^2+AD^2+2\,OA.AD\cos\alpha$

Substituting the values of OD, OA & AD we get

$$R^2=P^2+Q^2+2\,PQ\cos\alpha$$

Again $$\tan\theta = \frac{DE}{OE} = \frac{DE}{OA+AE}$$
$$= \frac{AD\sin\alpha}{OA+AD\cos\alpha}$$
$$= \frac{Q\sin\alpha}{P+Q\cos\alpha} \qquad (iv)$$

equations (*iii*) and (*iv*) determine the resultant in direction and magnitude

§5 **Triangle of forces**—If three forces acting at a point are in equilibrium they can be represented in magnitude and direction by

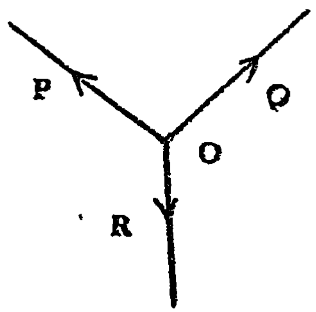


Fig 37.

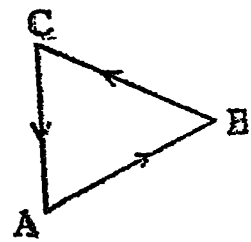


Fig 38

the sides of a triangle taken in order Consider a point O where three forces P Q and R are acting and O is at rest Draw a line AB equal and parallel to Q Draw BC equal and parallel to P Join AC Now AC will represent R in magnitude and direction This directly follows from the law of parallelogram of forces (*See* figs 37 and 38)

§6 **Moment of a force:**—If a body is fixed about a certain axis and a force is applied on it, it will rotate about the axis The rotating effect of the force upon the body depends upon the magnitude of the force as well as its perpendicular distance from the axis This is known as moment of the force and is *measured by the product of the force and the perpendicular distance of the force from the axis* Consider a force F acting on a body fixed at O Its moment will be $F\,a$, where a is the distance of F from O This will rotate the body in anti-clockwise direction and is taken as positive

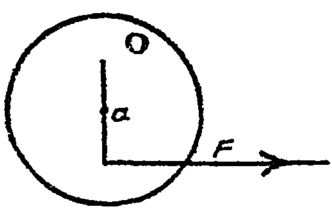


Fig 39

Law of moments:—If any number of forces acting on a body keep it in equilibrium, the algebraic sum of the moments is zero That is the sum of moments in clockwise direction is equal to the sum of moments in anticlockwise direction

57 **Verification:—**(*See Practical Physics by authors*) Take a metre scale and suspend it by a thread in such a way that it remains

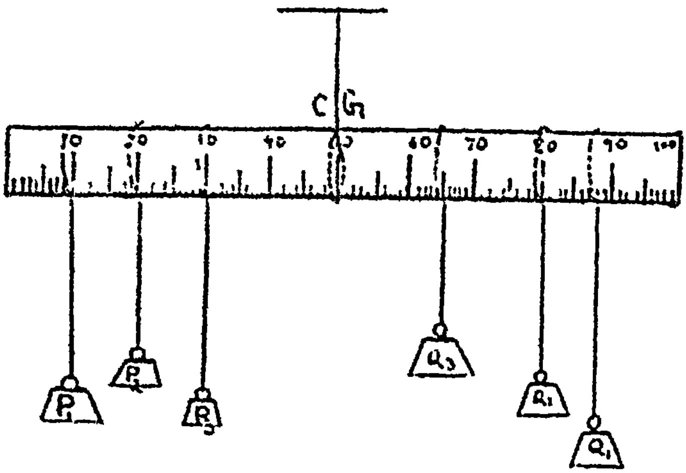


Fig 40

horizontal In this case the point of suspension coincides with centre of gravity of the scale Suspend two or three weights on one side and two or three weights on another Adjust the weights or their

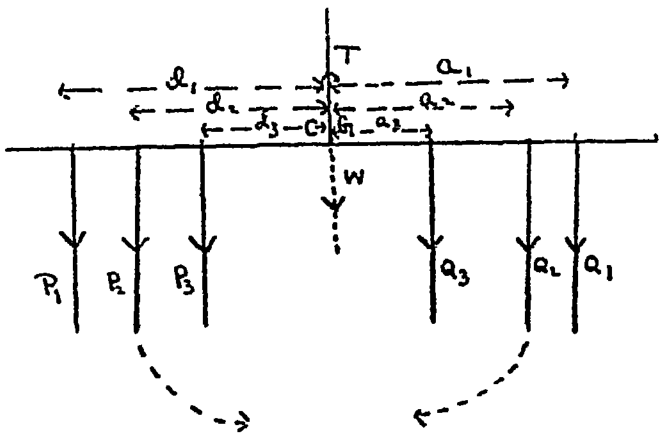


Fig 41

distances in such a way that the scale is again horizontal Note the weights and their distances from the point of suspension Multiply the weight by its distance This will give the moments of the forces on left and forces on right separately Add the moments on left and moments on right It will be found that the two sums are equal Thus the law is verified

Numerical Problems:—1 *A metre scale is suspended from a point at 30 cms on one side of centre of gravity which is at 50 cms A weight of 50 grams when suspended from a point at 10 cms keep the scale horizontal Find the weight of scale*

Let W be the weight of the scale. this will act at 50 cm point. Taking moments about the point of suspension (See fig. 42).

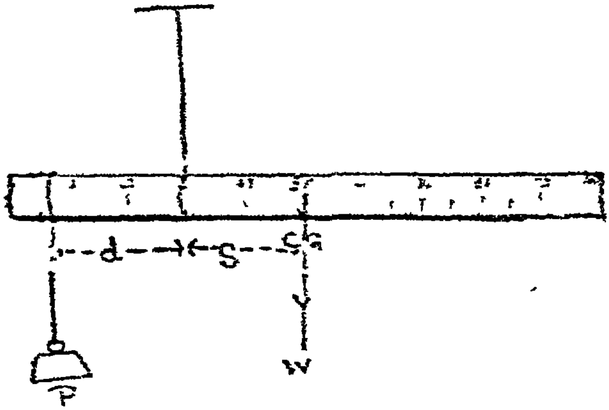


Fig. 42.

$$P \times d = W \times S$$
$$50 \times 20 = W \times 20$$
$$\therefore \quad W = 50 \text{ grams.}$$

2. *A metre scale is suspended from its centre of gravity. A piece of metal is suspended from one end and a weight is suspended at a dis-*

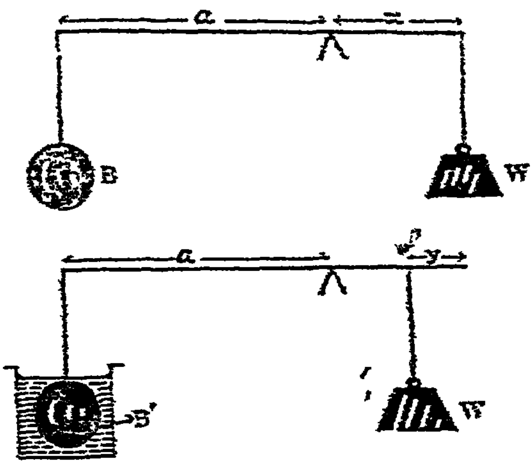


Fig. 43.

tance of 40 cms. from C.G. to make it in equilibrium. If the piece of metal is dipped in water the weight has to be shifted by 5 cms. to restore equilibrium. Find the sp. gravity of metal.

Taking moments in first case

$$B.a = W.x.$$

$$\therefore \quad B = \frac{Wx}{a}$$

in second case, let the weight of body in water be B'

$$\therefore \quad B'.a = W(x-y)$$

$$\therefore \quad B' = \frac{W(x-y)}{a} = \frac{Wx}{a} - \frac{Wy}{a}$$

$\therefore$ Loss in weight in water $= B - B'$

$$= \frac{Wy}{a}$$

$$\text{Now, sp gravity} = \frac{\text{Weight in air}}{\text{Loss in wt in water}}$$

$$= \frac{B}{B-B'}$$

$$= \frac{\frac{Wx}{a}}{\frac{Wy}{a}} = \frac{x}{y}$$

Substituting the values, we get,

$$\text{Sp gravity} = \frac{40}{5} = 8$$

3. *The arms of a balance are equal but its pans are unequal The weight of a body in one pan is W_1 gram and in another pan W_2 gram Show that the difference in weights of the pan is $\frac{W_1 - W_2}{2}$ gram*

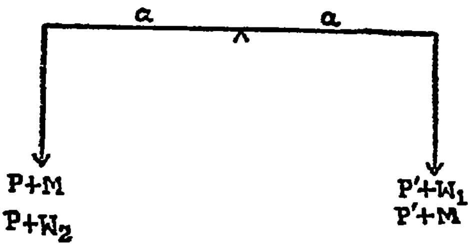


Fig 44

The weight of the body is shown in the figure in both the cases Taking moments in each case, we get,

$$(P+M)a = (P'+W_1)a \qquad (i)$$

$$(P+W_2)a = (P'+M)a \qquad \ldots \qquad (ii)$$

P and P' are the weights of the pans and a is the length of the arm

$$P+M = P'+W_1 \qquad (iii)$$

or $$P+W_2 = P'+M \qquad (iv)$$

or $$M = (P'-P)+W_1 \text{ from } (iii)$$

and $$M = (P-P')+W_2 \text{ from } (iv)$$

$$\therefore \quad (P'-P)+W_1 = P-P'+W_2$$

or $$2(P'-P) = W_2 - W_1$$

$$\therefore \qquad P'-P=\frac{W_2-W_1}{2}$$

4 *A body requires 20.61 grams to hold it in equilibrium when placed in one pan of a balance and 20·73 grams when placed in another pan. Find its correct weight.*

We know that true weight of a body is given by

$$W=\sqrt{W_1\ W_2}$$

Here in this example $W_1=20.61$ and $W_2=20.73$

$$\therefore \qquad W=\sqrt{20.61\times 20.73}$$

$$\text{Log } W=\tfrac{1}{2}(\text{Log } 20.61+\text{Log } 20.73)$$

$$=\tfrac{1}{2}(1.3141)$$

$$\underline{1.3166}$$

$$\therefore\ \tfrac{1}{2}(2.6307)=1.3153$$

$$W=\text{Ant log } 1.3153=20.66$$

$$\therefore \qquad W=20.66 \text{ grams}$$

§8. **Resolution of forces.**—Just as two forces P and Q can be compounded into a single force R by the law of parallelogram of forces in the same way any force R can also be divided into two forces P and Q in two given directions from R. These forces are known as components of R.

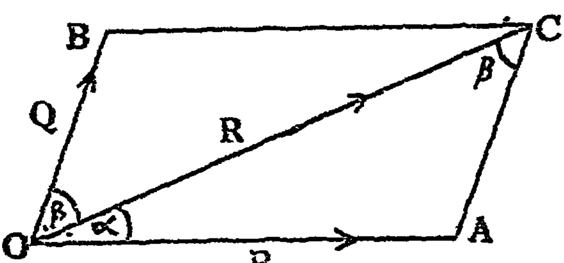


Fig 45

Let R be the force acting along OC. It is required to find two components of R along OA and OB inclined at an angle α and β from OC. Complete the parallelogram $OACB$. According to law of parallelogram of forces the resultant of P and Q is R. Therefore P and Q are the component parts of R.

To find P and Q.—

We know that in any triangle,

$$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$$

Applying this in triangle OAC

$$\frac{\sin\alpha}{Q}=\frac{\sin\beta}{P}=\frac{\sin(180-\overline{\alpha+\beta})}{R}$$

$$=\frac{\sin(\alpha+\beta)}{R}$$

Because $\sin\{180-(\alpha+\beta)\}=\sin(\alpha+\beta)$

$$\therefore \quad P=R\frac{\sin\beta}{\sin(\alpha+\beta)} \quad \text{and} \quad Q=R\frac{\sin\alpha}{\sin(\alpha+\beta)}$$

From these, P and Q can be calculated

§9. **Resolution in two mutually perpendicular directions:**—When P and Q are perpendicular to each other they are known as resolved parts In this case P and Q can be calculated by knowing the value of R and angle θ as shown in fig 46

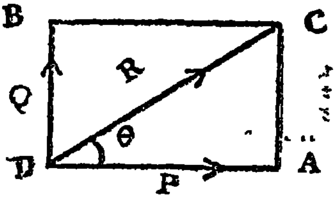


Fig 46

From the $\triangle OAC$

$$\frac{Q}{R}=\sin\theta \text{ or } Q=R\sin\theta$$

$$\frac{P}{R}=\cos\theta \text{ or } P=R\cos\theta$$

Numerical problems.—1 *Suppose $R=100$ and $\theta=30°$, $\sin 30=\frac{1}{2}$ and $\cos 30=\frac{\sqrt{3}}{2}$,*

$$\therefore \quad P=R\cos 30=100\times\frac{\sqrt{3}}{2}=50\sqrt{3} \text{ units}$$

$$Q=R\sin 30=100 \cdot \tfrac{1}{2}=50 \text{ units}$$

2 *Four forces are acting at a point as shown in fig 47(a) Find the resultant force*

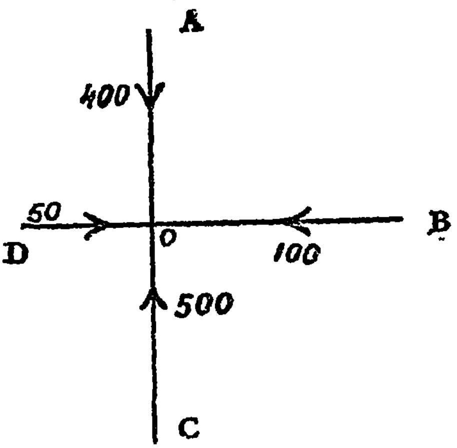


Fig 47 (a)

Force BO and DO are acting in opposite direction, therefore their resultant is equal to $(100-50)$ dyne along $\overrightarrow{BO}$

Similarly the resultant of $\overrightarrow{AO}$ and $\overrightarrow{CO}$ is equal to $(500-400)$ dyne along $\overrightarrow{CO}$

Now we have two forces as shown in Fig 47(b) They can also be represented as in Fig 47(c) complete the rectangle $OC'D'B'$, the resultant is given by,

$$R^2=50^2+100^2$$
$$=2500+100\times100$$
$$R^2=100(25+100)$$
$$R=10\sqrt{125}=50\sqrt{5}$$
$$\tan\theta=\frac{100}{50}=2$$

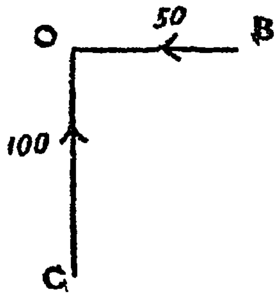


Fig 47 (b).

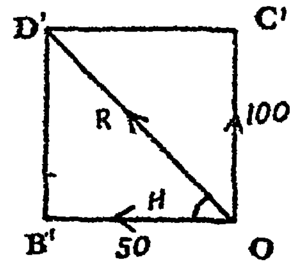


Fig 47 (c)

§10. Resultant of two parallel forces:—(i) When the forces are like.—Consider two parallel forces P and Q acting on AB The resultant R will be given by.

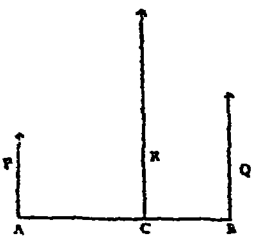


Fig 48(a)

$$R=P+R \quad \ldots \quad (i)$$

It will act at a point C such that

$$P.AC=Q.CB \quad \ldots \quad (ii)$$

It will be parallel to P and Q

Here AB is perpendicular P, Q and R If it is not so in any case, take the perpendicular distance

(ii) **When the forces are unlike:**—Consider two parallel and unlike (acting in opposite direction) forces P and Q acting at A and B Their resultant R will be given by

$$R=P-Q \qquad (i)$$

It will act at a point C such that,

$$P\ AC=Q\ CB \qquad (ii)$$

It will be parallel to P and Q

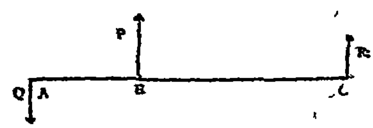


Fig 48(b)

§11. Resultant of two equal parallel and unlike forces, couple:—Consider two parallel and opposite forces P and P acting on a body

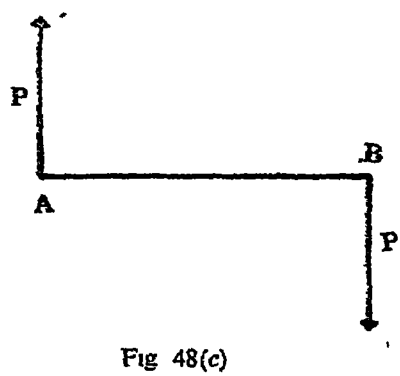


Fig 48(c)

Their resultant R will be zero In this case the body will not move as a whole from one place to another but will rotate about any point as axis. This combination is known as a couple. The rotating

effect of couple or moment of couple is found by multiplying any of the two forces by the perpendicular distance between the two forces

Moment of couple $= P \times AB$

If AB is not perpendicular to P, drop a perpendicular AC from A

Moment of couple $= P \cdot AC$

QUESTIONS

1 Distinguish between vector and scalar Give examples (*See* §1)

2 Define force and give its unit (*See* §2)

3 State and prove law of parallelogram of forces (*See* §3)

4. Define moment and state law of moments How will you verify it? (*See* §6 and 7)

5 What are resolved parts of a force? (*See* §9)

Numerical Questions —

1 Find the resultant of two forces equal to 15 and 10 lbs weights respectively acting at an angle of (*i*) 60° (*ii*) 90° and (*iii*) 135°

[*Ans* (*i*) $5\sqrt{19}$, (*ii*) $5\sqrt{13}$, (*iii*) $5\sqrt{19-6\sqrt{2}}$]

2 Find the component of a force 10 lbs wt in directions of 60° and 45° with the given force on opposite side [*Ans* $10(\sqrt{3}-1)$, $5\sqrt{6}(\sqrt{3}-1)$]

3 A force equal to 10 lbs weight is inclined at an angle of 30° to horizontal Find the resolved parts in horizontal and vertical directions

[*Ans* $5\sqrt{3}$ and 5 lbs wt]

4 Find the resultant of forces P and Q acting at A and B in a plane where $P=35$ lbs, $Q=30$ lbs, $AB=2$ ft and (*i*) P and Q are like or (*ii*) P and Q are unlike.

[*Ans* (*i*) 65 lbs wt $AC=\frac{12}{13}$ ft, $BC=\frac{14}{13}$ ft

(*ii*) 5 lbs wt $AC=12$ ft, $BC=14$ ft]

CHAPTER VIII

MOTION

§1. **Motion:**—Whenever the distance of a body changes with respect to its surroundings it is said to be moving Motion as such is relative The railway train appears to move because its distance from us changes The station buildings or the telegraph poles appear to be stationary because their distance with respect to earth is constant We know that the earth is rotating as well as revolving and along with it the station or the telegraph pole is also moving If we stand on another planet we can see the station and the poles moving In this way one thing may be stationary with respect to a certain thing but may be moving with respect to some other thing Generally whenever we say that a body is moving we take it with respect to earth

§2. **Speed.**—We generally say that a man is walking at the rate of 3 miles per hour, the cycle is going at the rate of 12 miles per hour etc These are known as speed *It is the rate of change of distance* It means the man is covering 3 miles in every hour If the body moves with a constant speed and describes D cm in t seconds Its speed is given by speed$=D/t$ cm per sec If the speed is not constant then we will get average speed by the above formula, average speed$=D/t$

§3. **Velocity:**—If we know the speed of a body we can find out its distance from the starting point after t sec But we cannot locate it unless we know in what direction it is moving This speed with direction is known as velocity It is defined as rate of change of distance in a given direction It has got magnitude as well as direction This is a vector quantity If a particular body moving with a constant velocity V describes S cm in t sec then V is given by, $V=S/t$ cm per sec in the given direction If it is not moving with uniform velocity, this will give average velocity

Average velocity$=S/t$ cm per sec

Velocity like other vectors can be represented by a straight line the length of the line is proportional to the velocity and the line is drawn in the direction of velocity

Addition of Velocities:—If a body is moving with two velocities inclined at certain angle, the resultant velocity can be found by the law of parallelogram of vectors Suppose a body is moving with a velocity u along AD and with a velocity v along AB Let the angle DAB be θ complete the parallelogram $ABCD$ (*See* fig 49) Then the resultant velocity R is given by AC According to law of parallelogram,

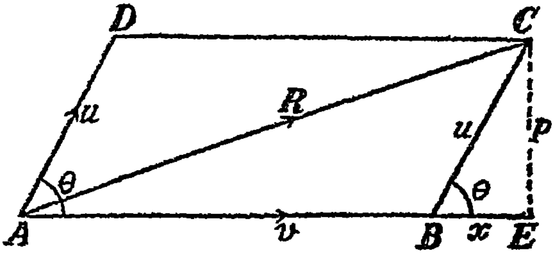


Fig 49

$$R^2 = u^2 + v^2 + 2uv\cos\theta$$

and $$\tan CAB = \frac{v\sin\theta}{u + v\cos\theta}$$

Similarly a velocity V in direction making at an angle θ from a particular direction OX can be resolved along OX and OY

The resolved part along $OX = V\cos\theta$

and the resolved part along $OY = V\sin\theta$

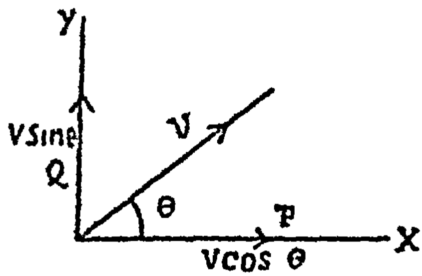


Fig 50

Unit:—The unit of velocity is cm. per sec or feet per sec

§ 4. **Acceleration:**—If the velocity of a body changes either in magnitude or in direction, we say it is moving under acceleration **Acceleration** *is defined as the rate of change of velocity* If the acceleration is uniform it can be calculated by the following formula,

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time taken}}$$

or $$a = \frac{v-u}{t}$$

where u is initial velocity v is final velocity and t is time taken, From this we get,

$$v = u + at \qquad \ldots(i)$$

Sometimes acceleration is also denoted by f

Unit of acceleration:—In metric system the unit of acceleration is cm per sec per sec and in British system it is feet per sec per sec

§ 5. **Equations of Motion**—Suppose a body moving with uniform acceleration a and initial velocity u, describe distance S in t sec and acquires velocity, v (*See* fig 51)

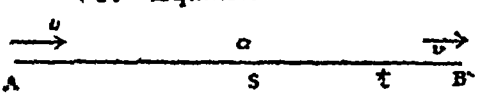


Fig 51

As shown above we have,

$$a = \frac{v-u}{t}$$

Let V be the average velocity then,

The average velocity $V = S/t$

or $$S = V\,t$$

The average velocity V is also equal to $\frac{u+v}{2}\quad t$

$$S = \frac{u+v}{2}\,t$$

Substituting for V from (*i*) we get,

$$S=\frac{u}{2}t+\frac{(u+at)t}{2}$$
$$=\frac{ut}{2}+\frac{ut}{2}+\frac{at^2}{2}$$
$$S=ut+\tfrac{1}{2}at^2 \qquad \ldots(ii)$$

Again squaring (*i*) We get

$$v^2=u^2+a^2t^2+2aut$$
$$=u^2+2a\,(ut+\tfrac{1}{2}\,at^2)$$
$$v^2=u^2+2as \qquad ..(iii)$$

with the help of these three relations any problem on motion can be solved

Numerical problems :—1. *A body starting from rest describes 96 feet in 4 sec. Find its acceleration.*

Here, $S=96$, $u=0$, $t=4$ sec

from relation (*ii*) we get,

$$S=ut+\tfrac{1}{2}\,at^2$$
$$96=0+\tfrac{1}{2}\;a\;\;16$$

or $$a=\frac{2\times 96}{16}=12 \text{ feet per sec per sec}$$

2 *A body moving with an acceleration of 4 feet per sec. per sec. acquires a velocity of 64 feet per sec. after travelling a distance of 224 feet. Find its initial velocity.*

From the relation (*iii*) we get

$$v^2=u^2+2as$$
$$64\times 64=u^2+2\times 4\times 224$$
$$u^2=64\times 64-2\times 4\times 224$$
$$=64\,(64-28)$$
$$=64\times 36$$
$$u=8\times 6=48$$
$$=48 \text{ feet per sec.}$$

§6. Distance described in t^th^ second :—Let S_1 be the distance described in t sec and S_2 be the distance described in $(t-1)$ sec from relation (*ii*) we have,

$$S_1=ut+\tfrac{1}{2}\,at^2$$
$$S_2=u\,(t-1)+\tfrac{1}{2}a(t-1)^2$$
$$=ut-u+\tfrac{1}{2}at^2-at+\tfrac{1}{2}a$$

Therefore $$S_1-S_2=u+at-\tfrac{1}{2}a$$
$$=u+\frac{2t-1}{2}\times a$$

QUESTIONS

1 Define velocity and acceleration. (*See* §3 and §4)

2 Prove $S=ut+\frac{1}{2}at^2$ (*See* §5)

3 Find out distance travelled in t^{th} second. (*See* §6)

Numerical Questions —

1 A body starting with initial velocity of 12 feet per sec moves with a uniform acceleration of 4 ft /sec ^{2}

(*a*) What is the velocity after 10 secs. ?

(*b*) How far will it go in 10 secs ? [*Ans* 52 feet/sec 320 ft]

2 A point, moving with uniform acceleration describes $\frac{8}{25}$ in the last second of its motion $\frac{9}{25}$th of the whole distance If it started from rest, how long was it in motion and through what distance did it move if it described 6 inches in the first second [*Ans* $t=5$ secs , $S=12\frac{1}{2}$ feet]

CHAPTER IX

NEWTON'S LAWS OF MOTION

§1 **Newton's Laws of Motion:**—In 1686 Newton formulated three foundamental laws of motion —

First Law or Law of Inertia:—*Everybody continues to be in a state of rest or of uniform motion in a straight line unless it is compelled by any external agent to change that state*

Second Law:—*The change of motion i e rate of change of momentum is proportional to the impressed force and takes place in the direction of the force*

Third Law —*To every action there is an equal and opposite reaction*

§2. **First Law:**—We know that a stone which is at rest will not move automatically unless we force it to move It will remain at rest Similarly if a body is moving it will continue to move unless it is stopped by a force That means bodies at rest or in motion show inertia *i e* they oppose any change in their state of rest or motion This law is, therefore known as law of inertia

So far as bodies at rest are concerned this part of the law requires no formal proof We know from our daily experience that unless some kind of force is applied a body will not move You must have observed that when a car takes a sudden start we lean backward That part of the body which is in contact with the seat moves along with the car but the upper part remains at rest due to inertia unless it is pulled in the forward direction. Therefore we experience a jerk in the backward direction So far as the second part of the law is concerned it is not self-evident nor we can experience it directly in our everyday observations We can prove this part only indirectly. According to this law any body set in motion should continue to move for an infinitely long time If we roll a mud ball on a rough floor it comes to rest after some time. If we take a smooth floor it will move for longer time Again if we take a glass ball and ice or glass surface, the ball will move for a considerable long time. It shows that some forces known as forces of friction act between the ball and surface which reduces the velocity of the ball As we take more and more smooth surface these forces go on reducing and therefore the ball moves for longer period If we can produce a perfectly smooth surface and roll a body in vacuum it will continue to move for ever But it is impossible to realise such a condition in practice This part can also be proved indirectly by the following examples

(*i*) When a car suddenly stops we lean in forward direction why? Our lower part of the body comes to rest with the car but the upper part continues to move with the same speed unless it is pulled back

(*ii*) If you get down from a moving train you may fall down in the forward direction when you jump from the train you are also moving with the velocity of train Suppose the train is moving with a velocity of 4 miles per hour When you touch the ground your feet will come to rest but your head will continue to move with 4 miles per hour and therefore you will fall down in the forward direction

(*iii*) You must have seen in any circus show that a man riding on a horse with great speed takes a jump and falls back again on the same horse This is possible because he continues to move along with the horse with the same velocity

(*iv*) As you all know earth rotates about its axis in 24 hours In 12 hours America will take the place of India If a person in India takes up a plane high up in the sky and continues to hold on there for twelve hours he will find himself over America and can land there But this is not possible Why? The reason is that when the plane is on the ground it is not at rest but it is moving along with the earth with the same velocity Therefore when it goes up in the sky it will continue to move along with the earth with same velocity and will remain over India always unless its engine works in opposite direction and destroys its velocity

Force:—This law also defines force That external agent which causes any change in the state of a motion of a body is known as force That is it changes or tends to change the position of rest or of uniform motion in straight line Force is a vector quantity

§3. **Second Law:**—Second law is also related with first law First Law states that unless force is applied no change in motion is produced a qualitative statement Second law gives the relation between force and motion produced *i e* a quantitative relation

Momentum.—It is defined as the product of mass and velocity Consider a body of mass m moving with a velocity u cm per sec A force F acts on it for t seconds so that its velocity changes to v cm per sec Then, we have

Momentum in the beginning $=mu$

Momentum after t sec $=mv$

Change in momentum $=mv-mu$

Rate of change of momentum $=\dfrac{mv-mu}{t}$

According to this law the impressed force F is proportional to the rate of change of momentum *i e*

$$F \propto \frac{mv-mu}{t}$$

$$F = K.m\frac{(v-u)}{t}$$
$$= K.m\,f.$$

where K is constant and f is acceleration i.e. rate of change of velocity. The unit of force is selected in such a way that K becomes unity. That is when $m=1$, $F=1$. $f=1$,

Therefore, $K=1$

$$F = m\,f \qquad \ldots(1)$$

This unit force is known as dyne or poundal.

Dyne:—That force which produces an acceleration of 1 cm per sec per sec in a mass of 1 gram is called 1 dyne.

Poundal:—That force which produces an acceleration of 1 foot per sec per sec in a mass of 1 pound is 1 poundal.

Relation between poundal and dyne —

1 Poundal = 1 pound × 1 foot per second per sec
= 453.6 gms 12 × 2.54 cm. per sec per sec
x = 13834.8 dynes

Third Law:—Whenever a force acts in a particular direction we call it an action. A force which is produced as a result of action

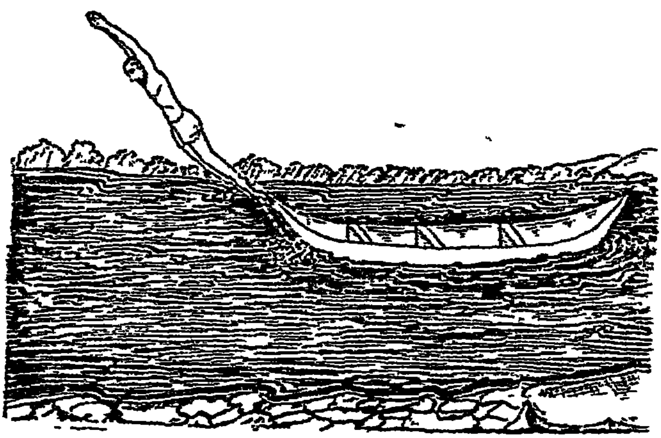


Fig 52

and which acts in opposite direction is called a reaction. According to this law action and reaction are equal and opposite.

If we press anything with our hand, that thing will also press our hand in opposite direction with the same force. If a book is placed on the table it presses the table in the downward direction and the table will press it in upward direction. If we suspend a weight from a thread the weight will pull the thread in downward direction and due to this a tension will be produced in the thread.

which will pull the stone in upward direction Whenever we walk on rough ground we push the ground in backward direction There-

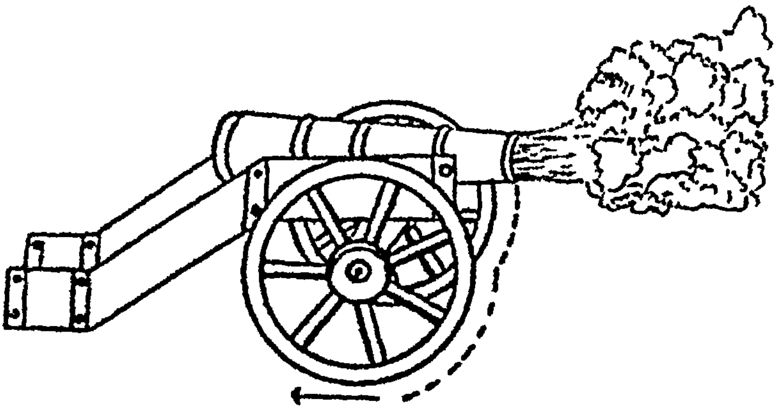


Fig 53.

fore ground applies a force on our foot in forward direction which enables our movement in forward direction, if the ground is smooth the reaction developed is very small and therefore it becomes difficult to walk

Law of conservation of momentum is also based on this law when we fire a gun, fig 53, the momentum of the shot is equal and opposite to momentum of recoil of gun Similarly, when we jump from a boat, fig 52, on the shore, the boat will be pushed back in the water The working of rocket fig 54 is also based on this A gas is discharged from the tail of the rocket with great velocity and due to the reaction the rocket moves in forward direction The working of steam turbines in the power house is also based on this Steam is discharged with a great velocity from the nozzles fitted along the circumference of a wheel and due to reaction the wheel rotates in the opposite direction

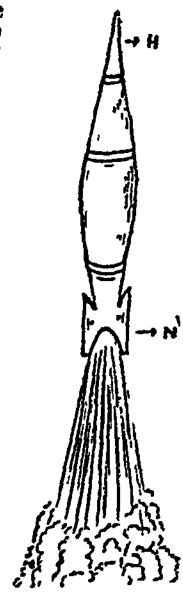


Fig 54

Numerical problems.—1 *A force of 100 dynes acts on a body of mass 10 grams at rest for 5 sec Find the velocity produced and distance travelled by the body.*

From the relation $F=m\ f$, we get,

$$100=10\times f$$

acceleration $f=100/10=10$ cm per sec per sec

From relation (*i*), $v=u+ft$, we get,

$$v=0+10\times 5=50 \text{ cm per sec}$$

From Relation (*ii*) $S=ut+\frac{1}{2}ft^2$, we get

$$S=0+\frac{1}{2}\ 10\times 5\times 5=125 \text{ cms}$$

2 *A force of 5 gram weight acts on a body of 98 grams for 5 seconds. Find the distance travelled*

Here the force is given in gram wt It should be converted into dynes before it is substituted in the relation $F=mf$

$$F=5 \text{ gram wt} = 5\times 980 \text{ dynes},$$

and
$$m=98 \text{ grams}$$

Therefore $5\times 980=98f$.

$$f=50 \text{ cm per sec per sec}.$$

From the relation $S=ut+\frac{1}{2}ft^2$

$$S=0+\tfrac{1}{2}\ 50\times 5\times 5$$

$$=625 \text{ cms}$$

3 *A bullet moving at the rate of 200 ft per second, is fired into the trunk of a tree into which it penetrates 9 inches If the bullet moving with the same velocity, were fired into a similar piece of wood 5 inches thick, with what velocity would it emerge supposing the resistance to be uniform.*

From the first case find out the value of f

Using relation (iii) we get,

$$v^2=u^2+2fs$$

$$0=(200)^2+2\ f\ 9/12$$

or
$$0=200\times 200+\tfrac{3}{2}f$$

Therefore $$f=-\frac{200\times 200\times 2}{3} \text{ feet sec}^2$$

In second case $$f=-\frac{200\times 200\times 2}{3} \text{ feet sec}^2$$

$$S=5/12 \text{ feet}$$

$$u=200$$

To find v

According to $$v^2=u^2-2fs$$

$$v^2=200\times 200-2\ \frac{200\times 200\times 2}{3}\ 5/12$$

$$=200\times 200(1-5/9)$$

$$=200\times 200\times 4/9$$

$$v=\frac{200\times 2}{3}=\frac{400}{3}=133 \text{ ft per sec.}$$

4 *The driver of a motor car, travelling at 30 miles per hour on a level ground, applies the brakes and comes to rest in a distance of 44-feet. If the weight of the car and load is 2,000 lbs and the acceleration is constant, calculate the retarding force*

Initial velocity of car $=30$ miles per hour

$$=\frac{30\times1760\times3}{60\times60}\text{ feet per sec}$$

$$=44\text{ feet per sec}$$

From relation $v^2=u^2+2fs$, we get

$$0=44\times44+2\ f\ 44$$

Therefore acceleration $f=-\dfrac{44\times44}{2\times44}=-22$ feet per sec ^{2}

Therefore force applied acting on the car is given by

$$F=m\ f$$

$$F=2000\times22\text{ poundals}$$

$$=44000\text{ poundals}$$

$$=\frac{2000\times22}{32}\text{ pound wt}$$

$$=1375\text{ pound wt}$$

5. *A bullet of 10 gram wt is fired from a gun of 5 K gram wt with velocity 400 metres per sec Find the recoil of the gun*

In such problems

Momentum of the gun $=$Momentum of the bullet

or $$M\ V=mv$$

Here $m=10$ grams $v=400\times100$ cms per sec

$M=5\times1000$ grams, $V=?$

Substituting the values we get,

$$5\times1000\times V=10\times400\times100$$

$$V=\frac{400\times100\times10}{5\times1000}$$

$$=80\text{ cms per sec}$$

6 *A rocket of mass 1 K gm. throws out through its jet gases with a velocity of 10 Km /sec. If the mass of the gas ejected per second is 100 gms find out the velocity with which the rocket would move*

Here, momentum of the rocket=momentum of the gas

or $$MV=mv$$

$V=?$ $v=10\,K$ metres/sec

$M=1000$ gms $m=100$ grams

Substituting the values,

$$V=\frac{mv}{M}=\frac{100\times10}{1000}=1\,K\text{ metre/sec}$$

7 *A 200 lb man stands on a lift What force does the floor exert on him when the lift is (a) stationary, (b) accelerating, upwards at 20 ft /sec ^{2}, (c) moving upward with a constant speed, (d) moving downwards with an acceleration of 20 ft /sec ^{2}*

(*a*) When a body is at rest on a plank, its reation R is equal to Mg the weight of the body (according to 3rd law) *See* fig. 55

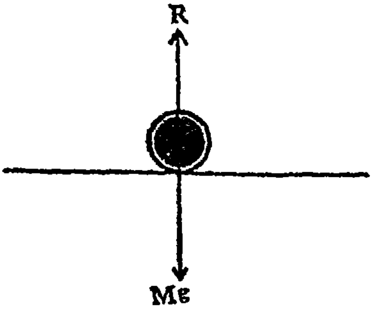


Fig 55

$$R = Mg = 200 \times 32 \text{ poundals}$$
$$= 200 \text{ lb wt}$$

(*b*) When the body is moving upwards with a constant velocity, net force acting on the body is zero and therefore, as above, $R = Mg$

$$R = 200 \text{ lb wt}$$

(*c*) When the body is moving upwards with an acceleration f net force acting on the body is equal to $R - Mg$

This must be equal to Mf according to second law

$$R - Mg = Mf$$
$$R = Mg + Mf$$
$$= M(g + f)$$
$$= 200(32 + 20) = 200 \times 52 \text{ poundals}$$
$$= \frac{200 \times 52}{32}$$
$$= 325 \text{ lb wt}$$

(*d*) When it is moving downwards, net force is equal to $Mg - R$

$$Mg - R = Mf \text{ or } R = Mg - Mf$$
$$= M(g - f), = M(32 - 20)$$
$$= 200 \times 12$$
$$R = 200 \times 12 \text{ poundals}$$
$$= \frac{200 \times 12}{32} = 75 \text{ lb weight}$$

QUESTIONS

1 State and explain Newton's laws of motion (*See* §1 and §2)

2 Define force and unit of force (*See* §2)

3 State Newton's second law of motion and derive the relation $P = mf$ (*See* §2)

4 Give a few examples of 3rd law. (*See* §2)

Numerical Questions :—

1 Find the magnitude of the force (*i*) in poundals (*ii*) in pounds weight which will produce in a mass of 10 lbs an acceleration of 20 ft /sec ^{2} [*Ans* 200 poundals ; $6\frac{1}{4}$ lb wt]

2 A force equal to the weight of a kilogram acts on a body continuously for 10 seconds and cause it to describe 10 metres in that time Find the mass of the body [*Ans* 49 05 k gram]

3 A mass of 10 lbs falls 10 ft from rest and is then brought to rest by penetrating 1 foot into some sand Find the average thrust of the sand on it [*Ans* 110 lbs]

CHAPTER X

LAW OF GRAVITATION

§1. **Introduction**:—Who is not familiar with the astronomical principles formulated by Kepler—He formed laws which give the path of a planet round the sun Newton formulated in 1687 his famous law of gravitation to explain Kepler's laws

§2. **Newton's Law of Gravitation**:—According to this law "every material particle attracts another particle with a force which

Fig 56

is directly proportional to the product of the two masses and inversely proportional to the square of the distance" Let m_1 and m_2 be two masses placed at a distance d cm apart Let F be the force of attraction between them, then according to this law, F is given by,

$$F \propto \frac{m_1 m_2}{d^2}$$

or
$$F = G\frac{m_1 m_2}{d^2} \qquad (i)$$

where G is a constant and is known as universal constant of gravitation Its value as determined by Boys in 1895 is 6 6576 $\times 10^{-8}$ C G S units If in the above equation $m_1 = m_2 = 1$ and $d = 1$ then $F = G$

Therefore G is numerically equal to force of attraction acting between two masses of 1 gram each when placed at a distance of 1 cm

When we consider attraction between two bodies d is the distance between their centres of gravity

§3. **Gravity**:—According to this law of gravitation earth also attracts everybody towards its centre This force of attraction is generally referred to as gravity, let Me and m be the masses of the earth and a body, respectively and let R be the radius of the earth If the height of the body above the surface of the earth is small as compared to the radius of the earth, (which is 4,000 miles) the distance of the body from the centre of the earth can be taken to be equal to the radius of the earth Then applying Newton's law we get,

$$F = G\frac{M_e\, m}{R^2} \qquad \ldots \quad (ii)$$

Here F is the force with which the body will be attracted towards the earth. We know from second law of motion that if a force F acts on a mass m the acceleration f is given by

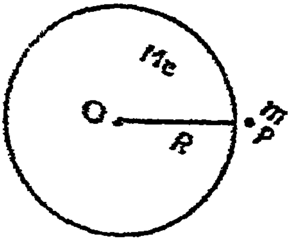


Fig. 57.

$$F=m.f. \qquad \ldots \quad (iii)$$

$$\text{acceleration}=F/M$$

Therefore the body will move towards the earth with an acceleration given by

$$f=F/M=\frac{G.M_e.m}{R^2}\times\frac{1}{M}$$ after substituting the value of F from equation (ii).

$$=G\,\frac{M_e}{R^2}$$

This acceleration of anybody towards the earth is known as acceleration due to gravity and is denoted by 'g'

$$g=\frac{G.M_e}{R^2} \qquad \ldots \qquad \ldots \quad (iv)$$

The value of g is 980 cms. per sec. per sec. It means if a body is dropped from a certain height its velocity will change by 980 cms./sec. after every second.

From (iv) we see that g does not depend upon mass 'm' of the body ; that is it will be the same for all bodies Hence. "All bodies having different masses will move with the same acceleration towards the earth."

Now we know that if a body is dropped from a certain height, the time taken in travelling that height depends upon the acceleration If acceleration is same, time taken will also be the same. for all bodies. Therefore we conclude, *"Time taken by different bodies in falling through the same height is the same."* This principle was first demonstrated by Galilio from the leaning tower of Pisa and he was put in jail for preaching against the religious belief according to which "the heavier body will fall first and lighter one afterwards."

(As explained above this force of attraction is mutual. Therefore earth is also attracted by the body with the same force but its mass being very large. acceleration produced will be very small and is imperceptible).

§4. **Density of Earth:**—If we know g, G and R we can calculate the density of earth. Because

$$g=G.\frac{M_e}{R^2}$$

If earth is considered as a sphere of mean density d, its mass Me is given by

$$M_e=\text{volume}\times\text{density}$$

$$=\frac{4}{3}\pi R^3\times d$$

$$g=G\left(\frac{4}{3}\pi R^3\right)d\times\frac{1}{R^2}$$
$$=\frac{4}{3}\pi GRd$$
$$d=\frac{3g}{4\pi GR}$$

From this we can find out the mean density of the earth, taking $g=980$/cms per sec per sec, $G=6.65\times10^{-8}$, and $R=4000$ miles

§5. **Variation in g:**—Though G is an universal constant, g is not so Its value goes on varying from place to place as discussed below.—

(*i*) **On the surface of the earth:**—As you all know, our earth is not a perfect sphere Its diameter at the equator is more than its diameter at the poles Therefore, if we consider a body on the equator, its distance from the centre will be greater than when the body is at the poles Therefore, the value of g, which *is inversely* proportional to the square of the distance will be less at the equator and more at the poles

Let g_e, R_e, be the acceleration and radius at the equator and g_p, R_p at the poles respectively

$$g_e=\frac{G\ M_e}{R_e^2}\text{ and }g_p=\frac{GM_e}{R_p^2}$$

$$\therefore\quad \frac{g_p}{g_e}=\frac{G\ M_e}{R_p^2}\times\frac{R_e^2}{G\ M_e}=\frac{R_e^2}{R_p^2}$$

Since, $$R_e>R_p$$
$$g_p>g_e$$

Hence "*As we move from the equator to poles g will increase.*" In other words, acceleration due to gravity increases with latitude

(*ii*) **Rotation of the earth about its axis:**—If a body rotates about a point in a circle, a force will act on it along the radius and away from the centre This force is known as centrifugal force This force is given by, centrifugal force$=\frac{Mv^2}{R}$ where v is the velocity of the body and R is the radius of the circle in which the body is rotating Now, we know that earth rotates about its axis in twenty-four hours and everybody placed on it will describe a

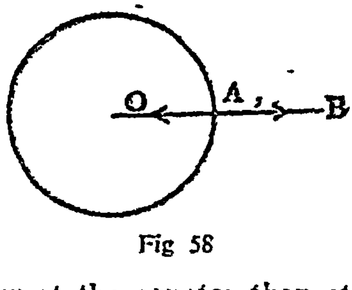


Fig 58

circle Since the velocity of earth is greater at the equator than at the poles, centrifugal force is greater at the equator than at the poles This centrifugal force acts in opposite direction to the gravitational force Therefore apparent force of attraction will be less at the equator than at the poles Hence "*As we go from equator to poles g will increase*"

(iii) **With height:**—If the height of a body above the surface is more we will have to consider it. and g will be given by,

$$g=\frac{G M_e}{(R+h)^2}$$

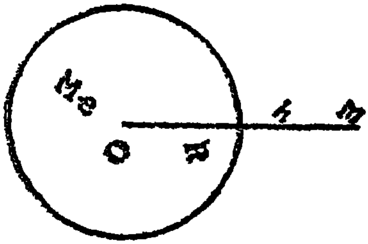


Fig 59.

where h is the height above sea level

From this as h increases g will decrease Hence, "*As we go higher and higher the value of g decreases*" or in other words g decreases with increase of altitude of a place at the same latitude

(iv) **Inside the earth:**—If we take a hollow sphere, the force of attraction on a body placed in it is zero Therefore, when we place a body at P inside earth at a distance r cm from the centre, only that part of the earth which lies within a sphere of radius r will exert a force and not that part which lies outside Therefore, in this case though R is less, M_e is also reduced and reduction in M_e is more than reduction in R Therefore g decreases Hence "*g decreases as we go inside the earth and becomes zero at the centre of the earth.*"

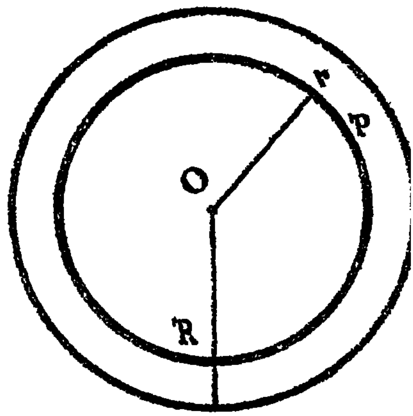


Fig 60

§6. **Weight of a body:**—Weight of a body is defined as the force of attraction exerted on it by the earth, while mass of a body is defined as the amount of matter contained in it Mass remains the same at all places Let m be the mass of a body and g be the acceleration due to gravity Then, according to 2nd law of motion, the force F acting on the body is given by

$$F=mg \text{ dynes}$$

As we have seen above, g varies from place to place and hence, F also varies and hence "*Weight of a body varies from place to place in the same manner as g varies*"

Thus, we see that weight and mass are two different quantities Sometimes we call mass also as weight because weight is proportional to mass and at a particular place

$$\frac{M_1}{M_2}=\frac{W_1}{W_2}$$

§7. **Unit of mass and weight:**—You have read that the unit of mass in metric system is gram and in British system it is pound, and the unit of force is dyne and poundal respectively. The practical unit of force is gram weight and pound weight The force of attraction acting on a mass of one gram is known as 1 gram weight and the force acting on 1 pound is known as 1 pound weight

Since force $= W = mg$

$\therefore$ 1 gram weight $= 1 \times 980$ dynes

and 1 pound weight $= 1 \times 32$ poundals

Remember that the value of g in f p s system is 32 ft per sec per sec Thus pound weight and gram weight which depend upon the value of g are known gravitational units and dynes and poundal are known as absolute units A force expressed in dyne or poundal will be the same at all places but if expressed in gram wt or pound wt it will vary with g

§8. **Value of g on different planets:**—Just as when we drop a body on the surface of the earth it falls towards the earth, in the same way, if we drop a body on the surface of the Moon or the Jupiter it will move towards the centre of moon or Jupiter with a certain acceleration depending upon their masses and radius Let g_e, M_e, R_e be the acceleration, mass and radius of the earth, and g_m, M_m R_m be the corresponding quantities for the Moon, then we get,

$$g_e = G\frac{M_e}{R_e^2} \text{ and } g_m = G\frac{M_m}{R_m^2}$$

$$\therefore \quad \frac{g_m}{g_e} = \frac{G\ M_m}{R_m^2} \times \frac{R_e^2}{G\ M_e} = \frac{M_m}{M_e} \times \frac{R_e^2}{R_m^2}$$

If mass of the earth is taken as 100 times the mass of Moon and its radius is taken five times that of the Moon, we get,

$$\frac{g_m}{g_e} = \frac{1}{100} \times \frac{25}{1} = \frac{1}{4}$$

$$\therefore \quad g_m = \frac{g_e}{4} \quad \text{If } g_e = 32,\ g_m = 8$$

A body on the Moon will fall with an acceleration of 8 feet per sec. per sec

Weight of a body on Moon:—Let m be the mass of a body

$\therefore$ W_e (weight on earth) $= m\,g_e$

W_m (weight on moon) $= m\,g_m$

$$\therefore \quad \frac{W_m}{W_e} = \frac{g_m}{g_e} = \frac{1}{4}$$

The weight of a body on the Moon will be 1 pound if it weighs 4 pounds on the earth

§9. **Highest height attained:**—Whenever we project a body in upward direction, its velocity continuously decreases on account of acceleration g which acts in the opposite direction The body will rise to a certain height h when its velocity will reduce to zero and then it will fall down again Greater is the velocity of projection greater will be the highest height attained But, in every case the body will come back This barrier which does not allow the bodies to escape

is known as gravitational barrier If we want to cross this barrier, the initial velocity of projection has to be very high. (7 miles per sec or 11 2 K metre per sec)

You must have heard the name of Russian luniks and American explorers They have been fired only at such great speeds so as to cross the gravity barrier and circle round the moon and the sun

Numerical example:—*A body is projected with a velocity of 96 feet per sec. Find the highest height attained and the time of flight. In this case $u=96, f=g=-32, v=0$ $S=?$ $t=?$*

From the relation $v=u+ft$

$$0=96-32\ t$$

$$\therefore \qquad t=\frac{96}{32}=3\text{ secs}$$

It will take 3 sec in reaching the greatest height It will come back again in 3 sec Therefore total time of flight is 6 sec

From the relation $v^2=u^2+2fs$

$$0=96^2-2(32)\ S$$

$$\therefore \qquad S=\frac{96\times 96}{2\times 32}=144\text{ feet}$$

§10. **Centre of gravity:**—If we consider a body of mass M it can be supposed to be made up of a large number of small particles of mass m Each particle will be attracted by a force mg towards the centre of the earth Thus, a large number of parallel forces will be acting on the body. The resultant force Mg will be equal to the sum of these forces, *i e*

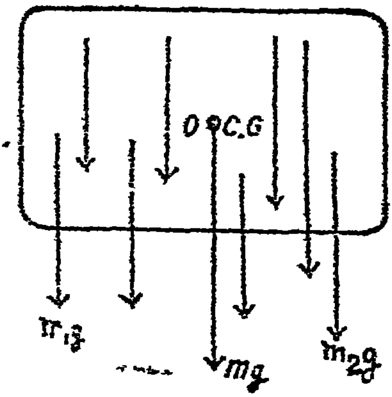


Fig 61.

$$Mg=m_1g+m_2g...$$

This resultant force will act at a certain point 0 of the body. This point is known as the centre of gravity This point depends upon the shape and size of the body The moment of all the forces m_1g, m_2g. about this point is zero The centre of gravity of a uniform metre scale is at the middle point The centre of gravity of a sphere is at its centre The body will remain in equilibrium when suspended from the centre of gravity.

§11. **To find the value of g:**—In order to find the value of g, we make use of simple pendulum Before we study simple pendulum we should understand simple harmonic motion.

Simple harmonic motion:—The motion of a body is simple harmonic if it satisfies the following conditions (For details see section IV):—

(i) *It should move along a straight line.*

(ii) *It should execute a to and fro i.e., oscillatory motion*

(iii) *The force of restitution should always be directed towards mean position*

(iv) *Acceleration should be proportional to its displacement from mean position*

These conditions can be represented by the following relation

$$\text{acceleration} = \omega^2 \text{ displacement} \qquad . \quad (i)$$

where ω is a constant

The periodic time of such an oscillation T is given by

$$T = \frac{2\pi}{\omega} \qquad .. \quad (ii)$$

§12 **Simple pendulum** :—It consists of a heavy bob suspended from a fixed support S by means of an inelastic, weightless string. When it is moved to B and left it will try to come back towards the mean position. When it reaches o it would acquire some velocity and therefore, it would not stop at o but would go to the other side A and will again come back. In this way it will execute to and fro motion till its amplitude (maximum displacement) reduces to zero due to air friction

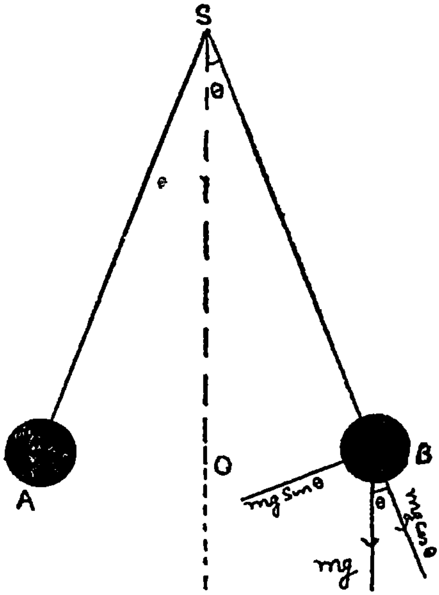


Fig 62

Consider the bob at B. A force equal to mg acts on the bob in a vertical direction. Here m is the mass of the bob and g is acceleration due to gravity. This force mg can be resolved into $mg \cos\theta$ along the string and $mg \sin\theta$ perpendicular to string as shown in the figure. $mg \cos\theta$ balances the tension and $mg \sin\theta$ moves the bob towards 0. This force is known as force of restitution

Force acting on the bob bringing it to its position

$$= mg \sin\theta$$

$$\therefore \text{Acceleration of the bob} = \frac{\text{force}}{\text{mass}}$$

$$= \frac{mg \sin\theta}{m}$$

$$= g \sin\theta$$

$$= g\,\theta \quad (\text{when } \theta \text{ is small, } \sin\theta = \theta)$$

$$= g\,\frac{x}{l} = \frac{g}{l}\,x \qquad \ldots \quad (iii)$$

$\therefore$ Acceleration $\propto x$ as $\frac{g}{l}$ is constant (l is the length of the pendulum)

Here $\theta = \frac{x}{l}$ where x=displacement OB and l=length SB of the pendulum

If the displacement is small it is also along a straight line. Hence, the motion of a simple pendulum is simple harmonic.

Comparing (*iii*) with (*i*) we get

$$\omega^2 = \frac{g}{l}$$

$$\omega = \sqrt{\frac{g}{l}}$$

$\therefore$ From (*ii*) $$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{g}{l}}} = 2\pi\sqrt{\frac{l}{g}}$$

or $$T^2 = 4\pi^2\frac{l}{g}$$

$\therefore$ $$g = 4\pi^2\frac{l}{T^2} \qquad \ldots \qquad \ldots \qquad (iv)$$

This is the required expression.

In order to find out g, displace the bob and find out time for 30 or 40 oscillations and find out time for one oscillation T. Measure

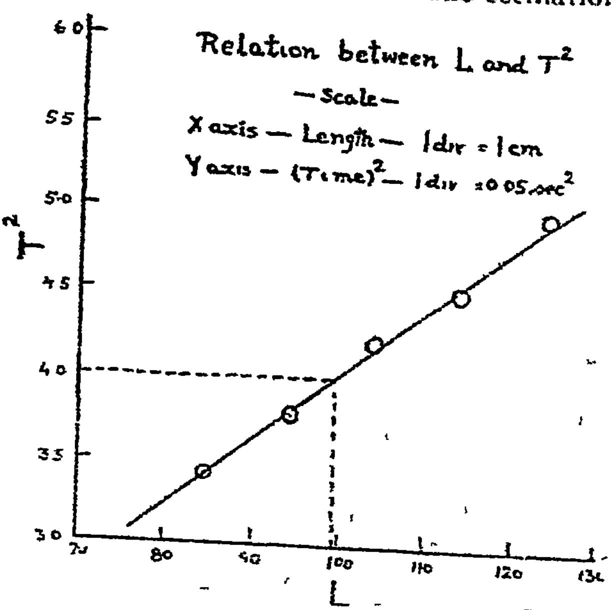


Fig 63

the length of the pendulum l from point of support to the centre of the bob. Substitute these values in (*iv*) and find the value of g. We

can take number of sets by changing l. Find out g from each set and find out mean g.

§13. **Seconds pendulum and its length:**—Seconds pendulum is that pendulum for which the periodic time is 2 sec. If we substitute this value of T in the equation $T=2\pi\sqrt{\frac{l}{g}}$. We can calculate l, the length of a seconds pendulum.

To find out this length experimentally:—Find out the periodic time T corresponding to various lengths, as explained above. Plot a graph between l and T^2 (Fig. 63). It will be a straight line. Find out the value of length l corresponding to $T^2=4$. This will give the length of seconds pendulum.

§14. **Factors affecting the periodic time of a simple pendulum.**—The maximum displacement of a bob from its mean position is known as *amplitude*. We have seen above that this amplitude should be small. For this the length of the pendulum should be large and the displacement should be small.

According to the given relation $T=2\pi\sqrt{\frac{l}{g}}$ we see that the periodic time T is directly proportional to the sq. root of length and inversely proportional to the square root of g, the acceleration due to gravity. Therefore, as the length increases periodic time will increase and as g decreases T will increase. We have already studied the variation of g on the surface of the earth, with height, or as we go inside the earth.

§15. **Free oscillations:**—In the absence of any external force, a pendulum once disturbed will go on oscillating for ever. At B its displacement is maximum and, therefore, its acceleration is also maximum. When it comes to 0 its displacement is zero. Therefore its acceleration is *o* but velocity is maximum. When it is at B its vertical height above 0 is h and, therefore, its potential energy is mgh and kinetic energy is zero. When it moves towards 0 it loses potential energy and gains kinetic energy. When it comes at 0, the whole energy is kinetic and potential energy is zero. In this way, the total energy of the bob is constant. It only changes from kinetic to potential and *vice versa*. Such a system in which there is no loss of energy is known as conservative system. In practice the friction due to air reduces the amplitude everytime till it comes to rest.

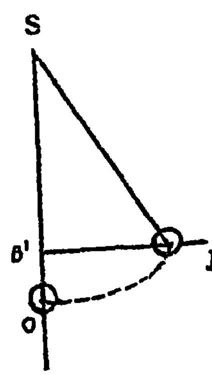


Fig. 64

Numerical Problems:—1. *Calculate the length of a seconds pendulum at a place where $g=981$ cms/sec^2.*

The periodic time of a seconds pendulum is 2 sec. Substituting this value in the equation,

$$T=2\pi\sqrt{\frac{L}{g}} \text{ we get,}$$

$$2=2\pi\sqrt{\frac{L}{g}}.$$

or
$$l=\frac{4\times 981}{4\times\pi^2}=\frac{4\times 981}{4\times 3\cdot 14\times 3\cdot 14}$$
$$=99\cdot 32 \text{ cms}$$

2 *A simple pendulum is formed by suspending a metal bob from a long string It swings through a small angle of 6° on either side of its mean position so that the vertical descent of the centre of the bob from the extreme to the lowest position is 5 mm., find the velocity and acceleration of the centre of the bob at the ends of the swing and when crossing the mean position (g=980 cm s/secs)*

(*i*) When the bob is in extreme position its velocity is zero

(*ii*) When the bob is in mean position, since its displacement is zero, therefore its acceleration is zero.

(*iii*) The vertical descent of the bob is 5 mm and therefore the velocity acquired is the same as that of a body falling vertically by 5 mm.

Accordingly $v^2=u^2+2gh$ since $u=0$

$$=2gh$$

$$\therefore \quad v=\sqrt{2gh}=\sqrt{2\times 980\times\cdot 5}=14\sqrt{5}$$

(*iv*) *Acceleration when the bob is in extreme position* —

Acceleration $=g \sin\theta$

$=g\,\theta$ (when θ is small)

$=\frac{g\times 6\pi}{180}$ (θ should be substituted in radians)

$=\frac{980\times 6\times 3\cdot 14}{180}$

$=102\cdot 67$ cms per sec per sec

4 *Gravity at the poles exceeds gravity at the equator in the ratio of 301 : 300. A pendulum regulated for the poles is taken to equator, calculate how many seconds a day it will lose or gain?*

Let T_1 and G_1 be the time and acceleration on the poles and T_2 and g_2 the corresponding quantities on equator

We are given that $\frac{g_1}{g_2}=\frac{301}{300}$

Let l be the length of a simple pendulum then we get

$$T_1=2\pi\sqrt{\frac{l}{g_1}}$$

$$T_2=2\pi\sqrt{\frac{l}{g_2}}$$

Squaring and dividing,

$$\frac{T_1^2}{T_2^2}=\frac{g_2}{g_1}$$

or
$$T_2^2=T_1^2\frac{g_1}{g_2}$$

Since the pendulum is regulated for poles $T_1=2$ secs.

$$\therefore \quad T_2^2=4\times\frac{301}{300}$$

$$\therefore \quad T_2=2\sqrt{\frac{301}{300}}=2\left(1+\frac{1}{2\times300}\right)$$

$$\therefore \quad T_2=2+\frac{1}{300}=2+.003$$

Since T_2 is more the pendulum will lose. It will lose .003 sec. in 2.003 sec.

$$\therefore \quad \text{It will lose} = \frac{.03}{2.003}\times\frac{24\times60\times60}{60} \text{ mins in 24 hrs.}$$

$$=2 \text{ mins. } 23.52 \text{ secs}$$

5. *Find the acceleration of the moon towards the earth assuming that the moon is situated at a distance which is 60 times the earth's radius, this distance being measured from the centre of the earth. The acceleration due to gravity at the surface of the earth is 32.2 ft./sec.²*

Let acceleration on the surface of the earth be g_1 and its radius be R_1 the acceleration of the moon be g_2 and its distance from the earth be R_2. Applying the formula of acceleration we get,

$$g_1=\frac{G\ M_e}{R_1^2} \quad \ldots \quad \ldots \quad (i)$$

$$g_2=\frac{G\ M_e}{R_2^2} \quad \ldots \quad (ii)$$

dividing (ii) by (i) we get,

$$\therefore \quad \frac{g_2}{g_1}=\frac{R_1^2}{R_2^2}=\left(\frac{1}{60}\right)^2 \left(\text{Since } \frac{R_2}{R_1}=60\right)$$

$$\therefore \quad g_2=g_1\left(\frac{1}{60}\right)^2$$

$$=32.2\times\frac{1}{3600}$$

$$=.00894 \text{ feet/sec}^2$$

QUESTIONS

1 What is Newton's law of gravitation? Show that two bodies of different masses when dropped from the same height will come down in same time. (*See* §2 and §3)

2 What do you understand by acceleration due to gravity? How does it vary? (*See* §3 and §5).

3 Compare the weights of a body on two planets (*See* §8).

4 Define Simple Harmonic Motion Show that motion of a pendulum is simple harmonic. Find the expression for its time period. (*See* §11 and §12)

Numerical Questions —

1 A body is projected from the earth vertically with a velocity of 40 feet per second. Find (*i*) how high it will go before coming to rest (*ii*) what time will elapse before it is at a height of 9 feet? [*Ans* 25 feet. $\frac{1}{4}$ and $2\frac{1}{4}$ secs]

2 A falling body in the last second of its fall passes through 224 feet Find the height from which it fell and the time of its falling [*Ans* 900 feet, $7\frac{1}{2}$ secs]

3 A stone is dropped into a well and reaches the bottom with a velocity of 96 feet per second and the sound of flash on the water reaches the top of the well in $3\frac{9}{70}$ secs from the time the stone starts Find the velocity of sound. [*Ans.* 1120 ft]

4. Calculate the length of a simple pendulum whose period of oscillation is exactly one sec at the sea level, $g=981$ Compare this length with that of a simple pendulum whose period is half second [*Ans* 24 83 cms, 4 . 1]

5 A pendulum beats seconds at a place where $g=980$ cms /sec ^{2} How would its length has to be changed so that it may beat seconds at a place where $g=685$ cm /sec^2? [Decrease by 29 87 cms]

6 If r_1 and r_2 be the radii of two planets and D_1 and D_2 their mean densities, show that value of acceleration due to gravity on the two planets will be in the ratio of $r_1 D_1$ to $r_2 D_2$

7 Calculate the mean density of earth from the following data —
$G=6\ 68\times 10^{-8}$ $C\ G\ S$ units, $g=980$, $R=6\ 4\times 10^3$ $K\ M$ [*Ans* 5·47 gms per c c]

8 A seconds pendulum loses 10 seconds in 24 hours, what changes should be made in the length of the pendulum so that it keeps correct time [Decrease by 023]

CHAPTER XI

PRESSURE OF LIQUIDS

§1. Properties of a liquid :—You have read about the pressure exerted by liquids and a few related properties of liquid in your previous classes A brief summary of these properties is given below.

Following are important properties related with the behaviour of liquids —

(*i*) Liquids have no definite shape They take up the shape of the container, though their volume remains constant

(*ii*) It can be divided in different parts

(*iii*) Whenever a body moves through a liquid it offers resistance to its motion

(*iv*) The surface of liquid behaves like a stretched rubber membrane, *i e* they show surface tension

(*v*) Liquids always take their own level

(*vi*) Liquids exert pressure

§2. Pressure exerted by a liquid column :—Liquids exert pressure on the sides of a vessel containing the liquid *This pressure acts*

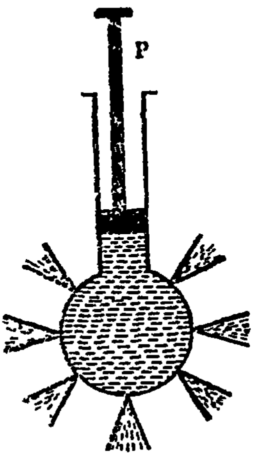


Fig 65

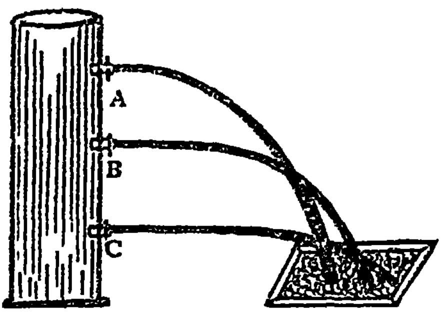


Fig 66

normally to the walls of the vessel and is equal in all directions. This can be demonstrated by taking a vessel as shown in Fig 65 When liquid is filled in it it comes out normally from all holes with some force This will be true so long as all the holes are at the same level. If we take a long cylinder having holes at different heights

(Fig 66) fill it with water we shall see that pressure of water coming out of different holes depends upon the height of liquid column above the hole

It shows that pressure of a liquid increases with depth.

These properties of a liquid can also be demonstrated by the following experiment Take a vessel containing water and in it place three tubes as shown in Fig. 67 When the mouth of the tubes is at *the same level*, the level of mercury in all tubes is the same It means pressure at the same horizontal level is the same Again, if we rotate the tubes so that their mouths point in different directions or put them at different points always keeping them at the same level, the level of mercury remains same It proves that liquids exert pressure at every point inside a liquid and this pressure acts equally in all directions

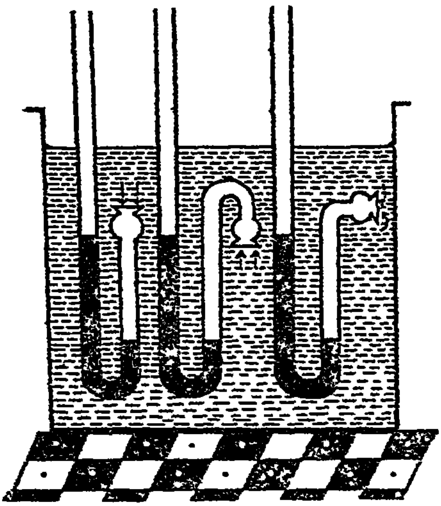


Fig 67

If we lower a tube we find that the height of mercury in it rises showing that pressure inside liquid rises with the height of liquid column above it

Thus we see that :—

(i) *Liqid exerts pressure at every point in its mass.*

(ii) *Pressure at any point acts equally in all directions*

(iii) *Pressure increases with the height of liquid column above it*

(iv) *Pressure is the same at same horizontal level in the same vessel or even in different vessels inter-connected*

§3. **Pressure exerted by a liquid column :—**Consider an imaginary cylinder of cross-section S sq cm containing a liquid column of height h cm

Volume of liquid in the vessel $= S\,h$ c c

∴ Mass of liquid contained in the vessel $= S\,h\,d$ grams

∴ Weight of the liquid *i e* force of attraction on it due to earth

$= S\,h\,d.g$ dynes

∴ Force acting on the bottom of the cylinder

$= S\,h\,D\,g$ dynes

∴ Pressure acting on the bottom$= \dfrac{\text{Force}}{\text{Area}}$

$$=\frac{Shdg}{S}$$

$$=Hdg.\ \text{dynes per sq cm}$$

Thus pressure exerted by a liquid column of height h and density d is given by hdg. dynes per sq cm

§ 4. **Transmission of pressure through a liquid column:—** If a pressure is exerted at any point in a liquid it would be transmitted equally in all directions and will act normally to the walls of the vessel

Consider a liquid contained in a vessel as shown in Fig 68 There are holes in the vessel carrying cylindrical tubes fitted with pistons, let the area of cross-section of three holes be S. $2S$ and $3S$ as shown in the figure Suppose we place a weight of F pounds on piston 1 Then, pressure exerted on the liquid is $P=\frac{F}{S}$ According to the above principle the same pressure will act on pistons 2 and 3 pushing them out If we wish to keep them fixed we shall have to apply the same pressure P in opposite direction Therefore, force needed to produce the same pressure on 2 is given by force=pressure×area

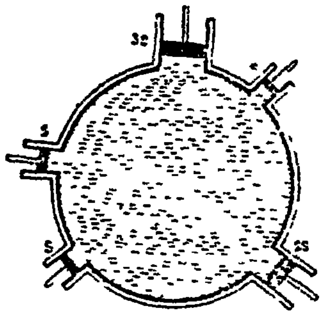


Fig 68

$$=P\times 2S=\frac{F}{S}\times 2S=2F$$

Similarly force on $3=P\times 3S=\frac{F}{S}\times\frac{3S}{1}=3F$

Thus, we see that force acting on 2 is $2F$ while on 3 is $3F$, *i e* a force of F pounds acting on 1 produces a force $2F$ pounds acting on 2 and $3F$ pounds on 3 proportional to their area of cross-section This is the principle of Bramah's Hydrolic Press (Pascal's principle)

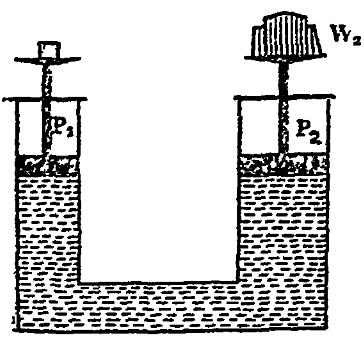


Fig 69.

The same principle can be proved by a vessel as shown in Fig 69 W_1 and W_2 are the weights placed on the piston P_1 and P_2 respectively so that the level of liquid is the same in both the limbs If S_1 and S_2 are the areas of cross-section of the two limbs, it can be shown that

$$\text{pressure}=\frac{W_1}{S_1}=\frac{W_2}{S_2}$$

$$\frac{W_1}{W_2}=\frac{S_1}{S_2}.$$

§5. **Bramah's Press:**—A simple sketch of the press is shown in Fig 70 A and B are two cylindrical vessels fitted with pistons P and Q The area of cross-section of B is larger than A P can be pressed down by means of a liver arrangement L Q moves through a strong frame S which carries a flat disc R at the top rigidly fixed to the frame Q also ends in a circular platform A and B are connected by a side tube D through a valve V which opens towards B. A is also connected to a tank T through a valve V which opens in A T is connected to D with a stop-cock C Tank T, A and B contain water Anything to be compressed is placed on the platform of Q.

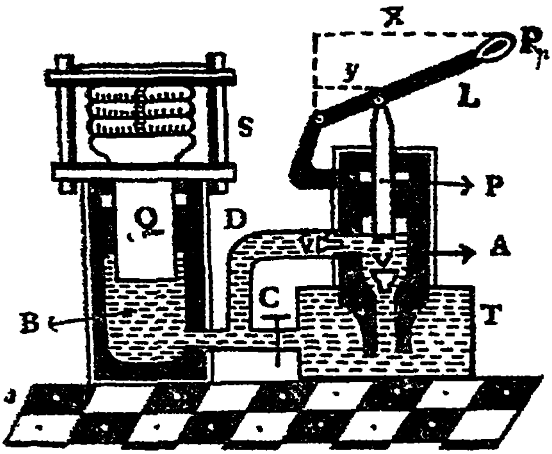


Fig 70.

Working:—When P_1 is pressed down, P moves downwards driving some water from A to B through V V (from T) is closed Q moves up When P_1 is moved up, P goes up V is closed. V (from T) opens and water from tank T enters A Again when P_1 is lowered down some water will be transferred to B and Q will rise up. In this way after every stroke Q rises up and the object between Q and R is compressed As the operations are repeated pressure on the object increases and it can be compressed to any extent When we want to release the pressure, open the stop-cock C, water from B will run into the tank T and Q will fall down Let the area of cross-section of P and Q be α and β and the distance of P_1 and P from fulcrum be X and Y Let F be the force acting on P_1 and F_2 on P then according to the principle of lever $F_1 x = F_2 y$

$$F_2 = \frac{X}{Y} F_1 \qquad ..(i)$$

Let F_3 be the force acting on the piston Q due to compression of the object, then,

Pressure exerted on $P = \frac{F_2}{\alpha}$

and pressure exerted on $Q = \frac{F_3}{\beta}$

Since, these two are equal, we have

$$\frac{F_3}{\beta} = \frac{F_2}{\alpha}$$

$$\therefore \qquad \frac{P_2}{P_2'} = \frac{\beta}{\alpha} \qquad\qquad P_2 = \frac{\beta}{\alpha} \cdot P_2' \qquad . \quad . \quad (ii)$$

$$= \frac{\beta}{\alpha} \cdot \frac{x}{y} \cdot P_1 \text{ [from } (i)\text{]}$$

$$\therefore \frac{P_2}{P_1} = \frac{\beta}{\alpha} \cdot \frac{x}{y}$$

This is mechanical advantage obtained by the press.

Let $\frac{x}{y}$ be 10 and $\frac{\beta}{\alpha}$ be 100, then a force of 10 lbs. exerted at P_1 will apply a force $P_2 = 10 \times 10 \times 100 = 10{,}000$ lbs. on the object.

QUESTIONS

1. Show that 'pressure exerted by a liquid depends on its height.' (*See* §2 and §3).

2. State Pascal's Principle and discuss its application in the construction of Bramah's Press. (*See* §4 and §5).

CHAPTER XII

ATMOSPHERIC PRESSURE AND ITS MEASUREMENT

§1. Atmosphere and its pressure:—Our earth is surrounded by an envelope of air on all sides It is a mixture of oxygen, nitrogen, hydrogen, carbon dioxide, water vapour and inert gases Nitrogen and oxygen form the major portion This envelope is known as atmosphere It extends up to 5 miles and in traces up to 200 miles and beyond The density of different layers of air goes on decreasing as we go up till it becomes negligible As you all know, air possesses weight and anything which possesses weight will exert some force when placed on other things, just as when we place a book on our hand we will feel its force If we place another book on it this force will increase, in the same way different layers of air exert force on the lower layers and the whole envelope exerts force on the surface of the earth *This force exerted per unit area of a surface is known as* **atmospheric pressure.** As we go up, the column of air above us will go on decreasing and consequently pressure also goes on decreasing. Imagine a cylinder of unit cross-section extending right up to the top of the atmosphere The force exerted per unit area due to the weight of air contained in this cylinder is known as atmospheric pressure This comes out to be 15 pounds per sq inch or 10^6 dynes per sq cm (approximately) The atmosphere is divided into various layers possessing characteristic properties These layers are shown in Fig. 71

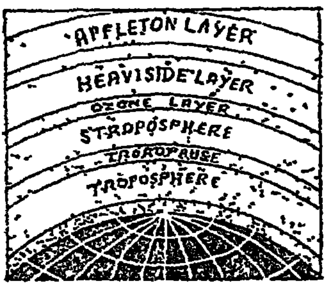


Fig 71

At any point this pressure of air acts equally in all directions.

§2. To demonstrate the presence of atmospheric pressure:—You must have read in your previous classes the various experiments which show the presence of atmospheric pressure A few have been enumerated here

(*i*) Take a glass and completely fill it with water Place a piece of cardboard or glass plate over its mouth. No air *bubble* should remain in it Gently turnover the glass as shown in Fig 72 It will be observed that even though water is exerting its weight on the cardboard, it does not fall down This can be explained if we suppose that atmospheric pressure pushes the cardboard in the upward direction

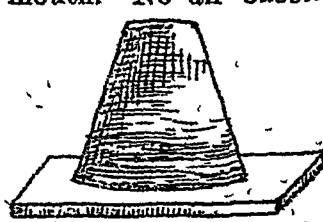


Fig 72

(*ii*) See Fig. 73 These are known as Magdeberg's spheres When they are fitted together they become

air-tight and can be separated by pulling apart so far as there is air inside them After fitting them together, exhaust the air with the help of a side tube and close the stop-cock Now it becomes difficult to separate them. This experiment was demonstrated by **Otto Von Guericke** before his emperor It required 6 horses on each side to separate them This can again be explained on the basis of atmospheric pressure So far as there is air inside, the pressure on both sides of the sphere is the same and is atmospheric pressure But, when they are exhausted, there is no pressure on the inner surface, while atmospheric pressure acts on the outer surface Therefore, a great pull is needed to separate them Similarly, the filling of fountain pen, working of kerosene pump or water pump, are all based on atmospheric pressure

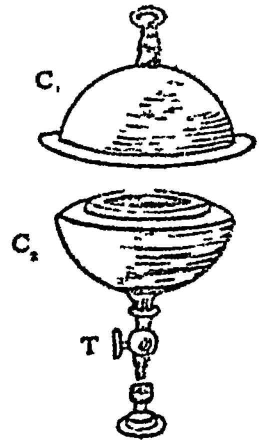


Fig 73

§3. We do not feel atmospheric pressure :— The pressure exerted by atmosphere is 15 lbs per square inch The average surface area of a man is 16 sq feet or 16 × 144 sq inch Therefore the total force exerted by air on a man's body is about **15 tons' weight**, but we do not feel it at all The reason is that there is air inside our body also and that air exerts the same pressure in opposite direction to atmospheric pressure Therefore, resultant pressure acting on our body is zero You must have read that whenever air is taken out of a barrel, it will be deformed on account of atmospheric pressure

§4. Measurement of atmospheric pressure :—Simple Barometer :— Measurement of atmospheric pressure has become a necessary part of our scientific times The instrument which is used for this purpose is known as barometer It was invented in 1643 for the first time by Torricelli, a pupil of Galileo

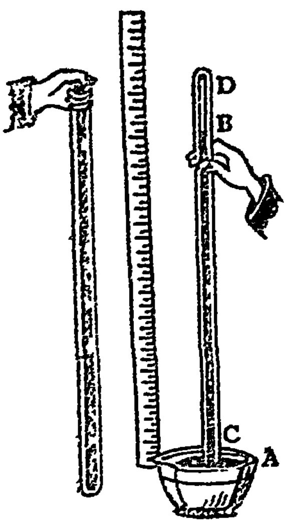


Fig 74.

Construction and working :— Take a thick glass tube about one metre long and 2 to 3 cms in diameter Fill it completely with mercury and close its mouth with your thumb Gently invert the tube in a trough of mercury and remove the thumb when the mouth of the tube is inside mercury (See Fig 74) When the thumb is removed, some mercury will come down and it will become steady at a certain height H According to the principle, liquid takes its own level, the level of mercury outside and inside should remain the same but it is not so, why ? When the tube is in vertical position let the height of mercury in the tube at *B* be *H* cm higher than the level of mercury in

trough at A. This column of mercury wants to run down on account of its weight, but it is supported in this position by atmospheric pressure acting on the level of mercury on trough at A. If we take a point C in the tube in level with A. we know that the pressure at A should be equal to pressure at C; otherwise the liquid will flow from high pressure to low pressure At A, atmospheric pressure is acting and at C, pressure due to mercury column is acting.

Therefore we have,

Atmospheric pressure P = pressure exerted by H cm of mercury column. ..(*i*)

Generally we say that pressure is 76 cms It means that atmospheric pressure is equal to the pressure exerted by a mercury column of height 76 cms What will be this pressure in absolute units, that is in dynes per sq cm ?

Let the radius of the tube be r cm Therefore its area of cross-section will be πr^2 sq cm The volume of mercury above C will be $\pi r^2 H$ c. c. Its mass will be $\pi r^2 H d$ gram Therefore its force (weight) exerted on the mercury level at C will be $\pi r^2 H d g$ dynes where g is acceleration due to gravity (force=mass $\times$ acceleration). This force acts on πr^2 sq cm Therefore, pressure P *i e*. force per unit area is given by

$$P = \frac{\pi r^2 H d g}{\pi r^2} = H d.g \text{ dynes per sq cm}$$

$\therefore$ Atmospheric pressure P in absolute units is given by $P = H d g$ dynes per sq cm .. (*ii*)

In the above example when H is 76 cms and d is 13 6 per c.c.,

Atmospheric pressure $P = 76 \times 13\ 6 \times 980$ dynes per sq cm
$= 1\ 03 \times 10^6$ dynes per sq. cm

76 cms is known as normal pressure It is observed at sea level We can calculate the pressure in absolute units for any other height in a similar manner

The space above mercury in the barometer tube is vacuum and is known as Torricelli vacuum.

§5. Following things affect a simple barometer:—(*i*) If the tube of the barometer is tilted, mercury will rise in the tube but its vertical height will remain the same (*see* Fig 75)

(*ii*) If a few drops of water or any other liquid are introduced in the space above mercury, the liquid will at once vaporise on account of low pressure and the height of mercury will fall down. Like air, vapours of different liquids also exert pressure and, therefore, now the pressure P of the atmosphere is equal to the pressure exerted due to mercury plus the pressure exerted by the vapours If H is the height of mercury when there is no vapour and h is the height of mercury after introducing the vapour, we have $P=H$, when there is no vapour and

$$P = h + p \text{ when there is vapour}$$

p is the pressure exerted by the vapours

$$\therefore \quad p = P - h = H - h \text{ cm. of mercury.}$$

For example, if, the height of mercury in the first case is 76 cms and after introducing the vapour it is 70 cms, then pressure exerted by the vapour is equal to 76—70 cm *i e*, 6 cms of mercury Such a barometer having a partial vacuum is known as a faulty barometer

(*iii*) If a hole is made at the top of the tube, the whole of mercury will fall down and the level inside and outside will become the same

(*iv*) If a hole is made in the middle of the tube, air will pass up and level will fall down

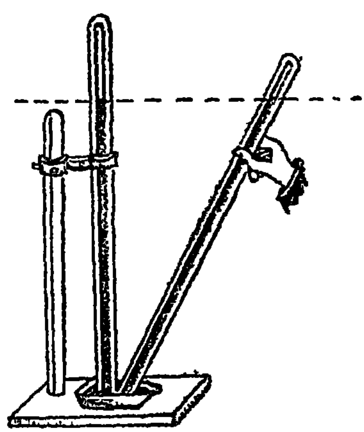


Fig 75

(*v*) If we take the barometer on a mountain or in an aeroplane, its height will fall as the pressure decreases This fall in pressure is generally utilised for measuring heights It is found that a fall of 1 inch takes place when we ascend a height of 990 feet approx This will hold good for ordinary heights These instruments used for measuring heights are known as altimeters

(*vi*) If we take the barometer inside a mine, its height will increase due to increase in pressure

(*vii*) If instead of mercury any other liquid is filled in barometer, its height will be different Let H_1 be the height of a liquid barometer of density d_1 and H_2 be the height of mercury barometer of density d_2 The atmospheric pressure P is given by

$$P=H_1d_1g$$

and

$$P=H_2d_2g$$

$$H_1d_1g=H_2d_2g$$

$$H_1=\frac{H_2d_2}{d_1} \qquad (iii)$$

From this we can calculate the height of any liquid barometer

Height of water barometer —

Suppose $H_2=76,\ d_2=13\ 6 \quad d_1=1$

$$H_1=\frac{76\times 13\ 6}{1}\text{ cm}$$

$$=\frac{76\times 13\ 6}{2\ 54\times 12}\text{ feet}=34\text{ feet approximately.}$$

Generally we take the height of water barometer as 34 feet It is 13 6 times the height of mercury barometer

§ 6. **Uses of barometer** (*i*) **To find out height :—** As explained above

(*ii*) **To forecast weather —** These days weather forecasts have become more important In that we should also know the atmospheric pressure Sudden fall of pressure indicates bad and stormy weather and it may be followed by rains If pressure increases it indicates clear and dry weather

§ 7. Fortin's Barometer:—Simple barometer as explained in § 4 *suffers from two* defects —(*i*) There is no permanent fixed scale for taking the reading of height, and

(*ii*) The level of mercury in the trough also changes which makes it difficult to fix a scale These two defects are overcome in Fortin's barometer.

Construction:—*See* Figs 76 and 77 It is an improved form of a simple barometer The glass tube filled with mercury is inverted over a trough of mercury. The glass tube is enclosed in a brass casing A long window *W* is cut in the brass tube in the upper part where the level of mercury stands in the glass tube A scale in inches is graduated on the one side of the window and in cm on the other side A vernier is fitted in the window and can be worked with a screw The mercury trough is also enclosed in a casing *MNOQ* and carries a leather bag at its bottom A screw *S* presses against the bottom of a leather bag. By moving the screw the bag can be raised or lowered and the level of mercury in the trough can be raised or lowered An ivory pointer *P* is fixed in such a way that the pointer lies at the zero of the main scale The whole thing is again enclosed in a glass casing.

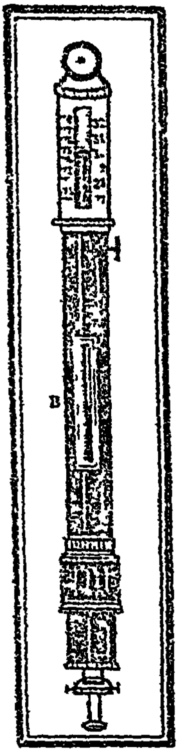
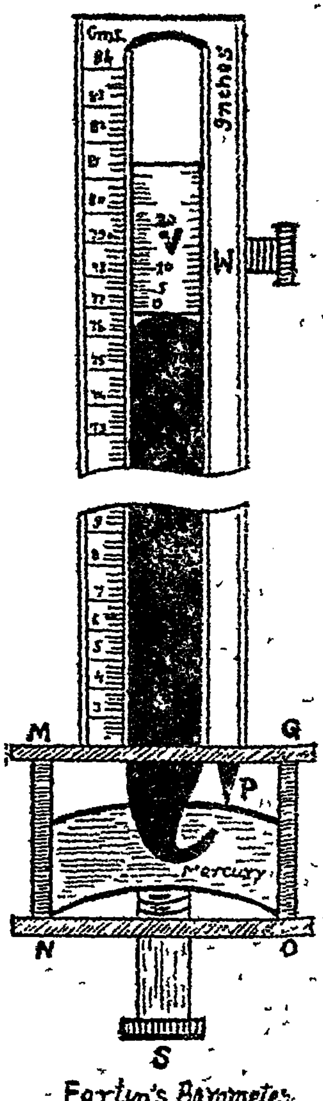


Fig 76



Fortin's Barometer

Fig. —

Working:—Level the barometer Move the screw *S* in such a way that mercury level in the trough touches the pointer. The image of the pointer will appear to touch the pointer. Move the vernier up and down till its lower edge coincides with the upper meniscus of the mercury Take the reading of mercury level in the tube

Correction of barometer readings:—There are a few errors in the reading so taken on account of the expansion in the scale and change in the density of mercury due to changes in temperatures These would be considered in the Chapter on Expansion in Heat

Defects of Fortin's Barometer:—This barometer is bulky and is not portable It needs vertical setting and, therefore, it is not convenient for taking it at different places or in aeroplanes for measuring height Hence, Aneroid barometer is used for this purpose But wherever accuracy is the main concern, Fortin's barometer is to be used

§8. Aneroid Barometer —No liquid is used in this and, therefore, it is known as aneroid barometer

Construction —(*See* Figs 78 and 79) It consists of a cylindrical chest *C* which is covered by a corrugated diaphragm *D* A pointer

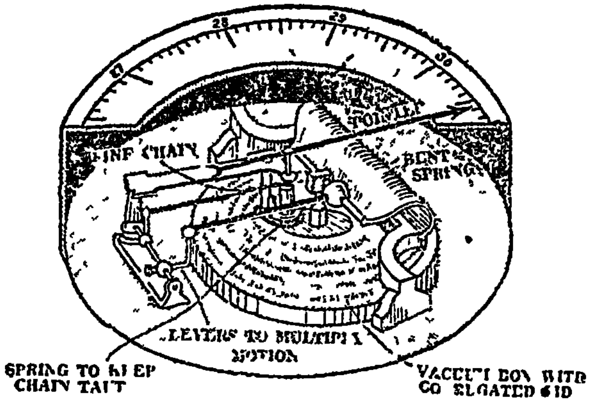


Fig 78

P is attached by means of a lever arrangement *L* Pointer *P* moves on a scale *S* graduated in inches of mercury to read pressure directly

Working —When atmospheric pressure changes, the corrugated lid moves accordingly These movements are magnified by the lever

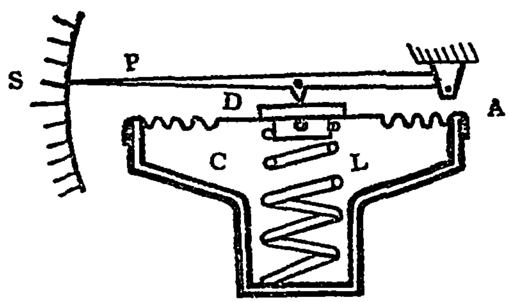


Fig 79

arrangement and finally pointer *P* moves along the scale The readings of the pointer give the pressure in inches directly

This instrument is light and portable and is commonly used Its readings are not very accurate

QUESTIONS

1 What is atmosphere ? What do you mean by atmospheric pressure being 76 cms of mercury at sea level ? Explain (*See* §1 and §4)

2 How is a barometer affected if

(*i*) a hole is made at its top

(*ii*) it is inclined

(*iii*) a hole is made in the centre

(*iv*) it is carried up in the air

(*v*) there is some vapour in the upper part of the tube

(*vi*) another liquid is used. (*See* § 5)

3 Explaining the defects of a simple barometer describe the construction and working of a Fortin's Barometer. Discuss its advantages and disadvantages. (*See* § 7)

4 Write a note on Aneroid barometer. How can it be used for measurement of height or forecast of weather (*See* § 8 and § 6).

$$P \propto \frac{1}{V}$$

$$P = \frac{K}{V}$$

Here K is a constant of proportionality.

$$\therefore \qquad PV = K$$

i.e. $$P_1V_1 = P_2V_2 = P_3V_3 = K$$

where P_1, P_2, P_3 are the pressures and V_1, V_2 and V_3 are its corresponding volumes. Thus, according to this law—*The product of pressure and volume of a certain mass of gas remains constant, so far as temperature is constant.*

For example consider m gram of gas at a certain temperature $t°C$ and let its pressure be P and volume V. The product of pressure and volume will be $P \cdot V$. Suppose now pressure is increased to $2P$, then its volume will become $V/2$ and again the product will be $2P \times \frac{V}{2} = PV$. In this way, product will always be the same.

If suppose at $t°C$ instead of m grams we take $2m$ grams of gas, its product of pressure and volume will be $2PV$ and so on. Thus, the product of pressure and volume is constant and depends upon mass of the gas.

§3. **Another form of Boyle's law :**—According to Boyle's law

$$PV = K$$

Let the mass of gas be m gram

Then, $$V = \frac{m}{d}$$

where d is the density of the gas

$$\therefore \qquad P\frac{m}{d}=K$$

or
$$\frac{P}{d}=\frac{K}{m}=K_1$$

Therefore for a given mass of a gas the ratio of pressure and its density remains constant at a constant temperature

84. Experimental verification—Apparatus and its description :— The Boyle's law apparatus is shown in fig 80 AB is a fixed tube of glass closed at one end. EF is an open tube which can be raised or lowered. AB and EF are connected by means of a rubber tubing. Some amount of gas is enclosed in AB and rest of AB and rubber tubing and part of EF is filled with dry mercury A scale is fixed on the stand on which we can read the levels of mercury in AB and EF for any position of EF.

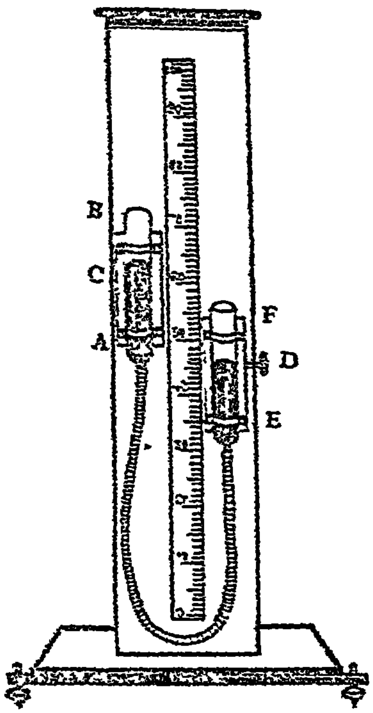


Fig. 80

Principle:—We measure the volume of the enclosed gas under different pressures and find out the product of pressure and volume If it comes out to be constant the law is verified. (*See* a 'Text Book of Practical Physics' by authors)

Working:—1 Find out the atmospheric pressure H by a barometer

2 Level the apparatus with the help of a spirit level.

3 Fix up the reservoir EF in some position; suppose the level in EF is lower than the level in AB. Take the reading of mercury levels in EF and AB. In this position the pressure of the enclosed gas P will be equal to $H-d$ where d is the difference in the levels of mercury in the two tubes The volume of the gas V will be $S\times l$ where S is the area of cross-section of AB and l is the length of air column in AB. l can be obtained by substracting the reading of B from the reading of mercury level at C.

4 Now raise the reservoir by 5 cms or so and again find out P and so on When the level of mercury in EF is higher than in AB pressure of the enclosed gas will be $H+d$.

5 Find out the product of P and the corresponding lengths l, *i.e* P_1l_1, P_2l_2 and so on If this comes out to be a constant the law is verified because.

If $\quad P_1l_1=P_2l_2$

$\therefore \quad P_1l_1S=P_2l_2S \quad$ or $\quad P_1V_1=P_2V_2$

A graph between P and $\frac{1}{l}$ will be a straight line, (Fig 81)

A graph between P and V is a rectangular hyperbola (Fig 81-A)

Numerical problems:—1 *The volume of air bubble increases 10 times in rising from the bottom of the lake to its surface If the height of barometer is 30 inches and if the temperature of air bubble remains constant, what is the depth of the lake? (Sp gr of mercury is 13 6)*

In such problems where certain amount of air is transferred from the bottom of a lake to the surface, the pressure on the air

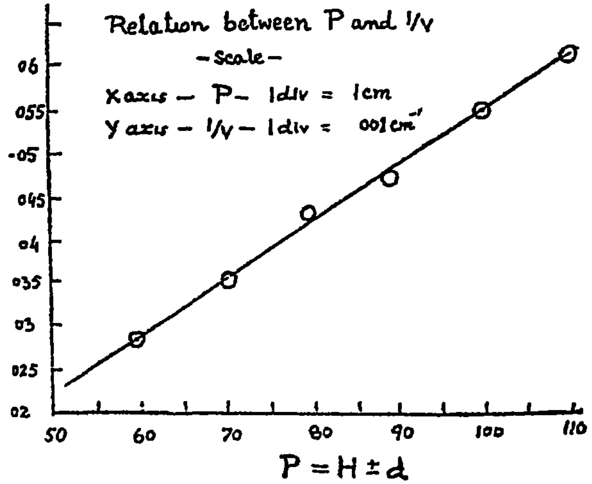


Fig 81

changes and therefore its volume also changes and we can apply Boyle's law When the bubble is on the surface the pressure acting

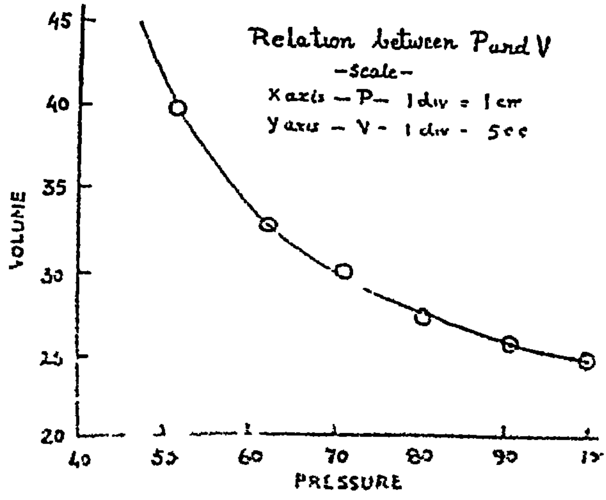


Fig 81-A.

on it is atmospheric pressure When it is at the bottom, the pressure acting on it is atmospheric pressure plus the pressure acting due to

a water column of height h Since we have to add these two pressures, they should be expressed in the same terms Atmospheric pressure in terms of water column is equal to 34 feet If the volume of air at the bottom is V c c, at the top it will be 10 V c c

Let the depth of the lake be h feet,

Then, applying Boyle's law we get,

$$34 \times 10V = (34+h)V$$

$$h = 340 - 34 = 306 \text{ feet}$$

2 *A cylindrical diving bell 14 feet high is lowered to the bottom of a lake (a) if the water rises 10 feet inside the bell how deep is the lake at that point (b) what minimum pressure would the compression pump have to produce to force water entirely out of the bell?*

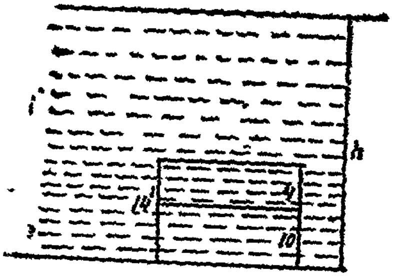


Fig 82

Height of water barometer=34 ft. Let the pressure and volume at the top be P_1 and V_1 and at the given depth P_2 and V_2 respectively

Here, $P_1 = 34$ ft $P_2 = (34+h-10)$

$V_1 = 14 \times S$, $V_2 = 4 \times S$

∴ Applying Boyle's law,

$$P_1V_1 = P_2V_2$$

$$34 \times 14 \times S = (34+h-10) . 4 \ S$$

or

$$476 = (24+h)4$$

or

$$24+h = \frac{476}{4} = 119$$

$$h = 119 - 24 = 95 \text{ feet}$$

Pressure required to throw out the water is equal to 95 ft. of water column

Numerical problem on Faulti barometer:—3 *Some air is introduced into the space above mercury in a barometer tube The height of mercury column is 29 inches and the space above is 4 inches. The tube is pushed down into the cistern, so that the space is reduced to 2 inches, the height of mercury becomes 28 inches What will be the height of mercury in air free barometer.*

When some air is introduced in the barometer its height falls down on account of the pressure exerted by the air In such a case,

Atmospheric pressure=Pressure of the enclosed air+the height of mercury

Pressure of the enclosed gas=Atmospheric pressure−the height of mercury.

Let p_1 and p_2 be the pressure of the gas in two cases, respectively

$p_1 = H - 29$ } H is atmospheric
$p_2 = H - 28$ } pressure.

Let V_1 and V_2 be the volume of the enclosed gas in the two cases. let area of cross-section be S

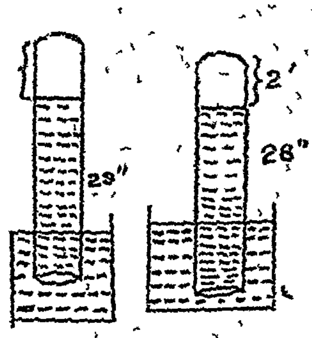


Fig 83

$$V_1 = 4 \times S \text{ cu inches}$$
$$V_2 = 2 \times S \text{ cu inches}$$

Applying Boyle's law we get,

$$P_1V_1 = P_2V_2$$
$$(H-29)(4S) = (H-28)\ 2\ S$$

or $$(H-29)2 = H-28$$

or $$2H-58 = H-28$$

$\therefore$ $$H = 30 \text{ inches}$$

QUESTIONS

1 State and explain Boyle's law (*See* §1)

2 How will you verify Boyle's law experimentally ? (*See* §2)

3 What are the important provisions of the law ? (*See* §1)

Numerical questions —

1 A cylinder filled with air at atmospheric pressure is lowered in water with its mouth downwards till it is 1/3 full of water To what further depth should it be lowered till it is 2/3 full of water Density of *Hg* is 13 6 and barometric height is 76 cm [*Ans* 15 504 metres]

2 On introducing a bubble of air of volume 3 c c the mercury in a barometer which orginally stood at 76 cms fell by 12 cms Find the space above mercury in air-free barometer, assuming the cross-section of the tube to be unity [*Ans* 7 cms]

3 At what depth in water will a bubble of air have the same density as water, given that water is in compressible and that air obeys Boyle's law for all pressures Density of air at ordinary atmospheric pressure is 1 25 gm litre [*Ans* 8258 46 metres]

4 The space above mercury in a faulti barometer measures 10 cms and the mercury column extends to 70 cms above the mercury in the cistern on depressing the tube into the cistern the mercury stands at 68 cms and the space above mercury measures 7 5 cms find the true atmospheric pressure (*Delhi 1948*) [*Ans* 76 cms]

5 Two chambers containing m_1 and m_2 gms of a gas at pressure P_1 and P_2 respectively are put into communication what will be the pressure of the mixture ?

$$\left[Ans\ \frac{P_1m_1d_2+P_2m_2d_1}{m_1d_2+m_2d_1}\right]$$

6 At what depth in a lake will a bubble of air have one half the volume it will have on reaching the surface ? [*Ans* 10 336 metres]

7 Calculate the ratio in which the volume of an air bubble increases when it rises to the surface of the sea from a depth of 2 Km given that the density of sea water is 1 05 and the atmospheric pressure is 10^6 dynes cm $^{-2}$ [*Ans* 205 8 · 1]

8 The space above *Hg* column in a barometer tube contains some air The *Hg* column is 28 40 inches long and the space above it is 3 05 inches long The tube is then pushed downwards into mercury so that the column is 28 14 inches long while the air space is 2 34 inches What is true height of the barometer ? [*Ans* 29 26 cms]

9. A narrow tube with uniform bore is closed at one end and at the other end is a thread of mercury of 8 cms length The tube is held vertical with the open end (*i*) up (*ii*) down The lengths of air column enclosed in the tube in the two cases are 34 and 42 cms respectively Calculate the barometric height [*Ans* 76 cms]

CHAPTER XIV

PUMPS AND SIPHON

§1. **Importance of pumps in life** :—Pumps play an important part in our daily life Right from a fountain pen to taking out water from deep wells, we use pumps Just as pumps are used for moving water from one vessel to another or from one height to another in the same way pumps are also used for producing vacuum. Many of the scientific discoveries and inventions would have remained obscure but for the production of high vacuum

§2. **Kinds of pumps**:—Pumps are divided into different categories depending upon their working. There are lift pumps, exhaust pumps or, compression pumps etc.

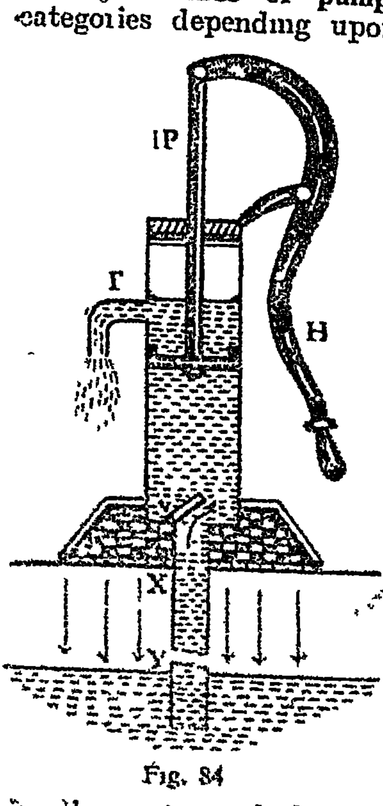


Fig. 84

Lift Pumps—Principle of Working :—You have already read that on account of atmospheric pressure, water can rise to a height of 34 feet in a tube if there is vacuum in it This is the principle of lift or suction pumps.

(*a*) **Water Pump—Construction** :—*See* Fig. 84 It consists of a cylinder fitted with a piston P which can be raised or lowered by means of a handle H A long tube XY is attached to the cylinder. The length of the tube is such that it reaches water level but it should not be greater than 30 feet for efficient working Theoretically, it can be up to 34 feet. If it is longer than 34 feet, water will not rise in the cylinder. This tube carries a wire gauge Z at the lower end At the upper end of the tube a valve V_1 is fixed which can open only in the cylinder Another valve V_2 is provided in the piston which also opens only in the upward direction.

Working:—(*See Fig 85*). Place the pump in such a way that the tube XY dips in water Suppose the position of the piston is at the lowest position of the cylinder Now raise the piston upwards valve V_2 will be closed due to atmospheric pressure and air from the cylinder will be thrown out Vacuum will be created between V_1 and V_2 and air from XY will come in the cylinder and some water will rise in XY Again when we lower down the piston V_1 will be closed due to pressure of enclosed air and V_2 will open and air from V_1 V_2 will be passed on Again when we move the piston in upward direction, the air above the piston will be thrown out and fresh air from XY will come in In this way in two or three operations good vacuum will be created and water will rise in the cylinder and like air it will also be thrown out of T.

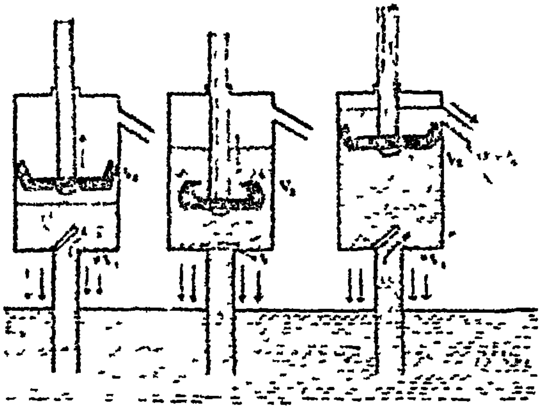


Fig 85

This pump can work only when water level is not below 30 feet Again, the outflow of water is not continuous Water will come out only during the upstroke

(*b*) **Force pump :**—In order to overcome the above defect we use a force pump It is shown in Fig 86 Its construction is the same as that of water pump described above except for the following differences (*i*) A side tube UW is attached to the cylinder instead of tube T at the top of the cylinder (*ii*) Value V_2 is provided inside tube instead of piston which also opens in the tube Piston is made air tight Cylinder C is fitted inside the well at a height lower than 30 feet from water level while the side tube UW comes to the top of the well

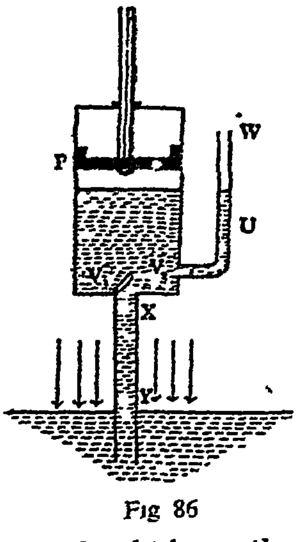


Fig 86

Working :—When the piston is raised up, the air of the cylinder passes out and air from XY enters in When it is lowered down V is closed and V_2 opens and air is thrown out through UW After

a good vacuum water comes up in the cylinder and is forced up the tube when the piston goes down. In this case also the flow is discontinuous. In order to make it continuous, a reservoir (R) or gas chamber is fitted in the side tube UW as shown in Fig 87. When the piston is raised or lowered rapidly, water collects in R and presses the air contained in it. When the piston is going down, the compressed air in R presses the water in GH. In this way water is forced up in the tube when the piston is going down as well as when it is going up.

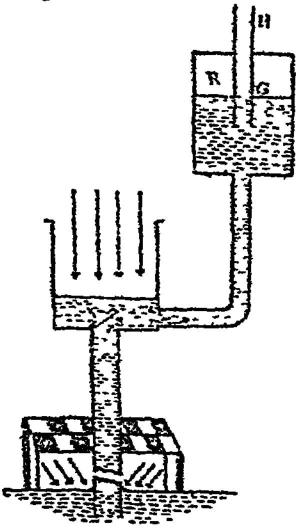


Fig 87

§ 3. **Some other kinds of pumps:**
(*i*) **Kerosene pump**:—You must have read about kerosene pump: its working is similar to water pump.

(*ii*) **Fountain pen**:—You are quite familiar with your fountain pen. When it is dipped inside ink and the spring S is raised it presses the rubber tubing and throws out the air from it. When S is lowered again rubber tube again

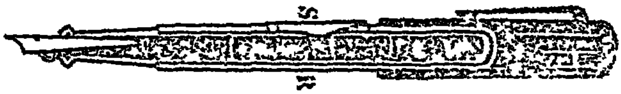


Fig 88

opens and vacuum is created in it. Ink due to atmospheric pressure rises in it. In this way it can be filled (Fig 88).

(*iii*) **Cycle pump**:—Its working can be understood by referring to Fig 89. When piston P moves up, V_1 is closed, V_2 opens and air from atmosphere enters the cylinder between V_1 and V_2. When the piston is lowered V_2 is closed due to pressure of enclosed gas and V_1 opens and the air is forced in the cycle tube. In this case V_1 is provided inside the cycle tube and not in the pump.

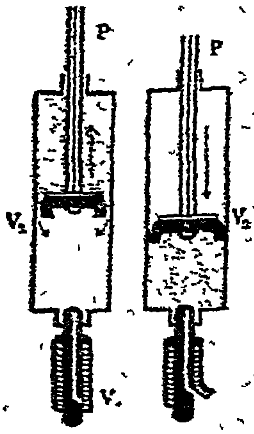


Fig 89

(*iv*) **Football pump**:—Its construction and working is similar to cycle pump except that V_1 is provided in the nozzle of the pump.

§4. **Vacuum pumps or exhaust pumps**:—For the first time in 1650 Otto van Guericke constructed a vacuum pump. His pump is shown in Fig 90. R is a reservoir to be evacuated. This is connected by means of a tube to the pump P. It is evident from the figure that the construction of the pump is similar to the water pump. When the position is moved up V_2 is closed and air from R enters into

the cylinder When piston comes down V_1 is closed and V_2 opens due to pressure of enclosed air. This air comes up in the cylinder above

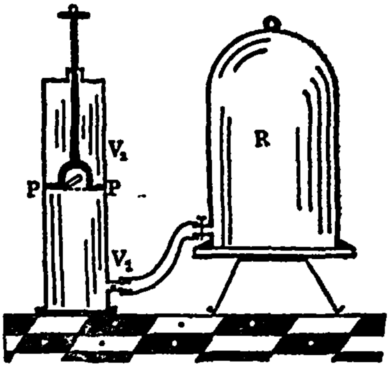


Fig 90

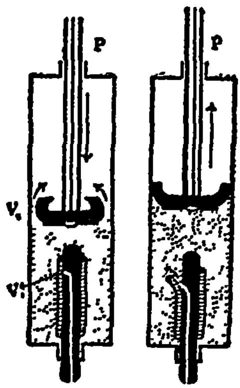


Fig 91

the piston See Fig 91 Again when the piston moves up this air is thrown out and some more air moves in from R Thus, after working the pump a number of times, good vacuum will be produced in R When the pressure inside R becomes so low that it is unable to lift the valve V_1, air from R will not enter into the cylinder and any further evacuation will not be possible

§5. **Pressure in the Reservoir after a few strokes** —Let V be the volume of a vessel and the tube up to valve V_1 and v be the volume of cylinder enclosed between the two extreme positions of piston When the piston goes up and down, every time v c c of air is thrown out

In the beginning, when the piston is in the lowest position, the volume of air in R is V c c and its pressure is P (atmospheric pressure) When the piston is moved up, let the pressure fall to P_1 and volume becomes $V+v$

According to Boyle's law, we know that the product of pressure and volume is constant *i e* $P_1V_1=P_2V_2$ Applying this in the case,

$$PV=P_1(V+v)$$

or

$$P_1=\left(\frac{V}{V+v}\right)P \qquad \ldots \qquad (i)$$

when the piston comes down, V_1 is closed and v c c of air passes up the piston and now the volume of air in R is again V c c but pressure is P_1, again when the piston goes up this volume becomes $V+v$ c c and pressure falls to P_2 Applying Boyle's law

$$VP_1=(V+v)P_2$$

or

$$P_2=\left(\frac{V}{V+v}\right)P_1$$

Substituting the value of P_1 from (i) we get,

$$P=\left(\frac{V}{V+v}\right)\left(\frac{V}{V+v}\right)P$$

$$=\left(\frac{V}{V+v}\right)^2 P \qquad \ldots \qquad \ldots \qquad (ii)$$

Similarly, pressure after n strokes will be given by

$$P_n=\left(\frac{V}{V+v}\right)^n P \qquad \dots \qquad \dots \ (iii)$$

From (iii) it is clear that whatever may be the value of n, P_n can never be zero. We know that the density of a gas is directly proportional to pressure, therefore, if ρ_0 is the density of the gas in the beginning and ρ_n after n strokes, we get,

$$\rho_n=\left(\frac{V}{V+v}\right)^n \rho_0 \qquad \dots \qquad \dots \ (iv)$$

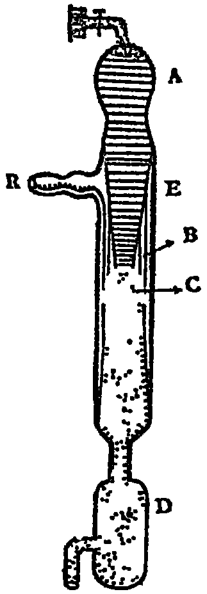


Fig 92

From (iii) and (iv) we can calculate the pressure and density of a gas after a given number of strokes.

§6. **Filter pump:**—This pump produces moderately low pressure but it is more convenient to work and is speedy. It is generally used for boiling a liquid under reduced pressure or filtering a liquid under reduced pressure.

Construction:—(Fig. 92) Water is flown in tube A. B is the nozzle at the other end of the tube. The velocity of water increases at B due to its being narrow. Water from this tube falls in another funnel-shaped tube C. B and C are enclosed in another big tube E. This tube is also connected to the reservoir R to be evacuated. On account of the force of water current, air contained in E is dragged along with the current. Fresh air from R comes in E and is in a similar way thrown out. In this way, pressure in R will fall.

§7. **Toepler pump:**—For producing good vacuum we use Toepler's pump. This pump was constructed by Toepler in 1862. It is shown in Fig 93. A is a big glass tube connected to other glass tubes, B, C, E and F. The length of B should be greater than barometer height, so that in the final stage mercury from B may not enter A. The length of tube C should also be greater than barometric height, so that when R is lowered, mercury in C may not clog the way to F. E is a bye-pass to air from V and F to A. The size of A is large so that in every operation larger volume of air is thrown out. A is also connected by means of a long rubber tubing to a reservoir D. V is one way valve and P is the vessel to be evacuated.

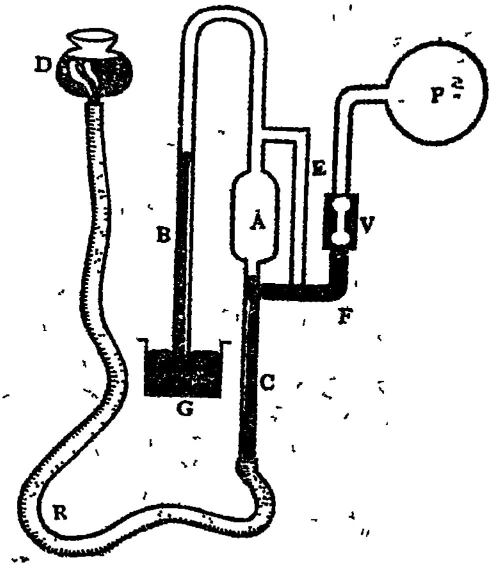


Fig 93.

Mercury is put in G and in the reservoir D and part of the tube C

Working:—Raise the reservoir D slowly. Mercury from C would enter in A and would move towards P, but when it reaches the valve, it will be closed due to pressure of mercury Mercury in A will go on rising and will throw out the air from A and E through B Raise D, till mercury reaches up to the top Lower D Again vacuum will be produced in A Air from outside cannot enter from B because of mercury in G. When mercury in F falls down, the valve will open due to its own weight and air from the vessel P will rush into A, F, and E After lowering the mercury up to F, raise the reservoir again and the air from A, F and E will be forced out through B In this way, after every operation part of the air would be thrown out and pressure in P will fall. In final stage, mercury in B and C would stand at the atmospheric height The pressure in V can be reduced to 10^{-3} mm of mercury

§8. **Rotary pump:**—The pump is shown in (Fig 94) It consists of the following parts.—

1 C_1 and C_2 are two metallic cylinders mounted along the axis of C_2 C_1 is mounted eccentrically such that it presses against C_2 along a line C_2 is stator *i e* it remains at rest C_1 is rotor *i e* it is rotated about the axis 0 The point of contact G moves along the circumference of C_2 with the rotation of C_1

2 C is a partition plate which keeps on pressing on C_1 due to a spring S This partition and the point of contact G, divides the space between two cylinders into two air-tight compartments (V_1 and V_2)

3 I and O are the two openings by the side of the partition C I is connected to the vessel to be exhausted and is called inlet, O is the outlet through which air is thrown out O carries a valve in it which opens in outward direction This valve allows the air to pass out but does not allow it to come back

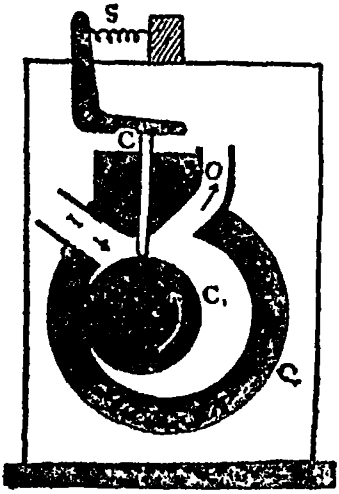


Fig 94

4 The whole thing is immersed in oil contained in a cast iron cylinder which also serves the purpose of a valve

Working:—Its working can be understood by referring to the Fig 95 *a, b, c* and *d* The vessel is connected to I Suppose G, is just at C The whole space is in contact with the vessel Fig 95(a) Now C, is rotated in anti-clockwise direction When it crosses I, the space between G_2 and C is cut out from the space between G_2 and O, as C moves on, the air in space V_2 is compressed and passes through O, while vacuum is created in V_1 and air from the vessel comes in Finally, Fig 95(d) V_1 becomes very large and V_2 very small Again, when

G crosses outlet and comes in the position G_1, the whole of the air which was present between the two cylinders has been thrown out and fresh amount of air from the vessel again fills the same space. Again, in second rotation this amount of air is thrown out.

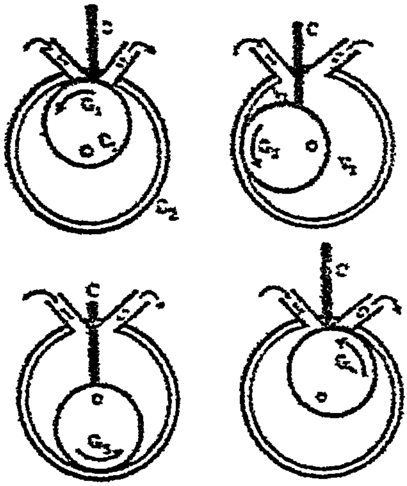


Fig. 95.

In this way, after a number of rotations, high vacuum is created in the vessel. The pressure can be reduced to 10^{-3} mm of mercury.

§ 9. **Siphon:**—When it is difficult to transfer liquid from one vessel to another we can use a device known as siphon.

Construction and working:—It is a tube bent twice at right angles or once at any angle as shown in Fig 96. Arm CE is longer than AB. AB should be less than barometric height. X is a vessel containing the liquid and it is to be transferred to Y. X should be slightly at higher level than Y. Fill up the tube by

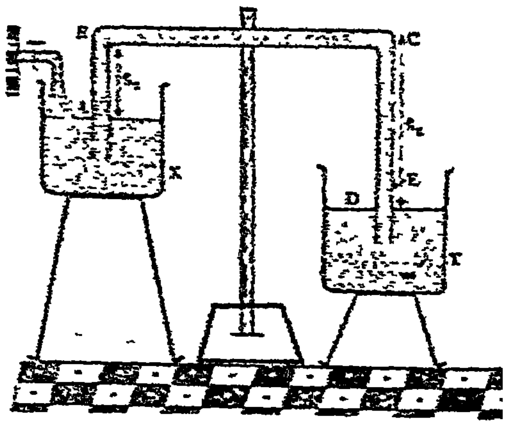


Fig. 96.

sucking in the liquid and then place it as shown in the figure. Liquid will flow from X to Y.

Principle:—Let the height of the tube above liquid levels in AB and EC be h_1 and h_2.

Let the atmospheric pressure acting on the liquid surfaces be H and let the pressure at B and C be P_B and P_C. Since, pressure on the liquid surface in X is H, the pressure at B is less than H by $h_1 d.g$.

$\therefore$ $$P_B = H - h_1 d\, g$$

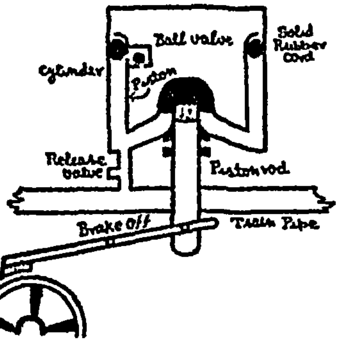
Similarly $$P_C = H - h_2 d\, g.$$

Therefore, $$P_B - P_C = (h_2 - h_1)\, d.\, g.$$

Because, h_2 is greater than h_1, P_B will be greater than P_C. Therefore, liquid will flow from B to C. And due to atmospheric pressure, liquid from X will rise in the tube AB. In this way, liquid will go on flowing.

If, h_1 is greater than barometric height, liquid will not rise up to B and the flow will stop. If, h_1 is greater than h_2, $P_B < P_C$ and liquid will not flow from B to C.

§ 10. **Siphon is also used in automatic flush system:**—When water in the vessel rises upto the level of BC the whole of the tube is filled with water. As explained above water from the vessel



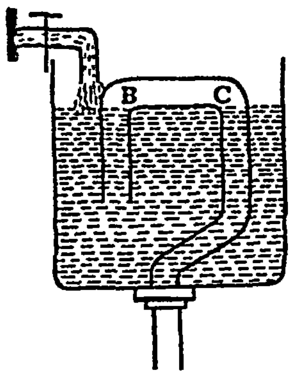


Fig 97

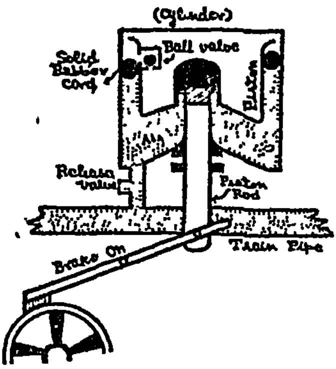


Fig. 98.

begins to run down till the whole of it is emptied. Again it will start filling up from the tap (*See* Fig 97)

§ 11. **Importance of vacuum in industry:**—Production of vacuum is of great importance in these days. Many of the discoveries and progress in science has been possible only an account of our capacity to produce high vacuum. The discovery of electron and other particles of the atom is directly related with discharge of electricity through gases at low pressure. Many of the most important things of our day are wireless telegraphy, telephony, radio, and many other such electronic devices. These have been possible only on account of highly evacuated valves. Everybody is familiar with the chain used for stopping the railway trains. This is possible on account of vacuum brakes (*See* Fig 98)

Vacuum Brake:—(*See* Fig 98) This is an arrangement by which brakes can be applied to every wheel of a railway train Each carriage is provided with iron pipe called the train pipe joined together by flexible and air-tight couplings A brake cylinder fitted with a piston is provided under every wheel and this is connected to train pipe The piston rod is connected by means of a system of levers to the brake shoes Between the inner wall of the brake cylinder and the outer wall of the piston there is a round rubber ring which forms an air-tight joint when piston moves up and down There is also a ball valve beneath the rubber ring fitted to the wall of the piston This valve permits air above the piston to be removed but prevents air from entering the space above the piston. So far as vacuum is maintained in the train pipe brakes are off but when vacuum is destroyed air rushes in and the piston moves up and brakes are applied

QUESTIONS

1 What is the principle of a pump ? Describe the construction and working of a lift pump Why can't it raise water above 34 ft Which pump is used for this purpose ? Explain its working. (*See* § 2A and 2B)

2 What is a vacuum pump ? Deduce a formula for the vacuum produced after a few strokes. (*See* § 4).

3 Describe the construction and working of a Toepler pump (*See* § 7).

4 Describe the construction and working of high Vacuum Rotatory pump. (*See* § 8)

5 Describe the principle and working of siphon (*See* § 9).

6 Describe the working of automatic flush siphon (*See* § 10).

7. Discuss the importance of vacuum in industry. (*See* § 10)

Whenever a force is applied on a body either it may move as a whole without any change between the molecular distance or the body may remain fixed but a change may take place between its relative parts. It may change in length, or in volume or it may change its shape. In such cases, we say that the body is deformed or its configuration is changed or it is strained. Those bodies which do not undergo this kind of change, whatever may be the force applied, are known as rigid bodies. As soon as the relative displacement takes place, new forces are set up between various particles which oppose or resist the deformation. They act in opposite direction to external force when the external force is removed, the body will gain its original form or configuration on account of these internal forces. These forces may also be called as forces of restitution

62 Elasticity:—*The property on account of which bodies regain their original shape or size when the deforming forces are removed is known as* **elasticity**.

Stress:—*The force of reaction per unit area of cross-section over which it acts is known as* **stress**. In the absence of any external force, the force of reaction is zero. As the external force is applied, the force of reaction begins to increase with the deformation and in equilibrium it is equal to the external force. Therefore, stress is also equal to the external force per unit area acting on the body. If the external force is F and area is A, stress is equal to F/A dynes per sq cm

Strain:—On account of the external forces, a body may change either in length or in volume or in shape. *The change in dimension compared with original dimension is known as* **strain**. For example, if, a wire of L cm increases in length by l cm, strain is l/L. This strain is known as linear strain. *See* Fig 99. If a solid, liquid or gas of volume V changes in volume by v the strain is $\frac{v}{V}$. This is known as **bulk strain.** If a cube or anybody is twisted by an angle θ, the strain is θ and known as shear strain. Strain is mere ratio and as such has got no unit

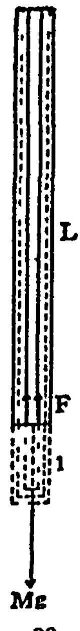


Fig 99.

§3. **Elastic limit and elastic fatigue:**—It is observed that when the deforming forces are small, the body regains its form completely; that is it is perfectly elastic. *But if we go on increasing the force after certain limit the body will not regain its original form, we say that the body is permanently stretched This limit is known as* **elastic limit.** If, we still go on increasing the load, after a certain limit it increases infinitely *i.e*, it breaks This is known as yield point

Sometimes, it is observed that bodies do not regain their original form instantaneously but after some time This is known as **elastic fatigue.**

§4. **Hooke's Law:**—After performing a number of experiments, Hooke concluded that *'within elastic limit stress is proportional to strain'*

i e

$$\text{Stress} \propto \text{Strain}$$

$$\text{Stress} = E.\ \text{strain}$$

$$\frac{\text{Stress}}{\text{Strain}} = E\ \text{(constant)}$$

This constant E is known as modulus of elasticity

If strain=1, E=Stress, therefore, *Modulus of elasticity is numerically equal to the stress which will produce unit strain in the body* Its unit is the same as that of stress *i e* dynes per sq cm or poundals per sq foot

§5. **Different kinds of Moduli—Bulk modulus or volume elasticity:**—(Fig 100). Whenever a body changes in volume without any change in shape, the modulus of elasticity is known as Bulk modulus and is denoted by K. Suppose, the initial volume of a body is V c c. and the change in volume is v c c; let the pressure applied be P.

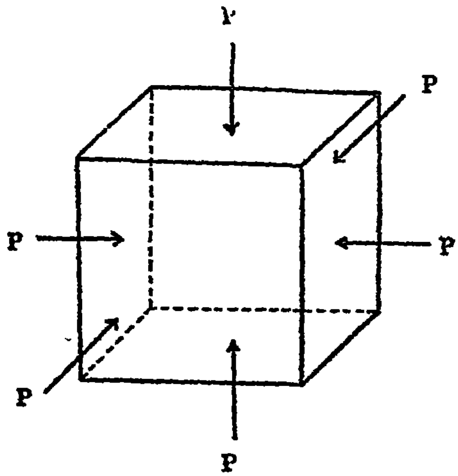


Fig. 100

Then, strain $=\dfrac{v}{V}$

Stress $= F/A = p$ dynes per square cm.

$$K = \frac{\text{Stress}}{\text{strain}} = \frac{P}{\frac{v}{V}} = \frac{P \times V}{v}\ \text{dynes}$$

per sq. cm.

Young's Modulus:—When the strain is linear, the constant E is known as Young's modulus and is denoted by Y Consider along wire AB of length L cm When a force Mg is applied on it, it increases in length by $BC = l$ *cm*. Therefore, we have; (*See* Fig 100)

$$\text{Stress} = \frac{Mg}{A}\ \text{dynes per sq cm.}$$

$$\text{Strain} = \frac{l}{L}$$

stress

$$Y = \frac{\text{Strain}}{\text{Stress}} = \frac{Mg}{A} \times \frac{L}{l}$$

strain

If $A=1$, $L=l$, $Y=Mg$. Here A denotes cross-section

Therefore, Young's modulus *is numerically equal to that force which will double the length of a wire of unit cross-section*

Modulus of rigidity:—Consider a cube $ABEF$ When a force T is applied tangentially on the top $EFGH$, the cube will be deformed as shown in Fig 101 In this case,

Stress$=T/A$

Strain$=\theta$

Modulus of rigidity $n = \frac{T}{A} \times \frac{1}{\theta}$

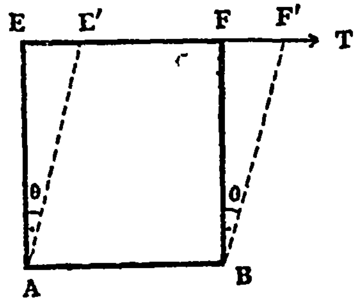


Fig 101

§6. To Measure Young's modulus.—In order to measure Y we use Searle's apparatus as shown in Fig 102 (For greater details *see* author's Practical Physics)

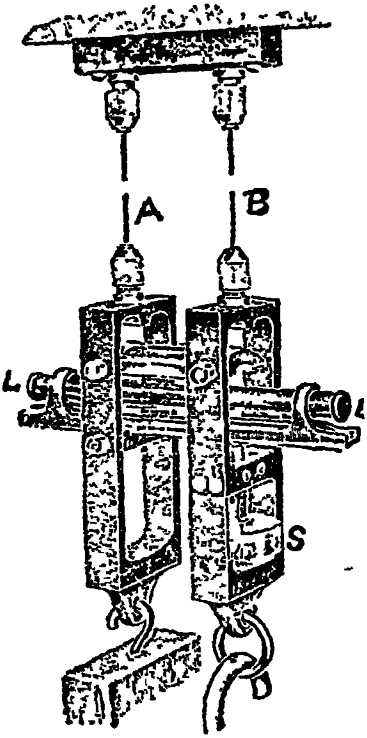


Fig 102

A and B are two identical long wires supported from a common support One wire is a experimental wire while the other is reference wire to eliminate the effect of temperature and yielding of support. Two rectangular steel frames are suspended at the lower ends of the wires One frame carries a fixed load and another a hanger on which weights can be placed LL is the spirit level One end of it is fixed in one frame while the other end rests on a micrometer screws which can be raised or lowered by moving the screw and this displacement can be read on the scale

Working:—Place 1 kilogram weight on the hanger and adjust the screw so that the bubble is in the middle Note the reading (r_1) of the screw Increase the weight by 1 k gram The wire will increase in length, this end of the spirit level will go down, so again move the screw up till the bubble is again in the middle Note the reading (r_2) again The difference of the readings r_1 and r_2 gives the increase in length for 1 k gram In this way, go on increasing the weights by 1 k gram every time to 5 or 6 k gm wts and again

reduce the weights one by one and again note the readings Find the mean of the two readings corresponding to the same load From this, find the extension for 2 k gram or 3 k gram wt by subtracting 1st from 3rd, 2nd from 4th or 1st from 4th, 2nd from 5th; 3rd from 6th readings and likewise

Find mean extension for 2 or 3 k grams etc. Find out the diameter of the wire at 10 or 12 different places by a screw gauge Also, find out its original length L Calculate Young's modulus by the following formula

$$Y=\frac{Mg}{\pi r^2}\times\frac{L}{l}$$ dyne per sq cm (where M=2,000 or 3,000 gms, r is mean radius of the wire)

§7. To measure Y by using a vernier:—This is another form of the apparatus (Fig 103) A and B are two long wires supported from the same support. A carries a rectangular metallic bar at its lower end A fixed load is suspended to keep the wire straight from the bar Main scale S is graduated on the bar A vernier scale V is attached to the lower end of the wire B Vernier carries a hanger on which k gram weights can be placed

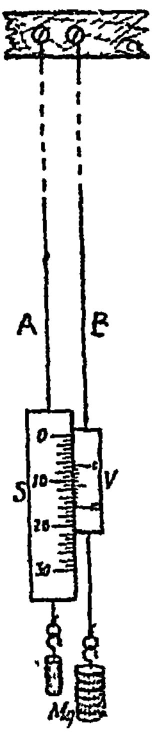


Fig 103.

Working:—Place 1 k gram wt on the hanger and take the vernier reading Go on increasing the wt. by 1 k gram and every time take vernier reading Rest of the method is the same

§8. To verify Hooke's Law:—From the above observations, find the extension for 1 k gram, 2 k grams, 3 k grams etc by substracting 1st reading respectively from 2nd, 3rd, 4th reading. Draw a graph between this extension and load It will come out to be a straight line It shows that extension is proportional to tension, which is Hooke's law (Fig 104)

Numerical problems:—1 *A stress of 1 k. gram per sq millimetre is applied to a wire of which the Young's modulus is 10^{12} dynes per sq cm. Find the percentage increase length.*

Here
$$Mg=1\times1000\times980 \text{ dynes}$$
$$A=1 \text{ sq mm} = \cdot01 \text{ sq cm}$$
$$L=100 \text{ cm. } Y=10^{12}$$

To find l?

$$Y=\frac{Mg}{A}\times\frac{L}{l}=\frac{1\times1000\times980}{01}\times\frac{100}{l}$$
$$l=\frac{1000\times980\times100}{01\times10^{12}}$$

Percentage increase $= 0008\%$, Because $L=100$

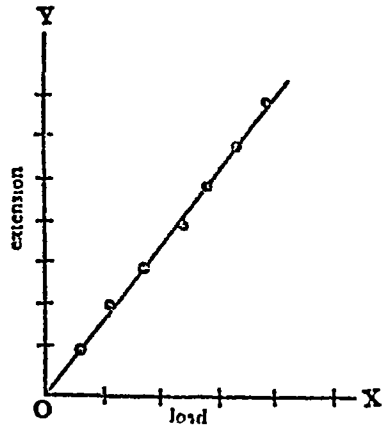


Fig. 104.

2 *A wire of a·4 cm diameter is loaded with 25 k. gram wt If length of 100 cms is found to be extended to 102 cms Calculate the Young's modulus of the material*

$$Y=\frac{MgL}{\pi r^2 l}$$

Here $M=25\times1000$ grams, $g=980$ dynes per sq cms $L=100$ cms, $l=102-100=2$cm, $r=0\ 2$ cms Then,

$$Y=\frac{25\times1000\times980\times100}{3\ 14\times\ 2\times\ 2\times2}$$

$$=\frac{25\times98\times10^{10}}{314\times8}$$

Calculations:—

log 25=1 3979	log 3 14=2 4969	
log 98=1 9912	log 8 = 9030	$= 9754\times10^{10}$
3 3891	3 3999	$=9\ 75\times10^{9}$ dynes per sq cm
3 3999		
$\bar{1}$ 9892		

Anti log $\bar{1}$ 9892=0 9754

QUESTIONS

1 Define elasticity, stress, strain and modulus of elasticity (*See* § 2)

2 State Hooke's law How will you verify it experimentally? (*See* § 4 and § 8)

3 Define Young's modulus Give its experimental determination (*See* § 5 and § 6)

Numerical Questions —

1 A wire of 0 4 cm diameter is loaded with 25 k gram wt a length of 50 cms is found to be extended to 51 cms calculate Young' modulus of wire [*Ans* $9\ 744\times10^9$ dynes per sq cm]

2 What force is required to stretch a steel wire of 1 sq cm cross-section to double its length $Y=2\times10^{12}$ dynes per sq cm [*Ans* 2×10^{12} dynes per sq cm]

3 An iron wire of diameter 0 4 mm is heated uniformly to 300°C and tightly clamped at ends while hot Find the pull exerted on the clamps when it has cooled to 20° C [α for iron $=1\times10^{-5}$ per degree C and $Y=1\ 1\times10^{12}$ dynes per sq cm

[*Ans* $3\ 872\times10^{6}$ dynes]

4 A spherical ball contracts in volume by 0 01% when subject to a normal uniform pressure of 100 atmosphere Calculate the bulk modulus K

[*Ans* 1013×10^{12} dynes per sq cm]

5 A stress of 2 k gram per sq millimetre is applied to wire of which the $y=10^{12}$ find the percentage increase in the length [*Ans* 0 0196%]

6 An iron wire of diameter 0 4 mm is heated uniformly to 330 C and rigidly clarified at its ends while hot Find the pull exerted on the clamps when it has cooled to 200° C [$\alpha=10^{-5}$ per° C, $Y=1\ 1\times10^{12}$ dynes per sq cm]

Ans $389{\cdot}7\times10^{4}$ dynes]

Section II
HEAT

CHAPTER I

HEAT AND TEMPERATURE

§1. **Meaning of Heat**:—When we feel extreme cold in winter either we stand in the sun or before fire By doing so we feel warmer We say that we are receiving heat from sun or the fire What is heat? It is not so easy to answer this question Formerly it was supposed to be a kind of fluid known as calorific fluid which flows in our body from the sun or the fire If we gain that liquid we feel warmer and if we lose it we feel cooler Later on this theory was given up

We know that when we rub our hands heat is produced similarly when we run or do some exercise, heat is produced In doing all these acts, we do some amount of work and spend energy Therefore, we conclude that heat is a kind of energy Energy of what? Every substance is made of molecules Every molecule vibrates about its mean position with all possible velocities The energy which is possessed by the body on accont of the motion of the molecules is known as heat energy If the motion increases heat increases If the motion decreases heat decreases If the molecules stop all motion heat energy will be zero You will read later on about this kinetic theory of heat For the time being we consider heat as a kind of energy which is related with the molecular motion of the body

§2. **Temperature**:—Whenever we sit on an iron chair on a summer day we feel it hot because we receive heat from the chair When we touch a piece of ice it appears cold because heat flows from our hand to ice This hotness or coldness of a body is known a temperature It is that property which governs the flow of heat Heat will always flow from higher temperature to lower temperature In the above example, the temperature of chair is higher than that of our body whole the temperature of ice is lower Temperature of a body depends upon the quantity of heat contained in it As a body gains heat its temperature increases

§3. **Difference between heat and temperature**:—The amount of a particular kind of energy is known as heat energy while the state of that energy is given by temperature The difference can be readily understood by taking a hydrostatic analogy Suppose there are two vessels containing water They are placed in such a way that level of water in one is lower than in another If we connect *B* these two vessels by means of a tube water will flow from the second to the first vessel and will continue to flow till the levels become equal This direction of flow does not depend upon the amount of water contained in the vessels Even if water in second is less than in the first vessel it will flow from second to first

Thus we conclude that water flows from higher level to lower level In the same way, if we take a copper vessel containing some amount of water and put in it a piece of metal at higher temperature metal piece will lose heat and water will gain heat and this flow will continue till the temperature becomes equal In this case amount of heat corresponds to the amount of water in the vessel and temperature corresponds to level As in the case of flow of liquids *the flow of heat is independent upon the quantity of heat contained in a body but it only depends upon temperature* A body at higher temperature will give heat to a body at lower temperature even though the amount of heat in it may be much less than the amount of heat in the receiver

Just as in the above example the same amount of water can be placed at different levels in the same way certain amount of heat energy may be present at different temperatures

§4. **Sources of Heat:**—In fact, the Sun is a vast source of heat for us Wood, coal, or other things formed on account of Sun's heat also give out heat when they are burnt The following are the different sources of heat —

(*i*) **Sun:**—The Sun is the greatest source of heat Some kinds nuclear reactions are taking place inside the Sun which produce this amount of heat

(*ii*) **Chemical reactions:**—When we burn coal, wood or kerosene, heat is produced In the process of burning these things combine with oxygen and a chemical action takes place

(*iii*) **Mechanical** —Whenever we rub two things heat is produced You must have seen people producing fire by a piece of steel and white stone

(*iv*) **Electricity:**—When current passes in wires they are heated up, like our heaters, electric irons etc.

(*v*) **Change of state:**—When steam condenses it gives out heat

§5. **Effects of heat:**—The following are the effects of heat :—

(*a*) Rise in temperature

(*b*) Expansion in length, area or volume

(*c*) Change of state

(*d*) Chemical change

(*e*) Physical change

QUESTIONS

1. Define temperature and distinguish it from heat (*See* §1, 2, and 3).
2. State the Sources of heat. (*See* §4)
3. What are the effects of heat? (*See* §5)

CHAPTER II

THERMOMETRY

§1. **Temperature** :—We have discussed in the previous chapter the meaning of temperature and its distinction from heat As the amount of heat increases in a body its temperature will also increase The flow of heat from one body to another will also depend upon temperature The measurement of temperature is most important in scientific studies

Ordinarily we can form some idea about the temperature of a body by merely touching it We can also feel that its temperature is increasing Similarly we can roughly say that temperature of one is higher than that of another But this preception by the sense of touch is neither always accurate as will be seen from the following experiment nor it is sensitive enough to note small differences of temperature

§2. **Temperature and sense of touch** :—Take three beakers Put cold water in one, tepid water in another and hot water in third Put one of your hands in cold water and the other in hot water After a while put both of your hands in tepid water To the first hand the water will appear hotter while to the second it will appear to be cold This clearly proves that the inference about the temperature by the sense of touch may be misleading It often depends upon the previous condition *i e* the temperature of hand

Again on a summer day touch an iron chair and a wooden one The former will appear to be more hot than the latter though both of them are at the same temperature The reason of this apparent difference is that iron being a good conductor gives out heat readily while wood does not

These two experiments prove conclusively that the measurement of temperature by the sense of touch is erroneous Again we cannot exactly say that the temperature of one body is how many times more than that of another

§3. **Temperature and its measurement** :—We must devise such an instrument for measuring temperature which may be free from above defects These instruments are known as thermometers We have seen that when things are heated their temperature also rises and along with it certain other changes also take place in them, for example, they increase in volume This increase in volume directly depends upon the rise in temperature We can, therefore, utilise this property of expansion for the measurement of temperature Later on you will read about some other changes caused by the temperature which are also used for the measurement of temperature The

instrument which is used for measuring temperature is known as thermometer.

§4. **Mercury thermometers**:—We shall read later on how liquids expand when heated Gases expand more and solids even less than liquids Therefore we generally use liquids and gases In liquids also we make use of mercury for the construction of thermometer

Construction:—Take a capillary tube of fine and uniform bore as shown in Fig 1 Put some mercury in the funnel K and, gently heat the bulb The air inside the bulb will be heated up and will expand. Some of it will pass out Again cool the bulb Vacuum will be created due to contraction of air On account of atmospheric pressure, some mercury will enter the tube In this way by alternate heating and cooling fill the tube completely with mercury Now place the bulb of the tube in a liquid whose boiling point is higher than the temperature up to which we want to use that thermometer Heat the liquid till it begins to boil, after some time heat the tube, just below the funnel with a burner and then pull out the funnel till it is separated from the tube at the same time the tube will also be closed Then allow the tube to rest for a number of days so that it returns to its normal form

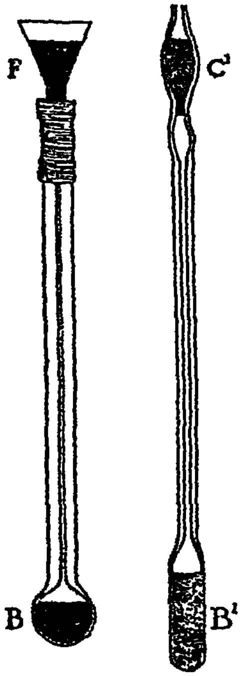


Fig 1.

Selection of a scale:—Before we actually graduate the thermometer we must select a proper unit for measuring temperature Just as for measuring length and mass we have different kinds of systems of unit in the same way we have different scales for temperature also

Before we decide a scale we select two fixed standard temperatures One is the melting point of ice and another is the boiling point of water. We mark these points on the thermometer

Lower fixed point:—Place the thermometer bulb in a funnel and cover it with ice as shown in Fig. 2 When the mercury becomes steady at some lower level, put a mark M on it This will be called the lower fixed point *i e.* melting point of ice

Upper fixed point:—For marking this point place the thermometer inside the hypsometer as shown in Fig 3 Generally some impurities are always present in water due to which its boiling point will be more but if we put the bulb in the steam produced the temperature will be the same This apparatus is specially suitable for this purpose The thermometer is completely surrounded by steam On boiling the water for some time, mercury in the tube will rise

high and will become steady at a certain point. Mark this point. This is denoted by *S* and is the upper fixed point

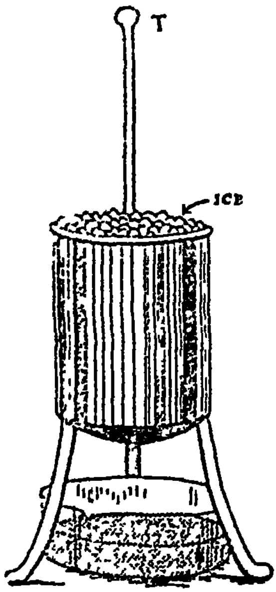


Fig 2

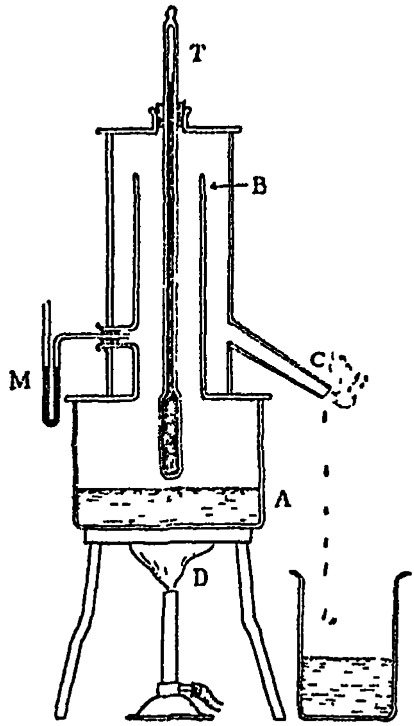


Fig 3.

To graduate the tube according to different scales :—Generally we have three kinds of scales on thermometers (*i*) Centigrade (*ii*) Fahrenheit, (*iii*) Reaumur

(*i*) **Centigrade scale :**—This scale was selected by Selcious On this scale the melting point of ice is taken as zero and the boiling point of water as 100 The interval between these two points is divided into 100 equal parts each part is known as 1 degree centigrade Each degree can further be sub-divided into 5 or 2 or 1 degree Graduation are marked below 0° also which indicate negative temperatures *i e* temperatures lower than 0°C Similarly graduations are marked above 100°C Generally thermometers reading up to 110° or 356° are constructed This scale is generally used all over and in scientific measurements

(*ii*) **Fahrenheit :**—This was selected by Fahrenheit in 1714 In those days temperatures lower than the melting point of ice could be produced by puting salt in ice These temperatures are taken as negative on centigrade scale He wanted to select some temperature as the lower point so that these temperatures may become positive According to this scheme he took 32 for the melting point of ice and 212 for the boiling point of water The interval is divided into 180

equal parts. Each part is known as degrees Fahrenheit (°F). This scale is generally used in United Kingdom This is also used in Doctor's thermometer (clinical thermometer)

Our government has decided to use only centigrade scale

(*c*) **Reaumur** :—This was selected by Reaumur This is used in a few countries in Europe On this scale the melting point of ice is taken as zero and the boiling point of water as 80 The interval is divided into 80 equal parts Each part is known as degrees Reaumer (°R)

A few specifications of the thermometer :—(*i*) The bore of the capillary tube should be uniform

(*ii*) The bore should be as fine as possible so that thermometer will be more sensitive *i e* it will be able to read smaller differences of temperature

(*iii*) The bulb should be larger so that it will contain greater amount of mercury which will make the thermometer more sensitive.

(*iv*) The bulb should be surrounded by thein walls while that of the capillary tube should be quite thick

§5. **Errors of the mercury thermometer** :—(*i*) When glass is subjected to heating or cooling it takes several days to come to its normal condition and as such its graduation becomes faulty

(*ii*) While marking the upper fixed point, if the pressure is lower than 76 cms, the boiling point will not be 100°C Generally it is found that a fall of 1°C takes place when pressure falls by 26 8 mm of mercury

(*iii*) On account of larger size of the bulb, it takes sufficient time to take the temperature of the bath and, therefore, it is not suitable for measuring changing temperatures

(*iv*) While noting the temperature of anybody most of the thermometer tube is outside the given body and that part of the tube is at lower temperature than that of the body and hence there will be less expansion of the mercury column This is known as ***exposed stem correction.***

(*v*) We cannot find out the temperature at any point with the help of this thermometer

(*vi*) If we place a thermometer in melting ice or boiling water it will not exactly read 0° or 100°C. This error is known as shifting of 0 or 100 point

§6. **Why mercury is used as a thermometric substance**:— On account of the following quantities mercury is used in thermometer .—

(*i*) It is a good conductor of heat and, therefore, takes up the temperature of the body very soon

(*ii*) It can be easily obtained in pure form

(*iii*) Its expansion is uniform i e it will increase by the same amount when heated by 1°C whatever may be the initial temperature

(*iv*) This does not stick to the sides of the tube

(*v*) Because it is opaque its position can be read conveniently

(*vi*) The freezing point of mercury is 139°C and its boiling point is 356°C and therefore mercury thermometers can be used from −39°C to 356°C and by putting a little air in the tube to still higher temperature (up to 500°C)

§ 7. **Relation between various scales of temperature :—** All the thermometers with proper graduations are shown in Fig 4 As explained above the interval between melting point of ice and boiling point of water is divided into 100°C, 180°F and 80°R Therefore we get

$$100°C = 180°F = 80°R$$

Dividing this equation by 20 we get

$$5°D = 9°F = 4°R \qquad \ldots(i)$$

This equation will enable us to compare the various scales. It may be slightly put in different form to make it convenient for use

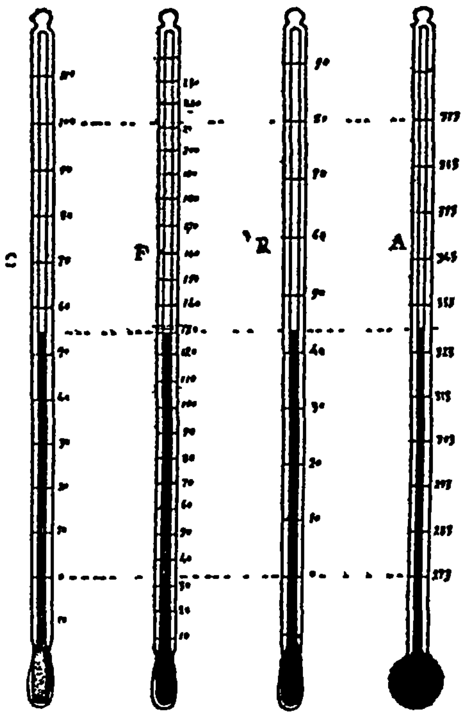


Fig 4

Suppose when all the thermometers are placed in a bath the mercury stands at X in each Let the reading of each be as C, F and R Let M and S be the lower and upper fixed points in each case Then the ratio of MX to MS will be same in each case Because the lower point in Fahrenheit thermometer starts from 32 No of degrees in MX is $F-32$,

$$\frac{C}{100} = \frac{F-32}{180} = \frac{R}{80}$$

or
$$\frac{C}{5} = \frac{F-32}{9} = \frac{R}{4} \qquad \ldots(ii)$$

with the help of equation (*ii*) we can convert any given temperature into its corresponding temperature in another scale

Numerical Problems :— 1. *Which is that temperature which reads same on two scales ?*

Let that temperature be $X°$. Substituting this X in equation (ii) we get

$$\frac{X}{5}=\frac{X-32}{9}.$$

From first relation, we get

$$9X=5X-160$$

or $$4X=-160$$

$\therefore$ $$X=-40$$

The required temperature is $-40°$.

2. *A patient's temperature is 104°. In what scale is this temperature denoted and what will be its value in other scale?*

These temperatures are recorded in Fahrenheit scale. Let C be the corresponding reading in centigrade scale

$\therefore$ $$\frac{C}{5}=\frac{F-32}{9}$$

or $$\frac{C}{5}=\frac{104-32}{9}$$

or $$\frac{C}{5}=\frac{72}{9}$$

$\therefore$ $$C=40°.$$

3 *The highest temperature of Jacobabad is 122°F. How much it will read in centigrade scale.*

Let the reading in centigrade scale be °C.

$\therefore$ $$\frac{C}{5}=\frac{122-32}{9} \quad \text{or} \quad \frac{C}{5}=\frac{90}{9}$$

$$C=50°.$$

§ 8. **Other thermometers**—(a) **Alcohol thermometer** :—As explained above we cannot use mercury thermometer below $-39°C$. In such cases we use alcohol in place of mercury in the thermometer. The freezing point of alcohol is about $-115°C$ while the boiling point is much lower about 70°C. Therefore alcohol thermometers are used in lower range. Its construction is similar to mercury thermometers. But they have certain disadvantages. (i) They cannot be used for higher temperatures. (ii) Since the expansion of alcohol is non-uniform, its readings are not accurate.

Clinical thermometer :—(Doctor's thermometer). This thermometer is graduated in Fahrenheit scale from 95°F to 110°F. Because the temperature of human body varies between this limit. There is a small bend at A. When the bulb is placed in the mouth or below the arm of the patient mercury expands and

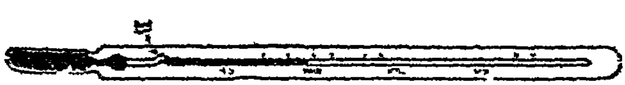


Fig 5.

rises through A After a minute or so it is removed and reading is taken This gives the temperature of the body. On account of the bend A mercury from the tube cannot go back to bulb easily and therefore it will give the same reading even after a long time When it is given a jerk mercury will come down and it can be used again This should not be placed in boiling water

The normal temperature of human body is 98 4°F It increases due to fever and falls down in weakness

(*c*) **Sixe's Maximum and minimum thermometers** :—You must have seen the columns of maximum and minimum temperatures recorded at various places during the last twenty-four hours They are recorded with the help of maximum and minimum thermometers The thermometer is shown in Fig 6 Bulb B and part of the bulb A is filled with alcohol These two columns of alcohol are separated by a mercury column S_1 and S_2 are two iron springs (index) placed inside the tubes They are such that alcohol can pass by their sides without moving them but mercury cannot When mercury pushes them they will move forward A scale is graduated on ED and CF in opposite direction

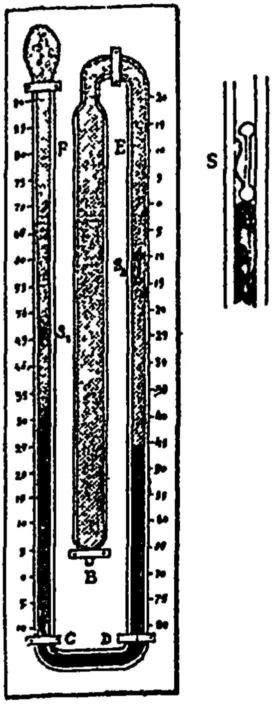


Fig 6

Working :—Bring the two index marks in contact with mercury with the help of a magnet Then leave the thermometer Suppose now temperature increases, alcohol in B will expand It will pass by the side of S_2 and press the mercury column Mercury column in CF will rise As the mercury rises it pushes the index S_1 in upward direction When the temperature begins to fall alcohol in B will contract, mercury in tube CF will fall but the index will be left behind at the highest point When mercury in DE rises index S_2 will go up Again when temperature begins to rise, mercury in ED will come down and S_2 will be left behind at the lowest temperature reached Then after 24 hours we take the readings of S_1 and S_2 and again set them In this way we can note the maximum and minimum temperatures reached during past 24 hours

QUESTIONS

1 What is a thermometer ? Why it is necessary ? Describe the construction of mercury thermometer (*See* § 1 to 4)

2 What are the advantages and disadvantages of mercury thermometers ? (*See* § 4, 5 and 6)

3 State the different scales of thermometer and establish the relation between them (*See* §4 and § 7)

4 Describe the clinical and Sixe's thermometer. (*See* § 8)

5 Why alcohol is suitable for thermometers ? (*See* § 8)

CHAPTER III

CALORIMETRY AND SPECIFIC HEAT

§1 **Introduction** :—We have read about the difference between heat and temperature in chapter I We have also read how to measure temperature Now we shall steady the measurement of heat

§2. **Unit of heat** :—For measuring any quantity we should select a proper unit As we have already said heat is a kind of energy and therefore it can be measured in terms of units of energy. The unit of energy is ergs or foot-pounds But it would be more convenient if we select another arbitrary unit for measuring heat This unit is known as calorie

Calorie :—It is defined as *the amount of heat required to raise the temperature of 1 gram of water by 1°C* preferably between 14 5° to 15 5°C

British Thermal Unit (B. Th U.) :—It is *the amount of heat required to raise the temperature of 1 pound of water by 1°F*

Centigrade Heat Unit (C H.U.) :—It is *the amount of heat required to raise the temperature of one pound of water through 1°C.*

Relation between B. Th. U. and calorie :—

1 lb of water$=$453 6 grams of water and $1°F=\frac{5}{9}°C$

$\therefore$ 1 *B Th U* $=453\,6\times\frac{5}{9}=252$ calories

§3. **Specific heat** :—It is found that amount of heat required by a body depends upon the mass of the body and the rise in temperature If H be the amount of heat required by m grams of a substance to raise its temperature by t°C we have,

$$H \propto m\, t$$

or

$$H = S\, m\, t$$

where S is a constant which depends upon the nature of the substance, and is known as specific heat If $m=1$, $t=1$, $H=S$, specific heat is therefore defined as *the amount of heat required by 1 gram of the substance to raise its temperature by 1°C*

Specific heat can also be defined as,

$$\text{Sp heat}=\frac{\text{Heat required by } m \text{ grams of the substance to raise its temp by 1°C}}{\text{Heat required by } m \text{ grams of water to raise the temp by 1°C}}$$

$$=\frac{m\times\text{Heat required by 1 gram of substance to raise the temp by 1°C}}{m\times\text{Heat required by 1 gram of water to raise the temp. by 1°C}}$$

$$\frac{\text{Heat required by 1 gram of the substance to raise its temp by } 1^\circ C}{\text{Heat required by 1 gram of water to raise its temp by } 1^\circ C}$$

= Heat required by 1 gram of the substance to raise its temperature by 1°C Because according to definition of caloric, heat required by 1 gram of water is 1 caloric

§4. **Thermal capacity:**—It is defined as *the amount of heat required by the whole body to raise its temperature by 1°C* Thus if m gram is the mass of the body, 1°C is the rise in temperature, and S is its specific heat

$$\text{Thermal capacity} = m\ s\ 1 = m\ s \text{ calories}$$

∴ Heat required to raise the temperature of the body by $t^\circ C$ = Thermal capacity × rise in temp

The mass of a calorimeter is 256 grams Its specific heat is 1 Find its thermal capacity

$$\text{Thermal capacity} = 256 \times 1 = 25\ 6$$

§5. **Principle of method of mixture:**—When two bodies at different temperatures are mixed up, heat will flow from the body at higher temperature to the body at lower temperature This flow of heat will continue till the temperatures are equal This common temperature is known as final temperature Since heat is a form of energy and we know that energy is never destroyed therefore in the above example, heat lost by one body must be equal to heat gained by another body provided no heat is lost by radiation or through any other source

Thus we conclude that in the case of mixture,

Heat lost = Heat gained

§6. **Calorimeter and water equivalent**·—In the experiments on heat measurements we generally use a copper vessel which is cylindrical in form (*see* Fig 7) This vessel is placed in a wooden box and some non-conducting substance like asbestos or glass wool is placed between the box and the vessel A copper stirrer is also provided in it with an insulating handle A lid covers this vessel There are two holes in the lid through which a thermometer and the stirrer passes This vessel is known as calorimeter In performing experiments calorimeter will also lose or gain heat If mass of the calorimeter is m_1 and its specific heat s_1, it will take $m_1\ s_1$ calories of heat when its temperature rises by 1°C.

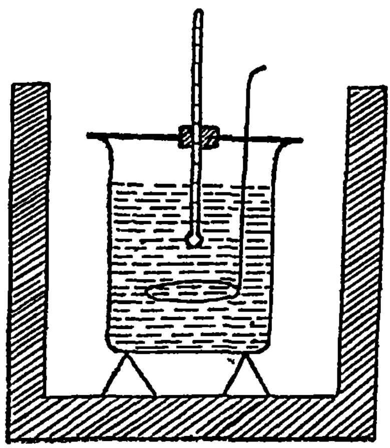


Fig 7.

$\therefore$ Heat required $= 310 \times \cdot 08 \times (75-25)$

$= 310 \times \cdot 08 \times 50$

$= 1{,}240$ calories

2. *A calorimeter contains 200 grams of water at 15°C. When 60 grams of water at 100°C is dropped in it, the temperature rises to 30°C. Find the water equivalent of calorimeter.*

Let the water equivalent of calorimeter be W gram

Heat lost by hot water $= 60\ (100-30) = 60 \times 70$

Heat gained by cold water and calorimeter

$= (200+W)\ (30-15) = (200+W)15$

Heat gained $=$ Heat lost

$(200+W)\ 15 = 60 \times 70$

$(200+W) = 4 \times 70$

$W = 280-200$

$= 80$ grams

§8. **To find the specific heat of a solid by Regnault's method :—** (*See* A Text Book of Practical Physics by authors) The principle of

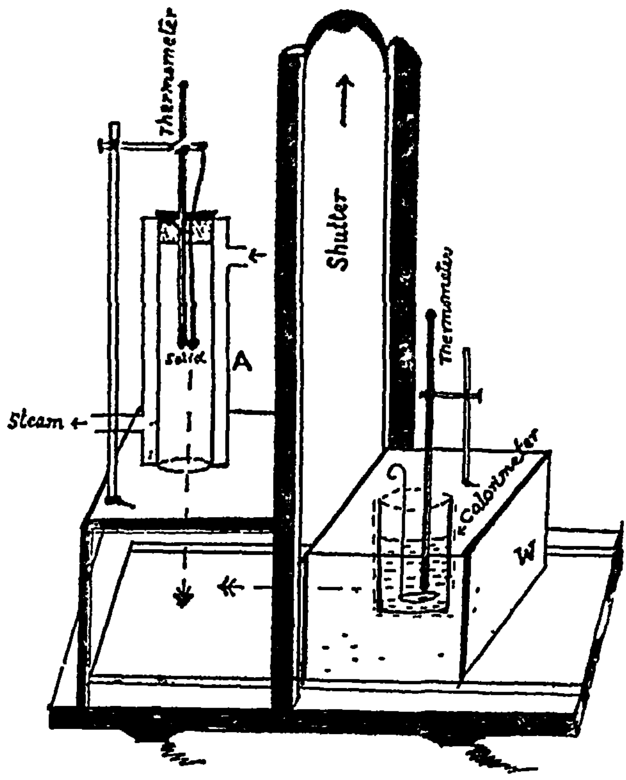


Fig 8

the method is the same as method of mixtures The apparatus is made in such a way that the solid can be heated conveniently

and heat loses while transferring the solid are minimised. The apparatus is shown in Fig 8. A is a double walled vessel in which the solid is suspended A thermometer is also placed in it just in contact with solid. Steam from a separate boiler is passed in the annular space of the vessel W is a wooden box in which a calorimeter with stirrer is placed This calorimeter is surrounded by felt or some non-conducting material There is a shutter which can be raised or lowered

Experiment :—Find out the weight of the given solid and suspend it in A Put the thermometer in A and close the upper mouth with a cork Connect A with the boiler and pass stream in it Weigh the calorimeter with stirrer Put some water in it and weigh it again Take a thermometer and find out the temperature of cold water Place the calorimeter in the box When the temperature of A becomes steady note it Raise the shutter and move the box just below A Gently drop the solid in the calorimeter Take out the box, put a thermometer in it stir it well and note the highest temperature reached Calculate the specific heat as shown below :—

1 Mass of the solid $=m$ gram.
2 Mass of calorimeter and stirrer $=m_2$ gram.
3 Mass of calorimeter and cold water $=m_3$ gram.
4 Initial temperature of cold water $=t_2°C$
5 Temperature of hot solid $=t_1°C.$
6 Final temperature of mixture $=T°C$
7 Specific heat of calorimeter and stirrer $=S_2$
8 Specific heat of liquid $=S_1$
for water $S_1=1.$

Calculations :—

Mass of water in calorimeter $=m_3-m_2=m_1$ gram
Heat lost by solid $=m\,S.\,(t_1-T)$ calories
Heat gained by water $=(m_1S_1+m_2S_2)\,(T-t_2)$

Heat lost=Heat gained

$$mS\,(t_1-T)=(m_1+m_2S_2)\,(T-t_2)\;..(S_1=1)$$

$$\therefore \qquad S=\frac{(m_1+m_2S_2)\,(T-t_2)}{m(t_1-T)}...$$

§9. **To find the specific heat of liquid :**—The principle and method is same Take the given liquid in calorimeter instead of water and perform the experiment in the same manner If the specific heat S of the solid is known, the specific heat S_1 of the liquid can be calculated

As shown above, we have,

$$mS(t_1-T)=(m_1S_1+m_2S_2)\,(T-t_2)$$

or $$m_1S_1(T-t_2)=m_2S_2(T-t_2)-mS\,(t_1-T)$$

$$\therefore \qquad S_1=\frac{m_2S_2(T-t_2)-mS\,(t_1-T)}{m_1(T-t_2)}$$

Numerical Problems :—1 *A ball of platinum whose mass is 200 grams is removed from a furnace whose temperature is 578 8°C and is immersed in 150 grams of water at 0°C If the temperature of water rises to 30°C, find the specific heat of platinum*

Heat lost by platinum $=200\ (578\ 8-30)\ S$ Cal

Heat gained by water $=150\ (30-0)$ Cal

Equating the two, we have,

$$200\ (548\ 8)\ S=150\times30$$

$$S=\frac{150\times30}{200\times548\ 8}$$

$$=\frac{15\times3}{2\times548\ 8}=\ 041\ \text{Cal}$$

2 *50 grams of iron (sp heat 112) at 90°C are dropped into a calorimeter weighing 60 grams and containing 50 grams of oil at 18 1°C The final temperature of the mixture is 29 5°C If the water equivalent of calorimeter is 4 8 grams, find the sp heat of oil*

Let the specific heat of liquid be S

Heat lost by iron $=50\times\ 112\times(90-29\ 5)$

$=50\times\ 112\times60\ 5$ Cal

Heat gained by oil and calorimeter

$=(50\times S+4\ 8)\ (29\ 5-18\ 1)$

$=(50\times S+4\ 8)\ 11\ 4$

Equating the two, we have

$$(50\times S+4\ 8)\ 11\ 4=50\times\ 112\times60\ 5$$

or $$50\times S+4\ 8=\frac{50\times\ 112\times60\ 5}{11\ 4}$$

or $$50S=\frac{50\times\ 112\times60\ 5}{11\ 4}-4\ 8$$

$$S=\frac{50\times\ 112\times60\ 5}{11\ 4\times50}-\frac{4\ 8}{50}$$

or $$S=\frac{112\times60\ 5}{11\ 4}-\frac{4\ 8}{50}$$

$$=\ 59-\ 096$$

$$=\ 494\ \text{Cal}$$

3 *A mass of 100 grams of copper (sp heat 1) heated to 90°C is dropped into a copper calorimeter weighing 250 grams and containing 300 grams of water at 20°C What will be the final temperature ?*

Let the final temperature be T°C

Heat lost by hot body$=100\times\ 1\times(90-T)$ Cal

Heat gained by water and calorimeter$=(300+250\times\cdot1)\ (T-20)$ Cal

Heat lost=Heat gained

$$100\times 1\times(90-T)=(300+25)\ (T-20)$$

$$10(90-T)=325\ (T-20)$$

$$900-10T=325T-6500$$

$$-10T-325T=-6500-900$$

$$335T=7400$$

$$\therefore \qquad T=\frac{7400}{335}=22\ 1°C$$

4 *Three liquids A, B and C are at temperatures of 30°, 20° and 10° respectively When equal part by weight of A and B are mixed, the temperature of the mixture is 26°C, and when equal parts by weight of A and C are mixed the temperature is 25°C Find the resulting temperature when equal parts of B and C are mixed*

Let S_1, S_2, and S_3 denote the specific heats of liquids respectively and M denote the mass of each liquid mixed

In the first case when A and B are mixed, we have

heat lost by liquid A $=M\times S_1\times(30-26)$,

heat gained by liquid $B=M\times S_2\times(26-20)$

$$\therefore \qquad M\times S_1\times(30-26)=M\times S_2\times(26-20)$$

or $$S_1\times 4=S_2\times 6$$

or $$S_1=\tfrac{3}{2}\ S_2$$

or $$S_2=\tfrac{2}{3}\ S_1 \qquad (1)$$

Similarly in the second case when A and C are mixed,

heat lost by A $=M\times S_1\ (30-25)$,

heat gained by C $=M\times S_3\times\ (25-10)$

$$\therefore \qquad M.S_1\ 5=M\ S_3\ 15$$

or $$S_1=3S_3$$

or $$S_3=\tfrac{1}{3}\ S_1 \qquad (2)$$

In the third case let T be the final temperature of the mixture,

Heat lost by B $=M\ S_2\ (20-T)$

Heat gained by C $=M\ S_3\ (T-10)$

$\therefore\ M\ S_2\ (20-T)$ $=M\ S_3\ (T-10)$

Substituting the value of S_2 and S_3 from (1) and (2), we have

$$\tfrac{2}{3}\ S_1\ (20-T)=\tfrac{1}{3}\ S_1\ (T-10)$$

or $$2\ (20-T)=T-10$$

or $$40-2T=T-10$$

$$-3\ T=-50$$

$$\therefore \qquad T=\tfrac{50}{3}=16\tfrac{2}{3}°C$$

QUESTIONS

1. Define —Specific heat, caloric, thermal capacity, water equivalent of calorimeter B Th U and give their units (*See* §2, 3, 5)

2 What is the law of mixtures ? (*See* § 5)

3 How will you find out the water equivalent of a calorimeter ? (*See* §6 and § 7)

4 How will you find out the specific heat of a solid or liquid by Regnault's apparatus (*See* § 8 and § 9)

Numerical Questions .—

1 A mass of copper weighing 700 grams at 98°C is put into 800 grams of water at 15°C, contained in a copper vessel weighing 200 grams and the final temperature is noticed to be 21°C Find the sp heat of copper [*Ans* 091]

2. A copper calorimeter of mass 100 grams and sp heat 09 contains 80 grams of heavy oil at 28°C A piece of copper of mass 100 grams and at a temperature 100°C is dropped into it and the contents well stirred If the final temp is 39°C Find the sp heat of oil [*Ans* 51]

3 A copper ball weighing 6 pounds is taken out of a furnace and plunged into 20 lbs of water at 10°C The temperature of water rises to 25°C Find the temperature of the furnace (Sp heat of copper = 095) [*Ans* 551 3°C]

4 A brass weight of 100 grams is heated so that a particle of solder placed on it just melts It is then put into 100 c c of water at 15°C, contained in a calorimeter of water equivalent 12 If the final temperature is 35°C, what is the melting point of solder ? (Sp heat of brass = 088) [*Ans* 289 5°C]

5 180 grams of water at 80°C is mixed with 50 grams of water at 14°C what will be the final temperature ? [*Ans* 65 65°C]

6 A calorimeter weighs 100 grams and is made of a substance whose specific heat is 1 Calculate the (*a*) thermal capacity (*b*) water equivalent of the calorimeter [*Ans* (*a*) 10, (*b*) 10]

CHAPTER IV

CHANGE OF STATE AND LATENT HEAT

§1. **Introduction:**—Take a piece of ice It is in solid state, when it is heated it melts and changes into water a liquid state. Again when water is heated it boils and is converted into steam a gaseous state. In this way generally all substances are found to exist in three states—solid, liquid and gas When solids are heated they are converted into liquid and on further heating liquids are converted into gases

§2. **Melting Point:**—Whenever we heat a solid its temperature increases up to a certain point and then it begins to melt. At this point temperature remains constant till the whole of solid is converted into liquid *The temperature at which a solid is converted into liquid is known as* **melting point.** It depends upon the nature of the substance, upon the presence of impurities in it and upon the external pressure acting on it Again when we cool a liquid it solidifies at the same temperature, this temperature is also known as freezing point

§3. **Determination of melting point:**—(For details *see* A Text Book of Practical Physics by authors)

(*i*) **Capillary tube method.**—Take a small capillary tube and close its one end by heating it in a flame Put a small amount of the given substance in it and tie it to a thermometer bulb with the help of a thread Suspend the thermometer a beaker of water as shown in Fig 9 Gently heat the beaker and observe the substance in the tube When the substance just begins to melt remove the burner and note the temperature Allow the beaker to cool when the substance again begins to solidify note the temperature again The mean of the two temperatures will give the melting point

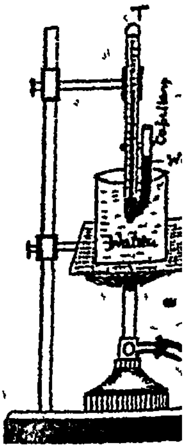


Fig 9

(*ii*) **Cooling curve method:**—This method is suitable when the substance is available in larger amount Put some substance in a test tube and place a thermometer in it and heat it by placing it in a water-bath When the substance melts and the liquid is also heated a few degrees above the melting point, remove the test tube and suspend it in a closed vessel Stir the substance with the thermometer and note its temperature after every half minute Go on doing this till the substance completely solidifies and cools further also Plot a graph between a temperature along Y-axis and time along X

axis Fig 11 Find out the temperature corresponding to horizontal portion of the curve This will give the melting point of the substance Because here as the time passes on temperature remains constant

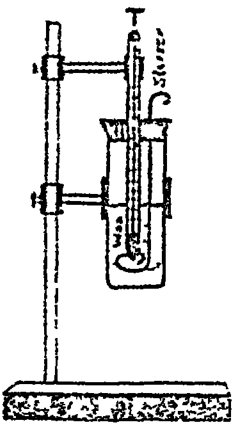


Fig 10

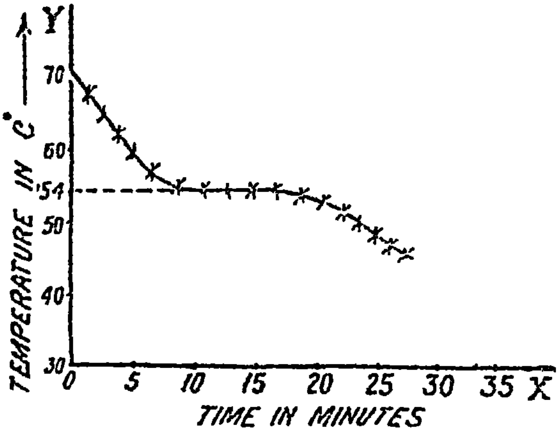


Fig 11

§4. **Effect of impurities on melting point:**—Whenever some impurity is present in the substance its melting point decreases This is the principle of freezing mixture You must have seen that while preparing ice cream or *kulfi* salt is added to ice Why ? Because on adding the salt the melting point of ice which is O will go down and at the existing temperature ice will melt In doing so it requires large amount of heat which it will take from the mixture and therefore the temperature of mixture will fall If we place a test tube of water or milk in such a mixture it will freeze

§5. **Change of volume on melting:**—There are two kinds of substances : ice type and wax type Ice type of substances decrease in volume on melting For example, when 10 907 c c of ice melts it will form 10 c c of water In the same way when water freezes it expands in volume During this expansion it exerts great pressure on the walls of the vessel This is the reason why water pipes in cold countries burst out at night, rocks in the colder region are broken into pieces Solids of this type will float on the corresponding liquids Cast iron, antimony, bismuth, brass etc belong to this class

Such metals are specially suitable for casting mould's when liquid metal is poured in the cast, it solidifies on cooling and due to expansion tightly fits in the cast

Wax type of substances contract on solidification and expand on melting. In this case solid is heavier than liquid and will sink down

§6. **Effect of pressure on melting point :**—You must have observed that when pieces of ice are pressed together, they combine to form one solid piece. Why so? Again if you take a block of ice and place a wire over it and suspend two weights from the two ends of the wire (Fig 12) It will be observed that the wire passes through the block of ice without breaking it This experiment was done by Tyndal.

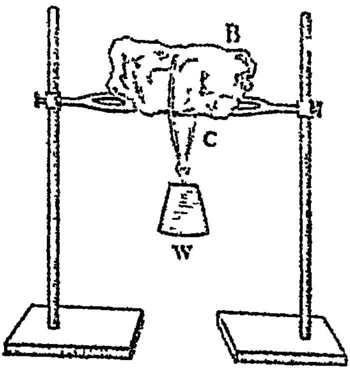


Fig 12.

The melting point of ice type of substances is lowered when pressure is increased In the above case, on exerting pressure, melting point is lowered and at the existing temperature ice is converted into water. On releasing the pressure again the melting point is raised and water freezes again forming one block

In the second case ice below the wire melts on account of pressure applied due to weights Water comes up from the sides of the wire and again freezes because now no pressure is acting on it. In this way the wire passes through the whole block and it is not broken It has been found that in case of water a pressure of 1,000 atmosphere is required to lower the melting point to 7·6°C. *The melting point of wax type of substances increases with the increase of pressure*

§7. **Latent heat of fusion :**—Whenever a body is heated the kinetic energy of the molecules increases consequently its temperature also increases But at melting point temperature remains constant The heat taken by a substance at the melting point is not shown by the thermometer, this heat is known as latent heat This heat is utilised in changing the state of the substance This amount of heat will depend upon the mass of the substance taken and therefore **Latent Heat** *is defined as the amount of heat required by 1 gram of the substance in melting at the same temperature.* This is known as latent heat of fusion If we wish to freeze 1 gram of the substance the same amount of heat should be removed from it The latent heat of ice is 80 calories, *i.e* 1 gram of ice will take 80 calories of heat in melting This depends upon the nature of the substance For example, latent heat of wax is 35 calories and that of zinc is 28 calories

§8. **To find the latent heat of ice :**—Take a calorimeter with stirrer and find out its weight (M_1) Put some amount of water in it and again weigh it (M_2) Place a thermometer in it and note its temperature (t_1) Take a piece of ice and dry it with the help of a blotting paper. Place it in the calorimeter and stir it Note the lowest temperature reached (T) Again weigh the calorimeter with its contents (M_3) From these, calculate the latent heat as shown below:

1 Mass of calorimeter and stirrer $= M_1$ gram
2. Mass of calorimeter + water $= M_2$ gram

3. Mass of calorimeter+water+ice $=M_3$ gram
4 Initial temperature of water $=t_1°C$
5 Final temperature $=T°C$
6 Initial temperature of ice $=0°C$
7 Latent heat of ice $=L$ cal

Mass of water M $=M_2-M_1$
Mass of ice m $=M_3-M_2$
Heat taken by ice $=m$ L cal
Heat taken by this water $=m\,(T-0)$
Heat lost by water and calorimeter

$$=M\,(t_1-T)+M_1\,S_1(t_1-T)$$
$$=(M+M_1S_1)(t_1-T)=(M+w)(t_1-T)$$

where w is water equivalent of calorimeter Applying the principle, heat lost=heat gained

$$(M+w)(t_1-T)=mL+m(T-0)$$

or
$$L=\frac{(M+w)(t_1-T)-mT}{m}.$$

In this method some water may be sticking to ice before transferring it and therefore some error will be introduced

Numerical problem :—*A copper calorimeter weighs 50 grams and contains 200 grams of water at 20°C , 20 grams of dry ice are added and stirred well. The final temperature is 11°C Find the latent heat of fusion (Sp heat of copper=·1)*

Heat gained by ice $=20\,L+20(11-0)$
$=20L+20\times11$ cal

Heat lost by water and calorimeter
$=(200+50\times\ 1)(20-11)$
$=205\times9$ cal

$\therefore$ $20L+220=1845$
$20L=1845-220=1625$

$\therefore$ $L=\frac{1625}{20}=81\ 25$ cal

§9. **Evaporation :**—If we heat a liquid in a beaker and place a thermometer in it, its temperature will rise in the beginning After some time the liquid will begin to boil and evaporate and the temperature will not rise till the whole of liquid evaporates *The temperature at which a liquid begins to boil is known as boiling point.* It depends upon the nature of the liquid, presence of impurities in it and the external pressure acting on it Different liquids have different boiling points and they can be identified by their boiling point

§10. To find the boiling point:—Take a hypsometer as shown in Fig 13 and put the given liquid in it. Place a thermometer in it. The bulb of the thermometer should not dip inside liquid. Heat the liquid till the temperature becomes constant. Note this temperature. This will give the boiling point at the pressure indicated by the manometer.

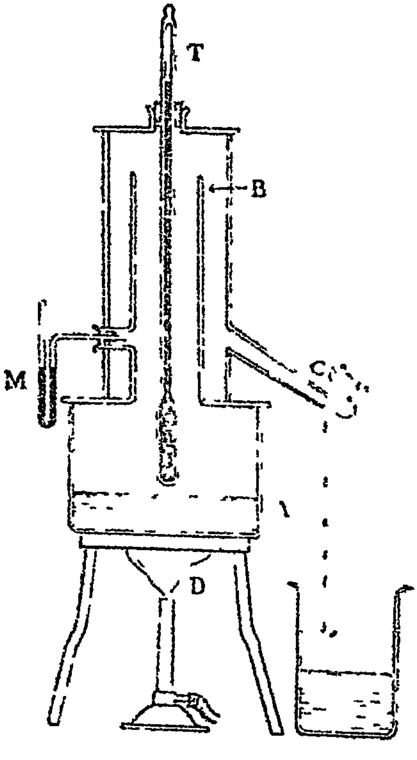


Fig 13

Change of volume on evaporation:—Every liquid expands in volume by large amount when it evaporates. 1 c.c of water at 100°C occupies 1,674 c.c. when it is converted into steam at the same temperature.

§11. Effect of pressure on boiling point:—As volume increases on evaporation, increase in pressure will oppose the evaporation and therefore boiling point will increase. If we lower the pressure boiling point will be lowered.

Water boils at 100° at sea level where the pressure is 76 cms of mercury. As we go up, pressure decreases and boiling point will also decrease. On the top of Everest boiling point is very low and as such cooking becomes difficult there.

Experimental proof:—Take a flask fitted with a tube carrying a stopper. Put some water in it and boil it so that the whole of the flask is filled with steam. Close the stopper and place the flask as shown in Fig 14. Sprinkle cold water over it. Steam will condense and vacuum will be created. Pressure inside will fall and water will begin to boil even if temperature is much less than 100°C.

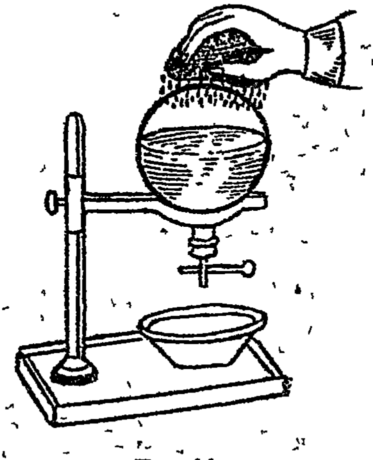


Fig. 14

§12. Latent heat of steam:—As in the case of melting, here also when the liquid is boiling temperature remains constant while the liquid is taking heat. This heat is known as latent heat of vaporisation. This is utilised in changing the state of the liquid. *Heat required by one gram of the liquid at the boiling point, to change into vapour at the same temperature is known as latent heat.* Latent heat

of steam is 536 calories. It means one gram of water will take 536 calories of heat at 100°C to evaporate.

§13. To determine the latent heat of steam—Principle:—Some amount of steam is passed in a calorimeter containing water and

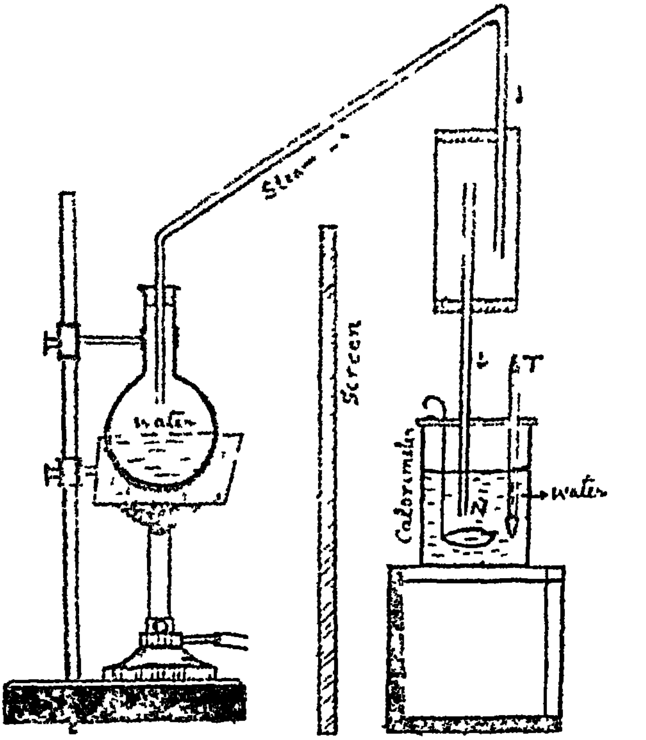


Fig 15

the rise in temperature noted. From the principle of mixtures latent heat of steam is calculated.

Apparatus and working:—The apparatus is shown in Fig 15. First of all put some water in the boiler and start it heating. Take a calorimeter and find out its mass. Fill about $\frac{2}{3}$ of it with water and again find out its mass. Put a thermometer in it and note its temperature. When dry steam begins to come out of the nozzle, dip the nozzle in the calorimeter and stir it well. When the temperature rises by 5 or 10°, remove the calorimeter and stir it well and note the highest temperature reached. Find out the mass of calorimeter with its contents again. Calculate the value of latent heat as shown below,

1. Mass of calorimeter and stirrer $=M_2$ gram
2. Mass of calorimeter and water $=M_3$ gram
3. Mass of calorimeter, water and steam $=M_4$ gram
4. Temperature of steam $=t_1°C$

5. Temperature of cold water $= t_2$°C
6. Final temperature $= T$ C
7. Latent heat of steam $= L$ calories
8. Specific heat of calorimeter and stirrer $= S_2$

Calculations:—

Mass of cold water $= M_1$ grams $= (M_2 - M_1)$
Mass of steam condensed $= m$ gram $= (M_4 - M_3)$
Heat taken by water and calorimeter $= (M_1 + M_2S_2)(T - t_2)$ cal
Heat lost by steam in condensing $= mL$ cal.
Heat lost by this water in cooling from t_1 to $T = m(t_1 - T)$ cal
According to the principle Heat lost = Heat gained, we have,

$$mL + m(t_1 - T) = (M_1 + M_2S_2)(T - t_2)$$

or

$$mL = (M_1 + M_2S_2)(T - t_2) - m(t_1 - T)$$

$$\therefore \quad L = \frac{(M_1 + M_2S_2)(T - t_2) - m(t_1 - T)}{m}$$

Sources of error:—Some water may also come along with the steam In order to reduce this a steam trap is used Some heat may be lost by radiation. For this radiation correction is applied.

Numerical problems:—1 *A copper calorimeter weighing 95 grams contains 310 grams of water at 25°C Steam at 100°C is passed into it till the temperature rises to 35°C. The mass of steam condensed is 5 grams Calculate the latent heat of steam. (Sp. heat of copper ·1 cal.).*

Let the latent heat of steam be L cal

Heat lost by steam $= 5L + 5(100 - 35)$

$$= 5L + 5 \times 65$$

$$= 5L + 325$$

Heat gained by water and calorimeter

$$= (310 + 95 \times \cdot 1)(35 - 25)$$

$$= 319.5 \times 10 = 3{,}195 \text{ cal.}$$

$$\therefore \quad 5L + 325 = 3195$$

$$5L = 3195 - 325$$

$$= 2{,}870$$

$$\therefore \quad L = \frac{2870}{5} = 574 \text{ cal}$$

2 *100 grams of ice at −10°C is heated till it evaporates and the temperature of steam rises to 110°C. Calculate the heat required for this change. [Sp heat of ice and steam is ·5 cal; latent heat of ice is 80 cal and latent heat of steam is 540 cal].*

Ice will take heat in the following stages —

Heat taken by 100 grams of ice to raise its temp. to 0°C

$$=100\times 5\times\{0-(-10)\}$$
$$=100\times 5\times 10=500 \text{ cal}$$

Heat taken by ice in melting $=100\times 80$

$$=8{,}000 \text{ cal.}$$

Heat taken by water at 0°C to raise its temp to 100°C

$$=100\times 100=10{,}000 \text{ cal}$$

Heat taken by water in evaporation $=100\times 540$

$$=54{,}000 \text{ cal}$$

Heat taken by steam in raising its temp to 110°C

$$=100\times 5\times(110-100)$$
$$=100\times 5\times 10$$
$$=500 \text{ cal}$$

∴ Total heat required

$$=500+8{,}000+10{,}000+54{,}000+500 \text{ cal}$$
$$=73{,}000 \text{ calories}$$

§14. **To find the specific heat of substance by joly's steam calorimeter.**—The principle of latent heat can be utilised for measuring the specific heat of a body The body is placed in a steam calorimeter and steam is passed in it From the knowledge of mass of steam and latent heat of steam sp heat can be calculated

Apparatus and working :—(*See* Fig 16) C is a steam chamber, one pan A of the balance is enclosed in it Two strips E and F are

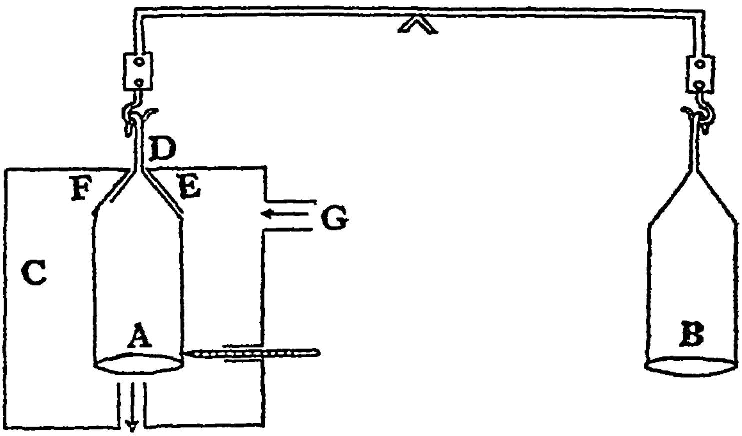


Fig 16

fixed at the upper end of the chamber so that steam condensed on any part will not fall on the pan A thermometer is inserted in the chamber which will give the temperature of the chamber Place the

body in the chamber and find out its mass. Note the temperature o the chamber. Pass steam in the chamber till its temperature become constant Some steam will condense on the body and due to whic A will go down Place weights in B till equilibrium is establishe again Calculate the latent heat as shown below -

Mass of the body $= M$ gram.
Temperature of the chamber $= t_1$ C.
Mass of steam condensed $= m$ gram
Temperature of steam $= T°C$.
Heat lost by steam $= mL$ Cal
Heat gained by body $= M S. (T - t_1)$

$$\therefore \quad M S \, (T - t_1) = mL$$

$$\therefore \quad S = \frac{mL}{M \, (T - t_1)} \text{ cal.}$$

In the above experiment some heat will also condense on th pan and pan holders of the balance In order to eliminate this, firs an experiment is performed without anybody in A and the mass c steam condensed is found out Then in the case of actual experi ment this mass is subtracted from the mass of steam condensed

§ 15. To find the sp. heat with the help of latent heat of ice:- If a body is heated to $t°C$ and dropped in a block of ice an the quantity of ice melted is measured, specific heat of the bod can be calculated This method will be clear from the numeric example given below

Numerical Problems :—1 *A metal sphere of 270 grams is su pended in a chamber at 0°C. Steam at 100°C is passed into tl chamber till the temperature rises to 100°C The mass of stea condensed is 5 grams Calculate the sp. heat of metal.* (Latent hea of steam=540 cal)

Heat lost by steam $= 5 \times 540$ cal
Heat gained by body $= 270 \times S \times 100$ cal

$$\therefore \quad 270 \times S \times 100 = 2700$$

$$\therefore \quad S = \frac{2700}{270 \times 100} = 1 \text{ cal}$$

2 *A piece of iron weighing 16 grams is dropped at a temperatu of 112 5°C into a cavity in a block of ice which it melts 2 5 grams. the latent heat of ice is 80 cals , find the sp.-heat of iron.*

Here final temperature is 0°C Let the sp heat of iron be S.
Heat lost by iron $= M. S \; t = 16 \times S \times 112\,5$
Heat gained by ice $= 2\,5 \times 80 = 200\,0$

$$\therefore \quad 16 \times 112\,5 \times S = 200$$

$$\therefore \quad S = \frac{200}{16 \times 112\,5} = \frac{200}{1800\,0} = \frac{1}{9}$$

$$= \cdot 11 \text{ cal.}$$

QUESTIONS

1 Define :—Melting point, boiling point, latent heat of ice, latent heat of steam and give their units (*See* §2, §9, §7, §12).

2 How will you find out the latent heat of ice or steam ? (*See* §8 and §13).

3 How will you determine the melting point of a solid and a boiling point of a liquid ? (*See* §3 and §10)

4 What is the effect of pressure on melting point and boiling point ? (*See* §6 and §11)

Numerical Questions —

1 A pond 50 sq metres in area is covered with ice at 0°C How much ice will be melt per hour if it absorbs heat from the sun at the rate of 25 cal per sq cm per minute ? [*Ans* 93 75 K gram]

2 Find the result of mixing equal masses of ice at 10°C and water at 60°C Sp heat of ice is 5 [*Ans* $\frac{11}{16}$ of total mass of ice melts]

3 A copper ball 56 32 grams in weight and at 15°C is exposed to dry steam at 100°C What weight of steam will condense on the ball before the temperature of the ball is raised to 100°C (Sp heat of copper= 093, L of steam=536 cals) [*Ans* 831 gram]

4 Steam at 100°C is passed into a copper calorimeter weighing 100 grams and containing 500 grams of water at 15°C until the temperature of the calorimeter and its contents rises to 25°C Calculate the weight of steam condensed (Sp heat of copper= 1 and latent heat of steam 536) [*Ans* 7 995 grams]

5 A mass of 500 grams of copper (sp heat 08) is heated in an oil bath and is passed in an ice calorimeter Find the temp of the bath if 10 K gram of ice melts [*Ans* 16°C]

CHAPTER V

EXPANSION OF SOLIDS

§1. **Introduction :**—You have already read in your previous Class that substances expand when heated This expansion may be in length, in area and in volume Study of this expansion is very important for our daily life How is the iron round the wheel of a bullock cart is fitted ? How do we remove the cork of a bottle by warming it ? Why are gaps left between the two rails ? Why a gap is left at the joints of an iron framework ? All these problems are related with expansion

§2. **Expansion of solids—Experiment 1 :**—(*See* Fig 17). *A* is a solid sphere and *B* is a ring This size of *A* is such that it can just pass through *B* If we heat the sphere, it will not pass through *B*. Showing that *A* has increased in size Again heat *B* and *A* will pass through it

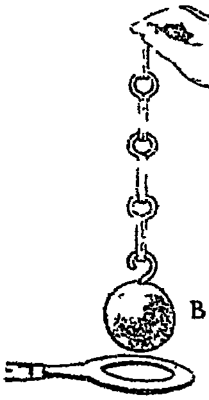


Fig 17.

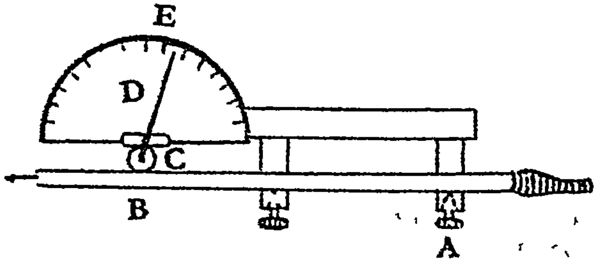


Fig 18

Experiment 2 :—*AB* is a metal rod whose one end *A* is fixed against a screw and another end *B* presses against a pointer *C* which moves on a graduated scale *E* As we heat the rod, it expands and the pointer moves on the scale On cooling again the pointer moves back A hole is provided in the rod at *B*. On placing a pin inside the hole, the rod is prevented from contracting The forces developed in this case are so strong that the pin will be broken

§3. **Linear expansion :**—Expansion in length of a rod depends upon the initial length of the rod, the rise in temperature and the nature of the material

Suppose the initial length of the rod at 0°*C* is L_0 and its length at t°*C* is L_t, then we have

$$L_t - L_0 \propto L_0 \; t.$$

or $$L_t - L_0 = \alpha\ L_0\ t \qquad (1)$$

or $$\alpha = \frac{L_t - L_0}{L_0\ t} \qquad (2)$$

where α is a constant which depends upon the nature of the material It is known as linear coefficient of expansion It is defined as *the increase in length per unit length for 1° rise in temperature* Linear coefficient of expansion of iron is 0000116 It means if we take a rod of iron of 1 cm length it will increase in length by 0000116 cm when heated by 1°C Since the value of expansion is small enough, we should take a long rod and heat it to a higher temperature in order to measure the expansion

$$\text{Unit .—}\alpha = \frac{L_t - L_0}{L_0\ t} = \frac{\text{Increase in length}}{\text{Original length} \times \text{rise in temperature}} \qquad (3)$$

From the above relation, since α is the ratio of two lengths therefore it will have no unit Therefore its value will be same in both the systems But its value will vary with the scale of temperature Smaller is the unit of temperature scale, smaller will be the value of α The unit of α is therefore put as per degree centigrade For a comparative study of the expansion a few coefficients have been given at the end of the book

§4. **Measurement of coefficient of expansion :**—[For greater details *see* A Text Book of Practical Physics by authors]

Pullinger's apparatus :—The rod is placed vertically in a jacket J A spherometer is placed at the top of the jacket such that its screw touches the upper end of the rod A thermometer is placed in the jacket

Working :—Measure the length of the rod and place it inside Move the screw till it touches the rod, note the reading of the spherometer Repeat this four or five times Note the temperature of the jacket Move the screw up and pass steam in the jacket When the temperature becomes steady again move the screw and when it touches the rod take its reading Repeat this also four or five times The difference of the two readings gives increase in length α can be calculated by the equation (3) given above

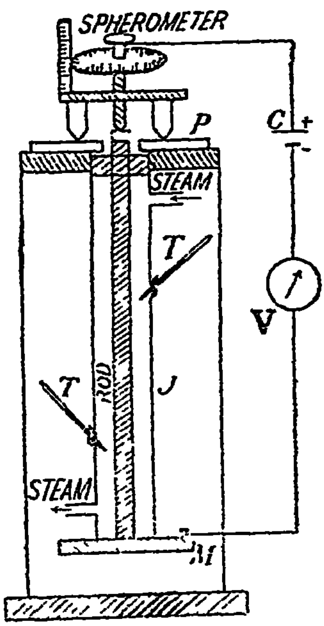


Fig 19

[Here in this case we have not started from 0°C but from t_1 °C Since L is small the value will came out to be the same Let L_{t_2} L_{t_1} and L_0 be the length of the rod at 0°C, t_1°C and t_2°C respectively According to relation (1)

$$L_{t_1} = L_0 + L_0\, \alpha\, t_1 \qquad (4)$$

and $$L_{t_2} = L_0 + L_0\, \alpha\, t_2 \qquad (5)$$

Subtracting (4) from (5) we get

$$L_{t_2}-L_{t_1}=L_0.\alpha.(t_2-t_1)$$

$$\therefore \quad \alpha=\frac{L_{t_2}-L_{t_1}}{L_0.\,(t_2-t_1)}=\frac{L_{t_2}-L_{t_1}}{L_{t_1}\quad(t_2-t_1)}$$

Because $L_{t_1}-L_0$ is negligible, therefore, we can put L_{t_1} for L_0]

Sometimes the apparatus is as shown in Fig. 19 (*a*). The working is the same as explained above In order to find out exactly when does the screw touches the rod, electrical connections are made as shown in the figure When the screw touches the rod the circuit will be completed and voltmeter will give deflection

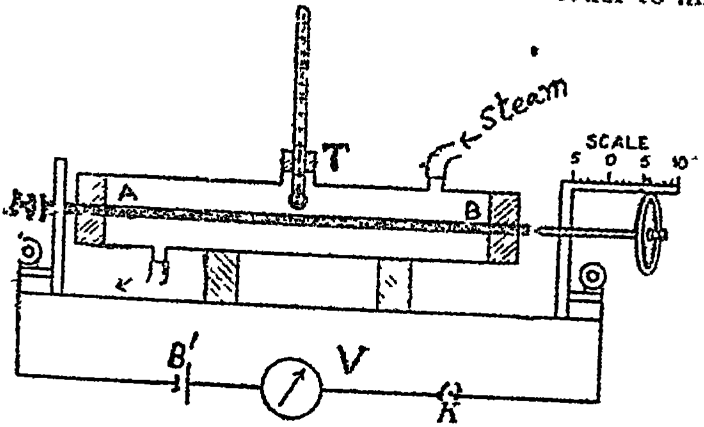


Fig 19 (*a*)

Sources of error:

1 Part of the rod is projecting outside the jacket and is therefore at lower temperature than the jacket

2 The rod is free to expand only in one direction

3 The screw also expands when it is in contact with the rod

Comparator Method :—The apparatus is shown in Fig 19 (*b*) A standard bar carrying two scratches at a distance of 1 metre is placed in the trough and the microscopes M_1 and M_2 are adjusted on the scratches Now the experimental bar with two scratches approximately at a distance of one metre is placed in the trough and the microscopes are adjusted on the scratches From the shifts of the microscopes the initial length of the bar at $0°C$ is found out Now the trough is heated to $t°C$ and again the microscopes are focused on the marks From the knowledge of the shift the length of the rod at $t°C$ is calculated knowing L_t and L_0, α can be calculated in the same way

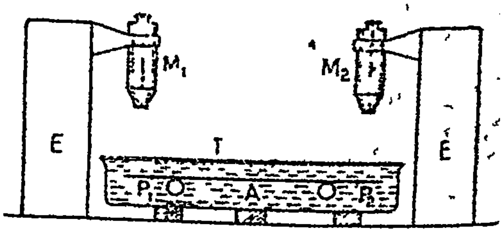


Fig 19 (*b*)

§5. **Superficial and cubical expansion :**—A body when heated also expands in area and volume Let S_0 and S_t be the area of a body at $0°C$ and $t°C$ Then as in the case of linear expansion, we have,

$$S_t-S_0=\beta\ S_0\ t$$

or

$$\beta=\frac{S_t-S_0}{S_0\ t}$$

where β is known as superficial coefficient of expansion It is defined *as increase in area per unit area per degree rise in temperature.*

From 7, $$S_t = S_o\,(1+\beta\,t) \qquad (9)$$

Similarly in the case of volume expansion,

$$V_t = V_0\,(1+\gamma\,t) \qquad (10)$$

Here γ is coefficient of cubical expansion *It is defined as increase in volume per unit volume per degree rise in temperature*

From equation 10, $\gamma = \dfrac{V_t - V_0}{V_0\,t}$

$$= \frac{\text{Increase in volume}}{\text{Original volume} \times \text{rise in temp}}$$

§6. Relation between α, β and γ :— Consider a square $A\,B\,C\,D$ (Fig 20) of side 1 cm When it is heated by 1°C it expands to $A\,B'\,C'\,D$ Considering linear expansion, $AB' = AB + AB\,\alpha\,1$

$$= (1+\alpha)$$

Similarly $AD' = (1+\alpha)$

$\therefore$ Area of the square $AB'C'D = (1+\alpha)^2$

$$= 1 + \alpha^2 + 2\alpha$$

Considering the superficial expansion

Area of the sq $AB'C'D'$ $S_t = S_0 + S_o\,\beta\,t$

$$= 1 + 1\;\beta\,1 = 1 + \beta$$

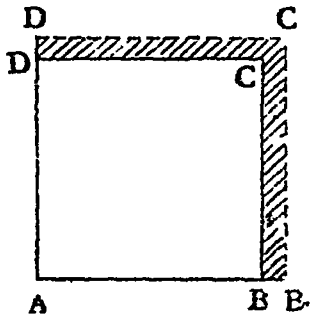


Fig 20

equalising both we get, values of area $AB'C'D'$

$$1 + \beta\,t = 1 + 2\alpha + \alpha^2$$

Since α^2 is still smaller and can be neglected

$$\therefore \qquad B = 2\alpha$$

Relation between α and γ :—Consider a cube of unit length Its volume v_0 will be 1 c c (Fig 21) When heated by 1°C each side becomes $1+\alpha$ cm and the total volume now becomes $(1+\alpha)^3$ Considering the volume expansion it becomes vt

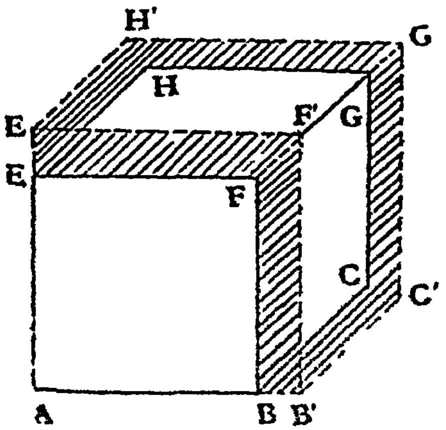


Fig 21

Now we have

$$vt = vo(1+\gamma t)$$

$$vt = (1+\alpha)^3 = 1 + 3\alpha^2 + 3\alpha + \alpha^3$$

$$\therefore\; 1 + \gamma t = 1 + 3\alpha + 3\alpha^2 + \alpha^3$$

neglecting $3\alpha^2$ and α^3 we get

$$\gamma = 3\alpha.$$

§7. Practical applications of expansion:—

(a) To put the rim on the bullock cart wheel:—First of all ...

rim is heated so that it expands then it is fitted on the wheel and cooled. On cooling it contracts and becomes tight on the wheel.

(b) **Some gap is left between two rails:**—Whenever two rails or any two girders in a bridge framework are joined some gap is left in between so that due to expansion in summer it may not bend and give way.

(c) **Fire bell:**—(Fig 21a) PQ and RS are two strips of different metals connected to a bell A and battery B. On account of any accident if fire breaks out in the house, the rods will be heated. RS expands more than PQ and so it will touch at X and the circuit will be completed and bell will ring

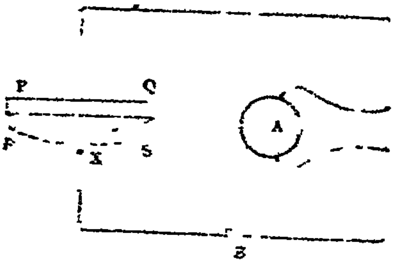


Fig 21(a)

(d) **Breaking of a glass:**—Whenever we put very hot tea or ice in a glass suddenly it breaks. Glass is a bad conductor of heat and therefore it does not spread in all directions. Some part of it expand or contract more than the other and therefore it breaks

(e) **To fix metallic wires in glass bulbs:**—In many electric goods we use metallic wires in a glass bulb. For this purpose we use platinum wires The expansion of platinum is equal to that of glass otherwise the glass will break.

(f) **Compensated rods pendulum:**—In case of pendulum clocks, the time of oscillation depends upon the length of the pendulum. The length of the pendulum changes with season and therefore clocks will go fast or slow. In order to eliminate this difficulty, compensated pendulums are used. It is shown in Fig 22 A, B and C are made of the same material and when they increase in length, pendulum will increase in length. D and E are made of different materials and when they expand bob will go up and the length of the pendulum will decrease. If the two expansions are equal the effective length of the pendulum will remain same. For this, following condition has to be satisfied—

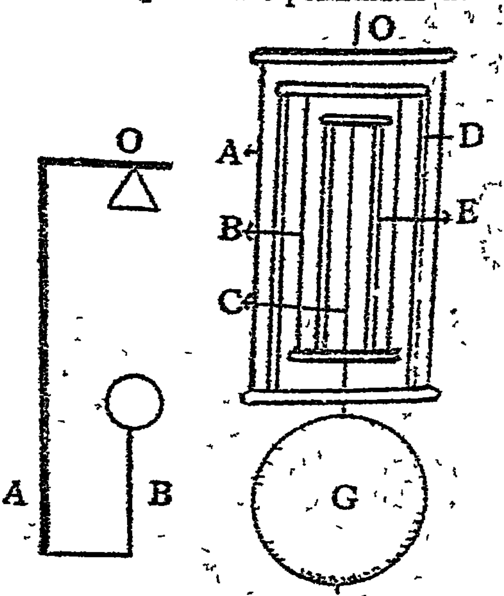


Fig. 22 and 22(a).

$$\frac{\text{Length of } (A+B+C)}{\text{Length of } (D+E)} = \frac{\text{Coeff of expansion of } D}{\text{Coeff. of expansion of } A \text{ or } B}$$

Another form of the pendulum is shown in Fig 22(*a*) here when *A* increases in length the bob will go down and when *B* increases in length the bob will go up If the two expansions are balanced the effective length will remain same

Balance wheel of the watch:—The wheel is shown in Fig 23 As the temperature increases the effective distance of the masses increases and the time of rotation of the wheel changes This is eliminated in such a way that the arcs of the wheel are made of two different materials, the outer one increases in length more than the inner one Due to this the masses come nearer and the effective distance remains the same

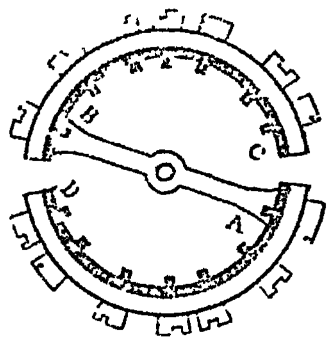


Fig 23

Correction due to expansion of scale.—(*See* Fig 24) Metallic scales also increase in length when their temperature rises Suppose graduations are marked when the temperature of the strip is 0°C Therefore, whenever, any length is measured with that scale it will give correct reading only when its temperature is 0°C If its temperature is higher, there will be some error in the readings taken by that scale

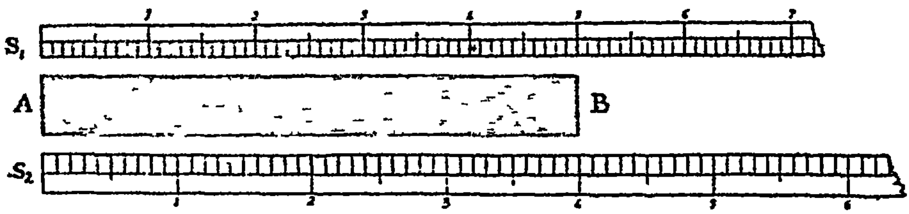


Fig 24

Suppose the coefficient of expansion of the material is α and the temperature of the scale is t°C Then every centimetre of the scale becomes $(1+\alpha t)$ cm due to expansion Therefore, when the length observed by that scale is 1 cm actual length will be $(1+\alpha t)$ cm and when the length observed by that scale is n cm actual length will be $n(1+\alpha t)$ cm Thus, we have,

Actual length=observed length $(1+\alpha t)$ *cm*

Numerical Problems:—1 *A zinc rod is measured with a copper scale which gives correct measurement at 0°C. At 10°C the length of the zinc rod is 1 0001 metres What is the true length at 0°C* [α *for zinc*=0 000029 *and* α *for copper*=0 000019]

In this question first we have to find the correct length of the zinc rod at 10°C by applying the correction due to expansion of the copper scale and then, we have to find the length of the zinc rod at 0°C

Observed length of the rod $=1{\cdot}0001$ metres $=100{\cdot}01$ cms.
Actual length of the zinc rod at 10°C $(L_t)=a(1+\alpha t)$

$$=(100{\cdot}01)(1+{\cdot}000019\times10)$$

$$=(100{\cdot}01)(1{\cdot}00019)\text{ cms.}$$

If the length of the zinc rod at 0°C is L_0, we have,

$$L_t=L_0(1+\alpha_2 t)\text{ where }\alpha_2={\cdot}000029$$

Substituting the values, we get

$$(100{\cdot}01)(1{\cdot}00019)=L_0(1+{\cdot}000029\times10)$$

$$\therefore \quad L_0=\frac{100{\cdot}01\times1{\cdot}00019}{1{\cdot}00029}=\frac{100{\cdot}01\times1{\cdot}00019}{1{\cdot}00029}$$

$$=100\text{ cms.}=1\text{ metre.}$$

2. *A pendulum made of brass beats seconds at 25°C. How many seconds a day will the clock gain if the temperature falls to 0°C (α for Brass$={\cdot}000019$)*

The clock beats seconds at 25°C, therefore, its periodic time at 25°C$=2$ sec. (T_{25}). Let the periodic time at 0°C be T_0. Let the length of pendulum at 0°C and 25°C be L_0 and L_{25} respectively.

Applying the formula of pendulum, we get,

$$T_0=2\pi\sqrt{\frac{L_0}{g}} \qquad \ldots(i)$$

$$T_{25}=2\pi\sqrt{\frac{L_{25}}{g}} \qquad \ldots(ii)$$

Dividing (*ii*) by (*i*), we get,

$$\frac{T_{25}}{T_0}=\sqrt{\frac{L_{25}}{L_0}} \qquad \ldots(iii)$$

But according to the formula for expansion, we get,

$$L_{25}=L_0(1+\alpha\,25)$$

or

$$\frac{L_{25}}{L_0}=1+\alpha\,.\,25=(1+{\cdot}000019\times25)$$

$$=1{\cdot}000475$$

Substituting this value in (*iii*), wet get,

$$\frac{T_{25}}{T_0}=\sqrt{1+0{\cdot}000475}=(1+0{\cdot}000475)^{\frac{1}{2}}$$

$$=(1+\tfrac{1}{2}\times0{\cdot}000475)$$

$$=1+0{\cdot}0002375$$

In 24 hours there are 86,400 seconds, therefore, number of oscillations made by the pendulum at 0°C and 25°C will be given by,

$$N_0=\frac{86400}{T_0}\text{ and }N_{25}=\frac{86400}{T_{25}}.$$

$\therefore$ Number of oscillations gained in one day at 0°C

$$N_0-N_{25}=\frac{86400}{T_{25}}\left[\frac{1}{T_0}-\frac{1}{T_{25}}\right]$$
$$=\frac{86400}{T_{25}}\left[\frac{T_{25}}{T_0}-1\right]$$
$$=\frac{86400}{T_{25}}[1+ 0002375-1]$$
$$=\frac{86400}{T_{25}}\times 0002375.$$

$\therefore$ No. of seconds gained $=\frac{86400}{T_{25}}\times 0002375\times 2$ secs

$\because$ $[T_{25}=2]$ $=\frac{86400}{2}\times 0002375\times 2$

$=20\ 52$ secs

3 *Temperature of a rod of iron of length one metre and cross-section one sq cm is raised by 100°C How much force will be required to prevent any increase in length? [$Y=20\times 10^{11}$ dynes per sq. cm, coeff of cubical expansion $=36\times 10^{-6}$ per deg C]*

In this problem we have to make use of expansion due to rise in temperature and elastic compression due to pressure applied According to the conditions of the problem, the two are equal

If L_0 and L_{100} are the lengths of the rod at 0°C and 100°C respectively, then we have,

$$L_{100}=L_0\ (1+\alpha\ 100)$$
$$=100(1+0\ 000012\times 100)$$
$$=100+100\times\ 000012\times 100$$
$$=100+0\ 12$$

If this length is to be compressed to L_0, contraction in length $(l)=L_{100}-100=\ 12$ Applying the formula for Young's modulus, we get,

$$Y=\frac{Mg}{A}\times\frac{L}{l}.$$

Here $Y=20\times 10^{11}$, $A=1$ sq cm., $L=100$ cms, approximately, $l=0\ 12$ cm

$$\therefore\ Mg=\frac{YAl}{L}=\frac{20\times 10^{11}\times 1\times\ 12}{100}=24\times 10^{8}\text{dynes}$$

QUESTIONS

1 Define the linear coefficient of expansion How will you measure it experimentally? (*See* §3 and §4)

2. Give the relation between (*i*) α and β and (*ii*) α and γ. (*See* §6).

3 Give some examples of practical applications of linear expansion (*See* §7)

Numerical Questions.—

1. Railway lines are laid with gaps to allow for expansion. If the gap between steel lines 66 ft. long is 0.[illegible] in. at 10°C, at what temperature will the lines just touch? (α for steel $= 11 \times 10^{-6}$ per deg C). [*Ans.* 67.4°C.]

2. A brass rod and a steel rod are both measured at 0°C and their lengths are found to be 120 and 120.2 cms. respectively. At what temperatures are their lengths equal? The coefficient of linear expansion of brass and steel are 0.0000197 and 0.000011 respectively. [*Ans.* 216.97].

3. A brass rod is measured by means of a zinc scale correct at 0°C and found to be 1 metre long at 0°C. What will be the observed length at 10°C, α for brass is 0.000019 and for zinc is 0.000029. [*Ans.* 99.99 cms.]

4. A brass pendulum keeps correct time at 0°C but loses 16 secs. at 20°C. Calculate the linear coefficient of expansion of brass. [*Ans.* ·0000185]

5. An iron rod 4 sq. cm. in cross-section is heated from 20°C to 300°C. If the temperature is now maintained constant, find the value of the forces which must be applied to the ends to compress it back to its original length.

[*Ans.* 1.23×10^{10} dynes]

CHAPTER VI

EXPANSION OF LIQUIDS

§1. Expansion of liquids :—Like solids liquids also expand when heated. Since their shape is not fixed we have only cubical expansion in the case of liquids. This coefficient of expansion of liquids is much larger than the coefficients of expansion of solids. You have read how this expansion of liquids is utilised in the construction of thermometers. Like solids in the case of liquids also expansion depends upon the nature of liquid. This can be shown by an experiment. Take three equal flasks as shown in Fig 25 and put three different liquids in them such that their level is same in all of them. Heat the water in the bath, you will find that the level of liquid becomes different in different vessels. (*See* Fig 25.)

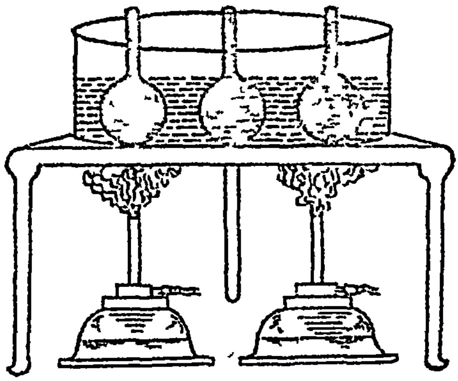


Fig 25

§2. Apparent and real coefficient of expansion :—Liquids are always heated in a certain vessel and the expansion in the volume is noted by reading its level. During the process of heating the vessel will also expand and, therefore, the level of liquid will change. In this way, the expansion of the vessel introduces the confusion.

If we imagine a liquid to be heated in such a vessel which does not expand, the expansion will be real expansion and the coefficient of expansion is known as real coefficient of expansion. *It is defined as real increase in volume per unit volume for 1° rise in temperature.*

Let V_0, V_t be the volumes of the liquid at 0° and t°C and C_r be the real coefficient of expansion.

Then, $$C_r = \frac{V_t - V_0}{V_0 \, t} \qquad ...(1)$$

or $$V_t = V_0(1 + C_r t) \qquad (2)$$

and real expansion $$V_t - V_0 = V_0 . C_r \; t \qquad ...(2a)$$

If we heat a liquid in a vessel which also expands but we do not take into consideration the expansion of the vessel, the expansion observed will be only apparent expansion, and the apparent coefficient of expansion is given by $$C_a = \frac{V_t - V_0}{V_0 \, t} \qquad (3)$$

or $$V_t = V_0\,(1 + Ca\,t) \quad \ldots (4)$$

and apparent expansion $$V_t - V_0 = V_0\,Ca\,t \quad \ldots (4a)$$

where V_0 and V_t are the apparent volumes of the liquid at 0°C and t°C and Ca the apparent coefficients of expansion.

To find the relation between Cr and Ca:—Take a long necked flask as shown in Fig 26 Put in it experimental liquid. Let the level of liquid be at A. Heat the flask At first the flask will be heated up and, therefore, it will expand and the level of liquid will go down Suppose, it falls up to B and then it begins to rise because now the liquid is also heated Suppose the final level of liquid is at C Then, the apparent expansion of liquid is equal to the volume AC, real expansion of liquid is equal to the volume BC while volume AB is the expansion in the vessel. Let the initial volume of the liquid and the vessel be Vo. Then we have,

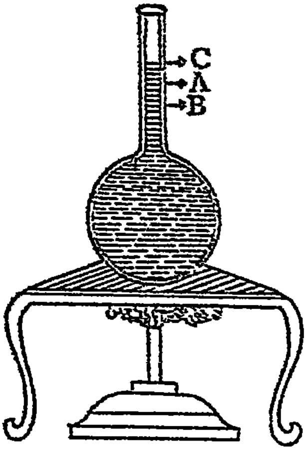


Fig 26

$$\text{Volume } BC = Vo\,Cr\,t \quad \ldots (5)$$
[from equation 2(*a*)]

$$\text{Volume } AC = Vo\,Ca\,t \quad \ldots (6)$$
[from equation 4(*a*)]

Similarly, considering the expansion of vessel,

$$\text{Volume } AB = Vo\ Cg.t \quad \ldots (7)$$

where Cg is the coefficient of cubical expansion of the vessel From the figure it is evident that.

$$\text{Volume } BC = \text{Volume } AC + \text{Volume } AB.$$

Substituting the values from eqns 5. 6 and 7, we get,

$$Vo\,Cr\,t = Vo.Ca\,t + Vo\,Cg\,t$$

or $$Cr = Ca + Cg \quad \ldots (8)$$

[This relation can also be derived as shown below.—

The vessel is graduated in c c These readings are correct at 0°C. When it is heated by t°C, every c c of it expands and becomes $1(1+Cg\,t)$ Therefore, the reading corresponding to C which we call Vat is actually $Vat(1+Cgt)$. If the real volume corresponding to C is Vrt we get,

$$Vrt = Vat(1 + Cg.t)$$

But we know that $Vat = Vo(1 + Ca\,t)$.

and $$Vrt = Vo(1 + Cr.t)$$

Substituting these values in the above equation

we get, $$Vo(1 + Cr.t) = Vo(1 + Ca.t)(1 + Cg.t)$$

or $$1 + Cr\,t = 1 + Ca.t + Cg.t + Ca.Cg.t^2$$

Because Ca and Cg are small, $Ca\,Cg$ can be neglected, therefore,

$$1+Cr\,t=1+Ca\,t+Cg.t$$

or
$$Cr=Ca+Cg.]$$

§3. Relation between density of a liquid at 0°C and t°C:— Let the mass of a liquid at 0°C be m grams, its volume be Vo c.c. and density do gram per c.c. Heat the liquid to t°C. Let its volume be now Vt c.c. and the density be dt gram per c.c. Since, mass remains the same we get,

$$m=Vo\,do=Vt\,dt$$

or
$$\frac{do}{dt}=\frac{Vt}{Vo}.$$

But
$$\frac{Vt}{Vo}=1+Cr\,t$$

Substituting this value in the above relation,

we get,
$$\frac{do}{dt}=1+Cr\,t$$

or
$$do=dt(1+Cr\,t) \quad \text{.. (9)}$$

or
$$dt=do(1+Cr\,t)^{-1}$$
$$=do(1-Cr\,t)$$

(neglecting higher powers $Cr\,t$) ..(10)

§4. To find out the apparent coefficient of expansion. (*i*) **Weight thermometer**—Weight thermometer is shown in Fig 27. It is a small bulb of glass with a narrow neck twice bent at right angles. Find out the weight (M_1) of the thermometer. Dip the mouth of the tube in the experimental liquid and gently heat it, air inside will expand and some of it will pass out. Again cool the bulb, due to contraction in air, vacuum will be created and due to atmospheric pressure some liquid will enter in. In this way, by alternate heating and cooling, fill the whole bulb with liquid. Finally, place it in melting ice and fill it. Again find out its weight (M_2). Now suspend the thermometer in a liquid bath maintained at t°C. Liquid will expand, bulb will also expand but since the expansion of liquid is more, some of the liquid will be expelled. Remove the thermometer and after cooling it find out its weight (M_3) again. Coefficient of apparent expansion Ca can be calculated as shown below —

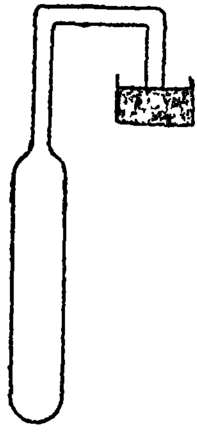


Fig 27

Mass of liquid expelled $m=M_2-M_3$ gram

Mass of liquid remained inside at t°C, $M=M_3-M_1$ gram

Mass of liquid present inside at 0°C $=M+m$ gram

Let the volume of the weight thermometer be V c.c. Here, we neglect the expansion of the thermometer.

$M+m$ gram occupy the volume V c.c. at 0°C.

$\therefore$ 1 gram occupies $\dfrac{V}{M+m}$ c.c. at 0°C.

Again, M gram occupy V c.c. at t°C.

$\therefore$ 1 gram occupies $\dfrac{V}{M}$ c.c. at t°C.

$\therefore$ expansion in volume of 1 gram of liquid

$$=\frac{V}{M}-\frac{V}{M+m}$$

$$=\frac{V(M+m)-Vm}{M(M+m)}$$

$$=\frac{VM+Vm-VM}{M(M+m)}$$

$$=\frac{Vm}{M(M+m)}.$$

Initial volume of 1 gram of liquid $=\dfrac{V}{M+m}$

Coefficient of expansion $=\dfrac{\text{Increase in volume}}{\text{Original vol.}\times\text{rise in temp.}}$

$$\therefore \qquad Ca=\frac{\dfrac{mV}{M(M+m)}}{\left(\dfrac{V}{M+m}\right)t}$$

$$=\frac{mV}{M(M+m)\times t}\times\frac{(M+m)}{V}.$$

$$=\frac{m}{Mt}=\frac{\text{Mass expelled}}{\text{Mass remained}\times\text{rise in temp}} \quad \text{....(II)}$$

In this way we can find the apparent coefficient of expansion of a liquid and if we know the coefficient of expansion of glass vessel we can find the true coefficient or real coefficient by the formula given in equation (3). But it is difficult to find the coefficient of expansion of glass; secondly it depends upon the sample of glass taken and upon its previous history. Therefore this method is not suitable for measuring *Cr*.

Numerical problems :—1. *A weight thermometer weighs 40 grams when empty and 490 grams when filled with mercury at 0°C. On heating it to 100°C, 6·85 grams of mercury escapes. Calculate the linear expansion of glass. (Cr for mercury=0·000182).*

Mass of the mercury at 0°C $=M+m=490-40=450$ grams.

Mass of mercury expelled $m=6{\cdot}85$ grams.

Mass of mercury remained $M=450-6{\cdot}85=443{\cdot}15$ grams.

Rise in temperature $t=100$°C

$$\therefore \qquad Ca = \frac{m}{M\,t} = \frac{685}{4431\,5 \times 100} = \frac{685}{4431500}$$

$$= \frac{685}{4432000}$$ 4431500

Calculations :—

log 685 $= 2\ 8357$

log 4432000 $= \frac{6\ 6466}{\bar{4}\ 1891}$

Ant log $\bar{4}\ 1891 = 0001545$

$\therefore\ Ca = 0001545$

$Cg \quad = Cr - Ca$

$= \quad 000182$

$-\ 0001545$

0000275

$\therefore\ \gamma g = \frac{Cg}{3} = 000009$

2 *A weight thermometer contains 510 gms of mercury at 15°C. On placing it in a hot oil bath only 500 gms are left in it. Find the temperature of the oil bath (Cr of mercury= 00018 and γg=·00001)*

Let the temperature of the oil bath be t°C

Mass expelled (m) $= 510 - 500 = 10$ gms

Mass remained (M) $= 500$ gms

Rise in temperature $= (t-15)$

$$\therefore \qquad Ca = \frac{10}{500\ (t-15)} \qquad \ldots (1)$$

Again, $Ca = Cr - Cg = 00018 - 3 \times 00001$

$= 00015$

Substituting the value of Ca in (1), we get,

$$00015 = \frac{10}{500\ (t-15)}$$

or $$(t-15) = \frac{10}{500 \times 00015} = \frac{10 \times 100000}{500 \times 15}$$

$= 133\ 3$

$t = 133\ 3 + 15 = 148\ 3$°C

3 *A specific gravity bottle holds 50 grams of liquid at 30°C. How much will it hold at 100°C? [Cr= 00051, Cg= 00003].*

Let the mass remained at 100°C $= M$ gram

$Ca = Cr - Cg = 00051 - 00003$

$= 00048$

Again, $Ca = \frac{m}{M\,t}$ Here $m = (50 - M)$ gram

and $t = 100 - 30 = 70$°C

Substituting the values, we get,

$$00048 = \frac{50 - M}{M\ 70}$$

or $\quad M.70\times\cdot00048=50-M$

$$M\,(1+0\cdot0336)=50$$

or $$M=\frac{50}{1\cdot0336}=48\cdot3 \text{ grams.}$$

(*ii*) **Dilatometer method** :—This consists of a bulb whose volume at 0°C is known. It is attached to a stem graduated in c.c. The given liquid is filled in the bulb and its volume (V_o) is noted at say 0°C. It is then placed in a bath maintained at t°C and again its volume (Vt) is noted. Apparent coefficient of expansion is calculated by the formula

$$Ca=\frac{Vt-Vo}{Vo\,t}.$$

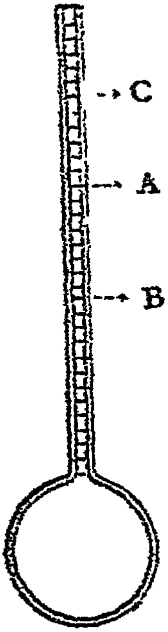


Fig 28

(*iii*) **Hydrostatic method** :—Take a solid and find out its weight in air. Suspend the solid in the given liquid at 0°C and again find out its weight. Heat the liquid to t°C and again find out the weight of the solid when it is inside the liquid at t°C. From these observations Ca can be calculated as given below.

Suppose the loss in weight of solid at 0°C$=mo$ gram and the loss in weight of solid at t°C$=m$ gram. Let the density of the liquid at 0°C and t°C be do, and dt respectively. Let the coefficient of cubical expansion of solid be Cg.

In both the cases, the apparent loss in weight is equal to the weight of liquid displaced.

. Weight of liquid displaced at 0°C$=mo$ gram

∴ Volume of liquid displaced at 0°C$=\frac{mo}{do}$ c.c.

Again, weight of liquid displaced at t°C$=m$ gram

Volume of liquid displaced at t°C$=\frac{m}{dt}$ c.c.

We know that the volume of liquid displaced is equal to the volume of the solid.

∴ Volume of the solid at 0°C$=\frac{mo}{do}$ c.c.

and volume of the solid at t°C$=\frac{m}{dt}$ c.c.

Substituting these values in the formula

$$Vt=Vo(1+C.t), \text{ we get,}$$

$$\frac{m}{dt}=\frac{mo}{do}(1+Cg\,t)$$

$$\frac{mo}{m}=\frac{do}{dt\,(1+Cgt)}$$

$$=\frac{1+Cr\,t}{1+Cgt}\left[\frac{do}{dt}=1+Cr\,t\right]$$

$$=(1+Cr\,t)(1+Cg\,t)^{-1}$$

$$=(1+Cr\,t)(1-Cg\,t\ \ldots)$$
$$=\{1+(Cr-Cg)t\ \ldots\}$$ neglecting higher powers of $Cg\,t$ and $Cr\,Cg\,t^2$

$$=1+Ca\,t$$

Numerical problem:—*A metal piece weighs 50 grams in air and 34 61 grams and 35 42 grams respectively when totally immersed in a liquid at 15°C and 65 C Calculate the coeff of linear expansion of solid [Cr of liquid= 00119]*

Here, $mo=50-34\,61=15\,39$ grams

and $m=50-35\,42=14\,58$ grams

We know that, $\frac{m}{mo}=\frac{1+Cg\,t}{1+Cr\,t}$

Subtituting the values, we get

$$\frac{14\,58}{15\,39}=\frac{1+Cg\,(65-15)}{1+\,00119(65-15)}$$

or $$1+50\,Cg=\frac{14\,58}{15\,39}\,(1+\,00119\times50)$$

$$=\frac{14\,58}{15\,39}\,(1\,0595)$$

$$\therefore\quad Cg=\frac{14\,58}{15\,39}\times\frac{1\,0595}{50}-\frac{1}{50}$$

$$=\,02007-\,02$$

$$=\,00007$$

Calculations:—$\sigma g=\frac{00007}{3}=\,000023$ *Ans*

log 14 58=1 1637	log 15 39=1 1873
log 1 060= 0253	log 50 =1 6990
1 1890	2 8863
−2 8863	
$\bar{2}$ 3027	

Anti log $\bar{2}$ 3027= 02007

§5. To find the true coefficient of expansion Cr:—On account of the difficulties given in §4, Dulong and Petit found out the real coefficient of expansion by the following method. *ABCD* is a long tube of glass bent twice at right angles as shown in Fig 29. This tube is filled with mercury. *AB* and *CD* are surrounded by outer jackets J_1 and J_2. Steam is circulated in jacket J_1 and ice cold water in J_2. In this way the temperature of mercury in *AB* is t°C and in *CD* 0°C. Let the height of mercury in *AB* above *B* be *Ht* and in *CD* above *C* be *Ho*. Since, *BC* is in the same horizontal line, pressure at *B* is equal to pressure at *C*. Now, pressure at *B* is equal to the atmospheric pressure plus the pressure exerted by the mercury column *AB*. We know that the pressure exerted by a liquid column is *hdg*. Let the atmospheric pressure be *P*, therefore, we have,

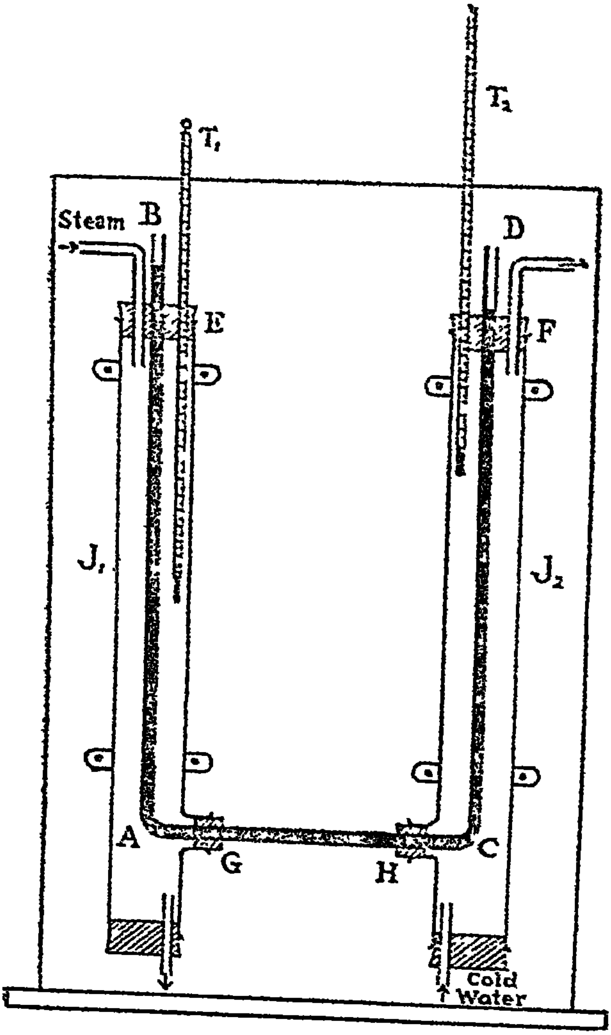


Fig 29

$$\text{Pressure at } B = P + Ht\,dt\,g$$

and $$\text{Pressure at } C = P + Ho.do\,g$$

equating the two, $$P + Ht\,dt\,g = P + Ho\,do\,g$$

or $$Ht.dt = Ho\,do$$

$$\therefore \quad \frac{do}{dt} = \frac{Ht}{Ho}$$

But $$\frac{do}{dt} = 1 + Crt$$

$$\therefore \quad 1 + Crt = \frac{Ht}{Ho}$$

or
$$C_t.t=\frac{Ht}{Ho}-1$$
$$=\frac{Ht-Ho}{Ho}$$

$$\therefore \qquad C_t=\frac{Ht-Ho}{Ho\,t} \qquad (12)$$

Knowing Ht and Ho we can find out C_t. Since the pressure of liquid depends only upon height it will not be affected by the expansion of the tube. Tube BC is kept narrow so that conduction of heat from B to C is minimised.

Errors in the Dulong and Petit's methods:—(*i*) The upper level of mercury is not at the same temperature and the curvature of the layers will be different, therefore, it will be difficult to read the level.

(*ii*) Temperature is recorded by mercury thermometer.

(*iii*) There is no stirring arrangement for maintaining the temperature constant.

These defects were removed by Callender in his method.

A U-tube contains mercury, its limbs are maintained at 0°C and 100°C respectively. If the cold column were 60 cms high and the hot one 1·08 cms higher, what would be the coefficient of absolute expansion of liquid?

Here $Ho=60$, $Ht-ho=1.08$ cms, $t=100°C$.

C_t ?

$$C_t=\frac{Ht-Ho}{Ho\,t}=\frac{1.08}{60\times 100}=0.0018.$$

§6. **Callender's method**—In this method a tube $HFBA\,CDEG$ as shown in Fig 30 is taken and mercury is filled in it. Every part of the tube except CD is maintained at 0°C by circulating or dropping ice cold water. CD is surrounded by a jacket in which some oil is filled which is maintained at t°C by an electric heater. It is kept well stirred by long electric stirrers. The temperature of the bath is noted by means of platinum resistance thermometers. The height of mercury columns is noted by a cathetometer which is a travelling telescope.

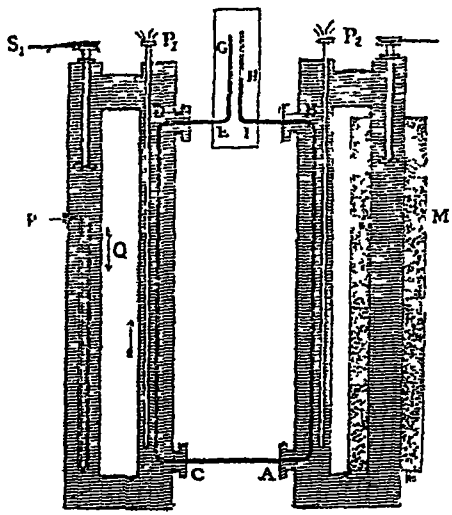


Fig 30

Let the height of AB, CD, HF, EG be Ho', Ht, ho, $h'o$ respectively.

As shown above, pressure at C
=pressure at A

or $\quad P+ho'\,dog+Ht\,dtg=P+ho\,dog+Ho\,dog$

or $$Ht\, dt.g = Ho\, do\, g + ho\, do.g - ho.'do\, g$$

or $$Ht\, dt = Ho\, do + ho\, do - ho'do$$
$$= do(Ho + ho - h'.o)$$
$$\frac{do}{dt} = \frac{Ht}{Ho + ho - h'o}.$$

But $$\frac{do}{dt} = 1 + Cr\, t$$
$$1 + Cr\, t = \frac{Ht}{Ho + ho - h'o}$$
$$Cr\, t = \frac{Ht}{Ho + ho - h'o} - 1$$
$$= \frac{Ht - Ho - ho + h'o}{Ho + ho - h'o}$$

$\therefore$ $$Cr = \frac{Ht - Ho - ho + h'o}{(Ho + ho - h'o)t} \quad ...(13)$$

In this way Cr can be calculated

§7. **Anomalous expansion of water :**—If we heat water from 0°C it contracts up to 4°C In this range, its coefficient of expansion is negative When the temperature increases beyond 4°C. it begins to expand Therefore, its density increases up to 4°C and then it begins to decrease It is greatest at 4°C This was proved by Hope in the following manner

Hope's experiment :—The apparatus is shown in Fig 31. It is a metal cylinder surrounded by another cylindrical jacket A and B are two thermometers placed in it It is filled with water and the outer vessel is filled with freezing mixture Before starting the experiment, the temperature in both the thermometers is the same Later on we find that temperature of B falls rapidly. Because, when the temperature of water in the upper part falls down, it becomes heavier and goes down This process goes on till it becomes 4°C Now, any further cooling increases the volume of water and decreases its density; so it becomes lighter and it does not come down Now the temperature of A will fall down and will reach 0 and even water in upper part may freeze

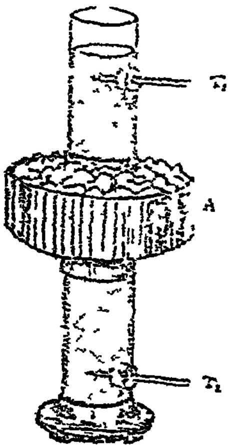


Fig 31.

How life is possible in frozen seas :—We know that large oceans in north and south are generally frozen throughout the year Yet there is life in them The reason is the anomalous expansion of water So far as temperature of water is above 4°C, cold water from the top layer goes down and hot water from the lower layers comes up In this way convection currents are set and the whole

water cools down But, when temperature becomes $4°C$, any further cooling makes the water lighter and it will not go down till it freezes, while the water below it is at $4°C$.

§8. **Practical applications—(*a*) Thermostat :—**This is an apparatus made of glass. It is filled with liquid and is constructed in such a way that when it is placed in a hot bath, the liquid expands and controls the supply of gas When the temperature rises to a fixed value the liquid expands in such a way that the supply of gas is cut off and the flame is almost extinguished When the temperature falls down again, the supply increases and temperature rises In this way temperature of the bath is maintained at a constant value

(*b*) **Barometric correction**—(*i*) *Correction due to expansion of a scale* —Generally the readings of a barometer are taken on brass scale which is correct at $0°C$ Therefore, when we take its reading at $t°C$, there will be some error When a scale is heated by $t°C$, every cm. of it becomes $1+\sigma t$ cm where σ is the linear coefficient of expansion Therefore, when observed reading with that scale is Ht cm, actual reading will be $Ht\ (1+\sigma.t)$ cm

(*ii*) *Correction due to variation of density* —When we take the reading of barometer at $t°C$, the density of mercury is less than its density at 0° and, therefore, the height of mercury column required to balance the atmospheric pressure will be more Unless the readings are converted in terms of mercury column at $0°C$ it is not taken as standard and cannot be compared with any other reading taken at some different temperature If Ho be the reduced height at $0°C$ we have,

$$Ho.do.g=Ht\ (1+\alpha t)\ dt\ g$$

or

$$Ho=Ht.\frac{dt}{do}(1+\sigma t)$$

But,

$$\frac{dt}{do}=(1-Cr\ t)$$

$$Ho=Ht\ \ (1-Cr\ t)\ (1+\sigma t)$$

$$=Ht\ \{1-(Cr-\sigma)t\} \qquad .(14)$$

neglecting $Cr\ \sigma\ t^2$

(*c*) **Exposed stem correction:—**When we use a thermometer, only its bulb and part of the stem are placed inside the liquid Part of stem projects outside The thread of mercury which is in the outer part of the tube is at lower temperature than the bath If this had also been at the temperature of the bath, the readings will be somewhat higher This correction is known as exposed stem correction

Suppose, the correct temperature is t, while the observed temperature is t_1 Suppose n divisions of the thermometer are exposed outside and suppose they are at room temperature $t_2°C$ If these n divisions are heated to $t°C$, they will expand by $n\ Ca\ (t-t_2)$, where

Ca is the apparent coefficient of expansion Therefore, the correct temperature t is given by

$$t=t_1+n\,Ca\,(t-t_2)$$

Since t and t_1 are nearly equal, we can put t_1 for t on right hand side and we get.

$$t=t_1+n\ Ca\,(t_1-t_2) \qquad \text{..(15)}$$

Numerical Problems:—1 *A mercury barometer has a brass scale which is correct at 0°C. If the barometer reads 75 cms. at 20°C. calculate its true height at 0°C (For brass* α*=·000018 and for mercury* Cr*=·00018)*

Here. $H_t=75$ cms $t=20$ $\alpha=\cdot000018$, $Cr=\cdot00018$ $H_0=?$

$$\begin{aligned} H_0&=H_t\{1-(C_r-\alpha)t\}\\ &=75\{1-(\cdot00018-\cdot000018)20\}\\ &=75\{1-0\cdot000162\times 20\}\\ &=75\times\cdot99676=74\cdot757 \text{ cms} \end{aligned}$$

2 *The stem of a thermometer from 40° upwards is outside the liquid, whose temperature the thermometer reads as 80°C If the average temperature of the stem outside the liquid is 50°C. and if* C_a*=·00016, find the true temperature of the liquid*

Here, $n=80-40=40$ $t_2=50°$, $t_1=80°$ and $C_a=0\cdot00016$ $t=?$

Substituting these values in the above formula.

$$\begin{aligned} t&=t_1+n\ Ca.(t_1-t_2)\\ &=80+40\ (0\cdot00016)\ (80-50)\\ &=80+40\times\cdot00016\times 30\\ &=80+\cdot19200\\ &=80\cdot192°C \end{aligned}$$

QUESTIONS

1 Define the coefficient of apparent expansion and real expansion of a liquid and derive the relations between them (*See* §2).

2 How will you measure the apparent coefficient of expansion by (*i*) Weight thermometer (*ii*) Hydrostatic method (*See* §4).

3 Prove $do=dt\,(1+Cr.t)$ (*See* §3)

Give Callender's or Regnault's method for measuring Cr (*See* §6)

4 Derive the expression for barometric correction and exposed stem correction for a thermometer (*See* §8).

5. What do you mean by anomalous expansion of water? How is it useful? (*See* §.....)

Numerical Questions:—

1. A weight thermometer weighs 6·34 grams when empty and 151·73 grams when filled with mercury at 99°C If 2·08 grams have been expelled in changing the temperature from 0 to 99°C, determine the coefficient of relative expansion of mercury in glass [*Ans* ·000144]

2 A weight thermometer contains 51 grams of mercury at 0 C When placed in an oil bath 9 grams is found to overflow Find the temperature of the bath (Cr 00018 and Cg= 000026) [*Ans* 1391 46°*C*]

3 A weight thermometer contains 500 grams of mercury at 0°*C* What mass of mercury will overflow if the temperature is raised to 100°*C* (Ca= 00015) [*Ans* 7 389 grams]

4 A column of mercury 76 35 cms at 100 *C* balances another column 75 cms at 0 *C* Calculate the coefficient of real expansion of mercury [*Ans* 00018]

5 A Fortin's barometer reads 75 38 cms at 30°*C* Find the reading of barometer corrected for temperature at 0 *C* (α for brass= 000018, Cr= 00018) [*Ans* 75 01 cms]

6 The bulb of a mercury thermometer and the stem up to 0° mark are immersed in hot water at 100 *C* while the remainder of the stem is in the air at 20 *C* What will be the reading of the thermometer ? [*Ans* 98 77°*C*]

7 A piece of glass weighs 47 grams in air, 31 53 grams in water at 4°*C* and 31 75 grams in water at 60°C Find the coefficient of expansion of water [*Ans* 0000276]

CHAPTER VII

EXPANSION OF GASES

§ 1. **Introduction :**—While considering the expansion of solids and liquids we have not considered the effect of changes in pressure on volume, because this effect is negligible. But when we come to the case of gases the effect of pressure is well marked. It has been shown by Boyle, that by increasing the pressure, volume can be reduced to $\frac{1}{2}$, $\frac{1}{10}$ etc and by decreasing pressure it can be made many fold. Therefore, the volume of a gas is not merely a function of temperature but of pressure as well. In fact, we can heat a gas in two ways—at constant pressure and at constant volume. In the first case, volume will increase and in the second case pressure will increase. In this way, we have two co-efficients of expansion (*i*) **Volume coefficient of expansion** α **and** (*ii*) **Pressure coefficient of expansion** β.

§ 2. **Volume Coefficient of expansion** (α) :—*It is defined as the ratio of the increase in volume* for 1°C rise in temperature to the original volume at 0°C pressure remaining constant. Let Vo and Vt be the volumes of the gas at 0°C and t°C respectively, then, we get,

$$\alpha=\frac{Vt-Vo}{Vo\,t} \qquad \ldots(1)$$

or

$$Vt=Vo\,(1+\alpha\,t) \qquad \ldots(2)$$

Pressure Coefficient of expansion β :—*It is defined as the ratio of the increase in pressure for 1°C rise in temperature to the original pressure at 0°C, volume remaining constant. If Po* and *Pt* are the pressures at 0°C and t°C respectively, then we have,

$$\beta=\frac{Pt-Po}{Po\,t} \qquad \ldots(3)$$

or

$$Pt=Po\,(1+\beta.t) \qquad \ldots(4)$$

§ 3. **Relation between** α **and** β :—Consider a certain mass of a gas enclosed in a vessel. Let its pressure, volume and temperature be Po, Vo and To respectively. Now, heat the gas at constant pressure Po so that its temperature becomes $T°$ i e $(To+t)$ and volume becomes Vt, then according to equation 2, we get,

$$Vt=V_o\,(1+\alpha t)$$

or

$$\frac{Vt}{Vo}=1+\alpha t \qquad \ldots(5)$$

Similarly, if, we heat the gas, keeping its volume constant so that its temperature is t°C and pressure becomes Pt, we get according to equation (4)

$$Pt=Po\,(1+\beta\,t)$$

or

$$\frac{Pt}{Po}=1+\beta\,t \qquad \ldots(6)$$

If we compare (2) and (3) state we find that temperature is constant and we can apply Boyle's law,

$$\therefore \qquad Po.\ V_t = Pt.V_o$$

or
$$\frac{V}{V_o} = \frac{P_t}{P_o} \qquad \ldots(7)$$

$\therefore$ From (5) and (6) we get

$$1+\alpha t = 1+\beta\, t$$

or
$$\alpha = \beta \qquad \ldots(8)$$

Thus α is numerically equal to β.

§ 4. **Absolute zero and absolute scale** —Experimentally the value of α and β comes out to be $\frac{1}{273}$ If we substitute $t=-273$ in equation (2) and (4) we get,

$$V_t = V_o\left\{1+\frac{1}{273}\times(-213)\right\}$$
$$= V_o\{1-1\} = 0$$

Similarly,
$$P_t = P_o\left\{1+\frac{1}{273}\times(-273)\right\}$$
$$= P_o\{1-1\} = 0$$

Thus we find that at $-273°C$ the volume occupied by the gas will reduce to 0 This is impossible and hence, this is considered to be the lowest temperature and is known as absolute 0 If we take this point ($-273°C$) as 0°, then 0°C will be read as 273° and 10°C will become $273+10$ *i e* 283° and so on This scale is known as absolute scale or Kelvin scale because it was first suggested by Lord Kelvin One degree of this scale is equal to 1°C but it differs only in its lowest point $t°C = 273+t°$ absolute Generally, absolute temperatures are denoted by capital T The relation between absolute and other scales is shown in Fig 4, Chapter 2

§5. **Charles' Law** —If temperature is recorded in absolute scale, equations (2) and (4) can be written as shown below —

$$V_t = V_o\left(1+\frac{1}{273}\,t\right)$$
$$= V_o\left(\frac{273\cdot t}{273}\right)$$
$$= V_o\,\frac{T}{T_o}$$

or
$$\frac{V_t}{V_o} = \frac{T}{T_o} \qquad \ldots(9)$$

where
$$T = 273+t°C$$

and
$$T_o = 273+0°C$$

Similarly,

$$Pt = P_o\left(1+\frac{1}{273}.t\right)$$

$$=P_o\left(\frac{273+t}{273}\right)$$

$$=P_o \cdot \frac{T}{T_o}$$

or
$$\frac{P_t}{P_o}=\frac{T}{T_o} \qquad .. (10)$$

equations (9) and (10) are known as Charles' law.

§5 **Relation between P, V and T of a certain mass of gas—Equation of state :**—Consider a certain mass of a gas contained in a vessel at temperature T_1, pressure P_1 and volume V_1 [*See* Fig 32(*i*)] Heat the gas to $T_2°$ so that its volume becomes V, pressure remaining P_1 Fig 32 *ii*) Now compress the gas to pressure P_2 so that volume becomes V_2, temperature remaining constant T_2 [Fig 32 (*iii*)]

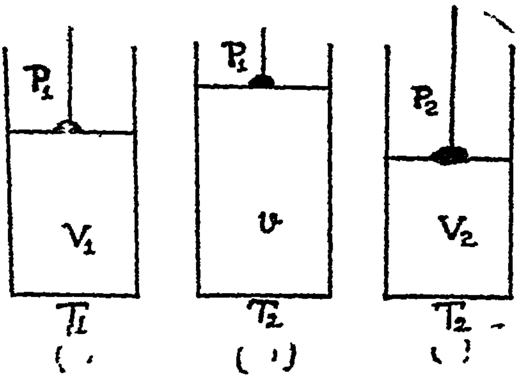


Fig 32

If we consider the change from (*a*) to (*b*), applying Charles' law, we get,

$$\frac{V}{V_1}=\frac{T_2}{T_1} \text{ or } V=V_1\frac{T_2}{T_1} \qquad .. (11)$$

From state (*ii*) to (*iii*) temperature is constant and we can apply Boyle's law

$$P_1.V=P_2 V_2$$

or
$$V=\frac{P_2 V_2}{P_1} \qquad .. (12$$

Comparing the value of V from eqns (11) and (12), we get,

$$V_1\frac{T_2}{T_1}=\frac{P_2V_2}{P_1}$$

or
$$\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}$$

It means the quantity $\frac{PV}{T}$ *for a given mass of gas remains* constant, *this constant is denoted by* R *and is known as* gas constant, and the relation $\frac{PV}{T}=R$ is known as gas equation

$$\frac{PV}{T}=R \qquad (13$$

The value of R depends upon the nature of the gas and quantity of gas taken. With the help of gas equation we can reduce a given volume of a gas at certain temperature and pressure to 0°C an

76 cm pressure These are known as normal temperature and pressure and are denoted by N T P Generally, constants like sp gravity, sp heat, etc of a gas are given at N T P

§7. To calculate the value of gas constant in a few cases:—

Numerical Problems:—1 *Calculate the constant for one gram of hydrogen (Given, the density of hydrogen at N T P is ·0899 gram per litre and density of mercury is 13 6 grams per c.c).*

According to gas equation, we have,

$$R = \frac{P_0 V_0}{T_0}$$

Here $P_0 = 76$ cm of mercury

$= 76 \times 13\ 6 \times 980$ dynes per sq cm

$= 1\ 01 \times 10^6$ dynes per sq cm.

$$V_0 = \frac{M}{d_0}$$

$$= \frac{1}{0899} \text{ litres}$$

$$= \frac{1000}{0899} \text{ c c}$$

$T_0 = 0°C$

$= 273$ absolute

Substituting these values, we get,

$$R = \frac{76 \times 13\ 6 \times 980}{273} \times \frac{1000}{0899}$$

$$= \frac{76 \times 136 \times 98}{273 \times 899} \times 10^7$$

$= 4\ 126 \times 10^7$ ergs per °C

Calculations:—

log 76 = 1 8808	log 273 = 2 4362
log 136 = 2 1335	log 899 = 2 9538
log 98 = 1 9912	5 3900
6 0055	
−5 3900	
0 6155	

Antilog 0 6155 = 4 126

2 *Find out the gas constant for 1 gram of air. (Given density of air at N T P = 1 293 gram per litre and density of mercury 13 6 grams per c c)*

As explained in above example,

$$R = \frac{76 \times 13\ 6 \times 980}{273} \times \frac{1000}{1\ 293}$$

$$= \frac{76 \times 136 \times 98}{273 \times 12930} \times 10^7$$

$= 2869 \times 10^7$ ergs. per degree centigrade.

3. *Find out the value of gas constant for 1 gram molecule of a gas*

We know that every gram molecule of a gas occupies 22·4 litre at N T P Substituting this value of V_o in the gas equation, we get

$$R = \frac{76 \times 13{\cdot}6 \times 980}{273} \times 22{\cdot}4 \times 1000$$
$$= 8{\cdot}3 \times 10^7 \text{ ergs.}$$

§8. Experimental determination of volume coefficient of expansion—Regnault's method :—Description :—The apparatus is shown in Fig (33) *B* is a large bulb of glass of known volume. It is connected by means of a capillary tube to a three way stop-cock and a graduated glass cylinder *D*. A side tube *F* is attached to *D* and a stop-cock *E* is fixed at the lower end. Cylinder *D* and part of the tube *F* is placed in a water bath maintained at constant temperature. Mercury is filled in *D* and *F*.

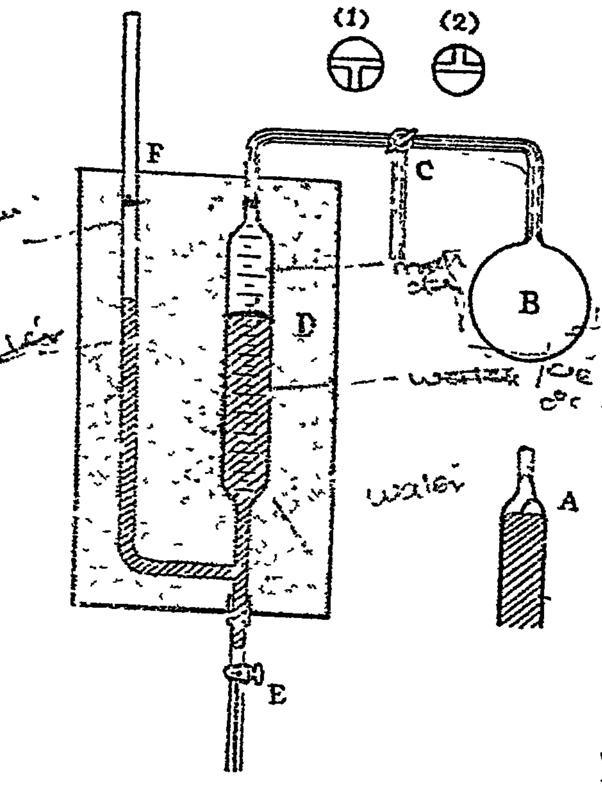


Fig. 33

Working :—First of all we connect the bulb *B* and cylinder *D* to the exhaust pump by putting the stop-cock in position (1) and take out the air Then fill the experimental gas in it Turn the stop-cock in position (2) so that *B* is in contact with *D* and cut out from external source The bulb is then placed in melting ice and by adding mercury in *F* or taking out from *E*, the level is adjusted at the top of the cylinder The difference in level of *F* and *D* is read by cathetometer The bulb is then placed in steam and after some time the level of mercury is so adjusted that the difference in level in *F* and *D* is the same; it means pressure before and after, remains the same Note the reading in the cylinder *D* The difference of the two readings gives the increase in volume. Original volume is known From these α can be calculated

$$\alpha = \frac{\text{Increase in volume}}{\text{Original volume at } 0^\circ \times t}$$

Sources of error :—(*i*) Part of the gas in the capillary tube and cylinder *D* is at lower temperature than the bulb. (*ii*) Some gas is

enclosed in tube D and capillary tube whose volume is not included in V_o (iii) Bulb also expands due to heating

§9. **Experimental determination of pressure coefficient**:—This can also be determined by the same apparatus In this case liquid bath is useless and the graduations are not necessary A pointer is attached in D as shown separately at A The gas is filled in the bulb in the same manner The bulb B is placed in melting ice and the level of mercury is adjusted in such a way that it touches the pointer The reading of the level in D and F is taken by cathetometer Then the pressure of the enclosed gas is equal to atmospheric pressure $\pm$ the difference in level in F and D, $P_o=(P\pm d_1)$ This is P_o Now place the bulb in steam and again adjust the mercury in such a way that it touches the pointer Read the difference in level again Find out the pressure of the gas P_{100} by the relation $P_{100}=(P\pm d_2)$ Calculate β by the following expression

$$\beta=\frac{P_{100}-P_o}{P_o\times t}$$

Note:—When the level of mercury in F is higher than in D, + sign is taken, when the level of mercury in F is lower than in D, − sign is taken

§10. **Constant volume air thermometer**:—The pressure of a gas increases with temperature This property of the gas can be utilised for measuring temperature in the same manner as expansion of the mercury has been utilised in mercury thermometers These thermometers are known as constant volume air thermometers

Construction:—The thermometer is shown in Fig 34 A is a big bulb of known volume connected to a cylinder C through a stop-cock V A pointer P is attached in C C is connected to another reservoir B through a rubber tubing As explained above the air from A and C is exhausted and then dry air is filled in A Mercury is filled in C and B

Working:—Place the bulb in melting ice and adjust the reservoir B in such a way that mercury touches the pointer P Take the reading of mercury at B and P on the scale fixed on the stand Find the difference in levels (d_1), then the pressure of the gas is $P\pm d_1$, where P is the atmospheric pressure Place the bulb in steam and again adjust

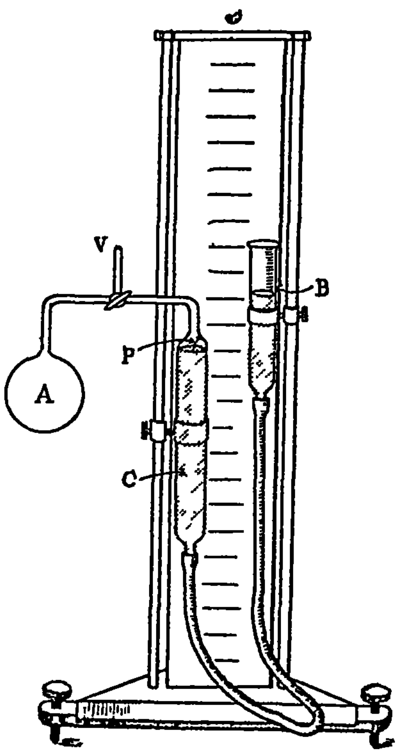


Fig 34

the reservoir in B till the mercury in C touches the pointer P Again note the difference in level d_2 and find out $P_{100}=P+d_2$ Now place the bulb in the bath whose temperature is to be determined Find out the pressure P_t in the same manner Then the corresponding temperature t can be calculated as shown below

Increase in pressure for 100° rise in temperature

$$=P_{100}-P_o$$

∴ Increase in pressure for 1° rise in temperature

$$=\frac{P_{100}-P_o}{100}. \quad \dots (i)$$

Again increase in pressure for t° rise in temperature

$$=P_t-P_o$$

∴ Increase in pressure for 1° rise in temperature

$$=\frac{P_t-P_o}{t}.$$

Equating (i) and (ii), we get

$$\frac{P_{100}-P_o}{100}=\frac{P_t-P_o}{t} \quad \dots (ii)$$

or

$$t=\frac{Pt-P_o}{P_{100}-P_o}\times 100 \quad \dots (14)$$

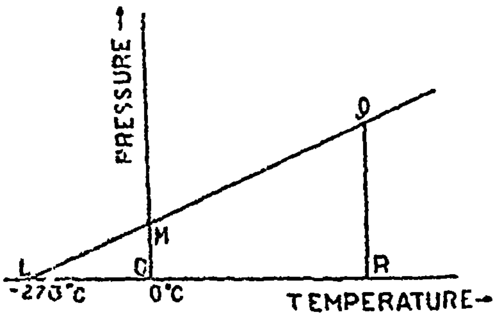


Fig 35

The above apparatus can be used for finding out β also $\beta=\frac{P_{100}-P_o}{P_o\times 100}$. We find out the value of pressure corresponding to various temperatures and plot a graph between pressure and temperature as shown in Fig 35 From the graph, P_o can be found out and then β can be calculated. [For greater details see author's Text Book of Practical Physics].

§11. Standard hydrogen constant volume thermometer:—Its principle and working is the same as that of air thermometer. There are a few modifications to make it more suitable for practical application The thermometer is shown in Fig. 36 B is one metre long bulb of an alloy of a platinum and iridium of capacity one litre It is connected to M by a narrow capillary. An ivory pointer P is fixed in M. M is connected to another cylindrical tube M' which is connected to a reservoir R which can be raised or lowered Dry hydrogen gas is filled in B and mercury is filled in M, M' and R A barometric tube bent as shown in the figure is filled with mercury and inverted in M' and is fixed in a stand.

This serves the purpose of a barometer. The height of mercury in the tube above M' gives the value of atmospheric pressure.

Principle:—Its principle is the same as that of an thermometer which has been discussed above. If P_o, P_{100} and P_t are the pressures of the gas corresponding to temperature 0°, 100° and t°, then the unknown temperature t is given by

$$t=\frac{P_t-P_o}{P_{100}-P_o}\cdot 100.$$

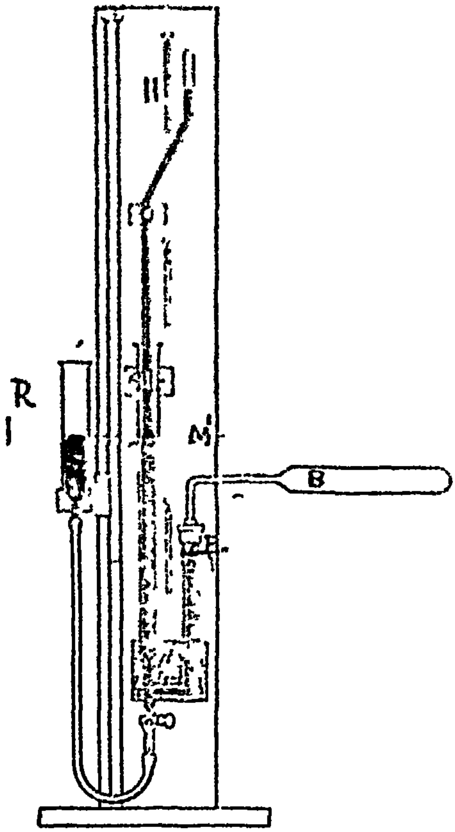


Fig 36

Working:—Place the bulb B in melting ice and adjust the reservoir R in such a way that mercury in M touches the pointer P. Now the pressure of the enclosed gas is equal to atmospheric pressure + the difference in level between M and M'. Atmospheric pressure is given by the difference in level between H and M'. Therefore, the pressure of the enclosed gas is directly given by the difference in level between H and M. This difference is directly read by means of a cathetometer. This gives P_o.

Place the bulb in steam at 100°C and again adjust the reservoir R such that mercury in M touches the pointer P. Again read the difference in level of mercury between M and H. This gives P_{100}.

Place the bulb in the bath whose temperature is to be determined and again adjust the level in M so that it touches P and read the difference. This gives P_t.

Calculate t by the following formula

$$t=\frac{P_t-P_o}{P_{100}-P_o}\times 100$$

Advantages of a gas thermometer over a mercury thermometer:—(*i*) The range of a gas thermometer is very large [from −258°C to 1600°C], while a mercury thermometer can be used from −39°C to 356°C only.

(*ii*) The coefficient of expansion of gases is more than that of mercury and, therefore, expansion of a gas will be more than that of mercury and hence, percentage error will be less in reading it.

(*iii*) Coefficient of expansion is the same for all gases and, therefore, purity of the gas will not affect the result. This is not the case with mercury.

(iv) Expansion of hydrogen is more regular than mercury, therefore temperature read on a gas scale will be more accurate than that read on a mercury scale.

(v) There is no question of change of zero reading

(vi) By providing the bent barometric tube the total pressure is known only by reading the level at H and thus the number of observations is reduced Thereby the chances of errors are eliminated.

Disadvantages :—(i) It is very big and cumbersome and cannot be used conveniently

(ii) It is not direct reading.

(iii) The bulb is very large and, therefore, requires large amount of substance and in that case it becomes difficult to maintain the temperature of the substance uniform throughout

(iv) The whole gas is not at the same temperature as that of the bath Here the bulb is made large enough so that the amount of air in the capillary tube may be negligible in comparison to the bulb.

This thermometer is therefore not used for day to day purposes but is employed in national laboratories to standardise mercury thermometers and is, therefore, known as standard thermometer

Numerical Problems :—1 *The pressure of air in the bulb of a constant volume air thermometer is 73 cm. 100·3 cm and 77 8 cm. at 0°C, 100°C and t°C respectively. Calculate the value of t.*

$$t=\frac{P_t-P_o}{P_{100}-P_o}\times 100=\frac{77\ 8-73}{100\ 3-73}\times 100=17\ 6°C.$$

2. *A sample of gas occupies 100 c.c at 18°C and 72 cm. pressure and a volume of 200 c c at 90°C and 45 cm pressure Calculate the coefficient of expansion, assuming that the gas obeys Boyle's law and expands uniformly at constant pressure.*

Since we have to find out volume coefficient of expansion, we should know the volume of the gas at 90°C and 72 cm pressure

Applying Bolye's law, we get,

$$P_1 V_1=P_2 V_2$$

$$72\ V_1=45\times 200$$

$$\therefore \quad V_1=\frac{45\times 200}{72}=\frac{5\times 200}{8}=125 \text{ c c}$$

Now, $V_{90}=V_o\ (1+\alpha\ 90)$

and, $V_{18}=V_o\ (1+\alpha.18)$

$$\therefore \quad \frac{V_{90}}{V_{18}}=\frac{1+\alpha\ 90}{1+\alpha\ 18}$$

Here $V_{90}=125$ c c $V_{18}=100$ c c therefore, substituting the values, we get,

$$\frac{125}{100}=\frac{1+90\alpha}{1+18\alpha}$$

or $$5(1+18\alpha)=4(1+90\alpha)$$

$$90\alpha-360\alpha=4-5$$

or $$-270\alpha=-1$$

$\therefore$ $$\alpha=\frac{1}{270}$$

3 *The volume of a certain mass of air is 100 c c at 20°C. What will be its volume at 50°C.*

We know that $\frac{V_1}{V_2}=\frac{T_1}{T_2}$

Here $T_1=273+20=293$, $V_1=100$ c c

$T_2=273+50=323$, $V_2=?$

Substituting the values in the above equation, we get,

$$\frac{100}{V_2}=\frac{293}{323}$$

or $$V_2=\frac{100\times323}{293}$$

$$=110\ 3 \text{ c c}$$

4 *A flask containing air at the atmospheric pressure is corked up at 35°C and heated in an air bath. A pressure of 3 atmospheres inside the flask will force the cork out Find the temperature at which this will happen*

We know that $\frac{P_1}{P_2}=\frac{T_1}{T_2}$

Here $P_1=1$ atmosphere, $T_1=273+35$°C

$P_2=3$ atmospheres, $T_2=?$

$$\frac{T_2}{T_1}=\frac{P_2}{P_1}$$

$\therefore$ $$T_2=\frac{P_2}{P_1}\times T_1=\frac{3}{1}\times308$$

$$=924$$

$\therefore$ $$t_2\text{°C}=924-273=651\text{°C}$$

5 *At N T P the volume of some oxygen is 175 c c Find the volume at 51°C, when the barometer reads 75 cm*

According to gas equation,

$$\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}$$

Here $P_1=76$, $T_1=273$, and $V_1=175$ c c

$P_2=75$ cm, $T_2=(273+51)$°C and $V_2=?$

$$\frac{P_2V_2}{T_2}=\frac{P_1V_1}{T_1}$$

or
$$P_2=\frac{P_1V_1}{T_1}\times\frac{T_2}{P_2}$$

Substituting the values, we get,

$$P_2=\frac{76}{75}\times\frac{324}{273}\times\frac{175}{1}$$
$$=210\cdot4 \text{ c c}$$

Calculations:—

log 76 = 1 8808	log 75 = 1 8751
log 324 = 2 5105	log 273 = 2 4362
log 175 = 2 2430	4 3113
6 6343	
−4·3113	
2 3230	

Anti log 2 3230 = 210 4 c c

6. *Calculate the change in volume during the expansion of one gram of hydrogen when heated from 25°C to 26°C against the atmospheric pressure of 1 million dynes per sq. cm. ($R=8\cdot3\times10^7$ ergs/gram molecule).*

From the gas equation, we have.

$$PV_1=RT_1$$
$$PV_2=RT_2$$

$\therefore$
$$P(V_2-V_1)=R(T_2-T_1)$$

or
$$V_2-V_1=\frac{R\,(T_2-T_1)}{P}$$

Here
$$R=\frac{8\,3\times10^7}{2},$$
$$P=10^6,\ T_2-T_1=26-25=1.$$

Substituting these values in the above equation, we get,

$$V_2-V_1=\frac{8\cdot3\times10^7\times1}{2\times10^6}$$
$$=\frac{8\,3\times10}{2}$$
$$=41\,5 \text{ c.c.}$$

QUESTIONS

1. Define volume coefficient of expansion How will you determine it experimentally? (See §2 and §8).

2. Define pressure coefficient of expansion How will you determine it experimentally? (See §2 and §9)

3. Prove that $\alpha=\beta$. (See §3).

4 Derive the gas equation and find out the value of gas constant for 1 gram molecule of any gas. (See §6)

5. Describe the construction and working of a constant volume air thermometer or a constant volume hydrogen thermometer. What are its advantages over mercury thermometers and what are its disadvantages? (See §10 and §11).

6 What is absolute scale or Kelvin scale (*See* §4)

Numerical Questions :—

1 A volume of 50 c c of air at 15°*C* is expelled from the bulb of a constant pressure air thermometer by changing the temperature from 0°*C* to 100°C Calculate the temperature when 10 c c are expelled neglecting the expansion of the bulb ($\alpha=\frac{1}{273}$) [*Ans* 3°*C*]

2 In a constant volume gas thermometer, the pressure is 54 6 cm at 0°*C* and at 100°*C* it is 74 4 cm If the coefficient of volume expansion glass is 00003 Find the coefficient of increase of pressure of gas [*Ans* 0 0036]

3 The mercury level in the closed limb of a constant volume air thermometer stands at 30 cm and that in the open limb at 32 4 cm when the bulb is in melting ice, 61 1 cm. when in steam and 22 4 cm when in freezing mixture Find the temperature of the freezing mixture [*Ans* −34 84°*C*]

4 A gram of perfect gas at 27°*C* has the pressure on it halved and is then cooled until it occupies the same volume as at first, what is the final temperature ? [−123°*C*]

5 1000 c c of a gas has a temperature of 21°*C* and a pressure of 798 mm Find the mass of the gas if the density of the gas at N T P is 1 2 gram per litre [*Ans* 1 17 gram]

CHAPTER VIII

VAPOUR PRESSURE

§1. Introduction :—Every liquid boils at a particular temperature known as boiling point but it evaporates more or less at all temperatures When we put over wet clothes even in shade they are dried up The water from lakes and oceans is constantly evaporating.

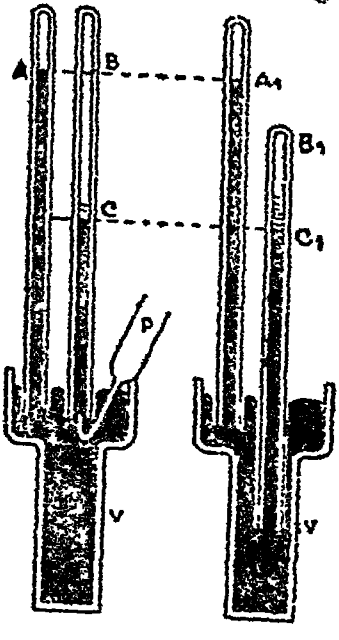


Fig 37

§2 Vapour pressure :—Just as air exerts pressure in the same way vapours of every liquid exert pressure. This can be demonstrated by means of the following experiment *A* and *B* are two barometric tubes filled with mercury [*See* Fig. 37(*a*)] The height of mercury in each tube is equal to atmospheric pressure. Now introduce a few drops of a liquid, say water, by means of a bent pipette in tube *B*. We shall see that the drops of water go up and then evaporate in the upper space The level of mercury in *B* falls down. It shows that the vapours of water exert pressure. This pressure is equal to the difference in level of the mercury in the two tubes This pressure is vapour pressure or vapour tension

§3. Saturated and unsaturated vapour :—In the above experiment as we introduce more and more water the level in *B* will fall more and more This means that the vapour pressure of a liquid depends upon the amount of vapour contained in a given space If we go on introducing water in the tube *B* it will be found that after a certain stage no more water evaporates. The space above mercury has become saturated with water vapour Any extra amount of water will remain floating on the mercury. *The pressure exerted by the vapour in this condition (when the space is saturated) is maximum and is known as saturated vapour pressure.*

Saturated vapour pressure and temperature :—In the above experiment when the space becomes saturated with vapours, if we gently warm the tube we will find that some more liquid evaporates and the level goes on falling down till again the space becomes saturated with vapours at that temperature

It shows that saturated vapour pressure increases with temperature

If we start from the condition when the space above is unsaturated *i e* no water drops are present on the mercury surface

and go on cooling the tube we shall find that after sometime drops of water will appear to condense on the surface and the space will become saturated at a particular temperature *Thus unsaturated vapours can be made saturated at lower temperature*

If we take unsaturated vapour and heat it, volume remaining constant, pressure will increase In the same way if, we cool the unsaturated vapour, pressure will decrease till at a particular temperature the vapour pressure is equal to the saturated vapour pressure and below this the pressure falls rapidly with temperature according to fall in saturated vapour pressure with temperature *It follows, therefore, that volume remaining constant, unsaturated vapour pressure varies with temperature (Charles' law)*

Saturated vapour pressure and the space available :—In the above experiment when the space above mercury is saturated at a particular temperature, if we push the tube *B* inside the cistern so that the space above is reduced, more liquid will condense but the difference in level of the two tubes will remain the same, [*See* Fig 37 (*b*)] If, on the other hand, we pull out the tube, more liquid will evaporate and again the difference will remain the same *It shows that saturated vapour pressure does not depend upon the amount of space available i e*, upon its volume so far as temperature remains constant *That is saturated vapour pressure is independent of volume Therefore, it does not obey Boyle's law*

If we start our experiment when the space above is unsaturated and change the available space, we shall find that when we increase the volume, pressure will decrease and when we decrease the volume pressure will go on increasing till a stage is reached when the space becomes saturated with that much amount of vapour and any further reduction in volume will not change the pressure *Thus we find that unsaturated vapours obey Boyle's law*

If we take unsaturated vapours in a cylinder and increase the pressure, temperature remaining constant, volume will decrease as in the case of Boyle's law After a certain stage, when the volume is reduced to such an extent that the space becomes saturated, any further increase in pressure will start liquification

Thus, we conclude the following results.—(*i*) *Every vapour exerts pressure which depends upon the nature of the liquid*

(*ii*) *Unsaturated vapour pressure depends upon the amount of vapour, amount of space available i e, volume and temperature*

(*iii*) *Unsaturated vapours obey Boyle's law and Charles' law*

(*iv*) *Unsaturated vapour can be made saturated by adding more vapour, by decreasing temperature or by decreasing volume*

(*v*) *Saturated vapour pressure is the maximum vapour pressure at that temperature.*

(*vi*) *Saturated vapour pressure does not depend upon the amount of liquid or amount of space available but depends only upon temperature*

(vii) *Saturated vapour does not obey Boyle's and Charles' law.*

(viii) *Saturated vapour can be made unsaturated by increasing the space or temperature.*

Isothermal curves :—If we study the relation between vapour pressure and volume at a constant temperature, we shall get curves as shown in Fig. 38 The curves are shown for CO_2 These are known as isothermal curves If we start from *A* as we increase pressure volume decreases as in the case of gases till we reach *B* Here condensation begins, and a slight increase in pressure will result in a large decrease in volumes because liquids occupy much less space than vapour. At *C*. the whole of the vapour has been converted into liquid Any further increase in pressure results into a very small decrease in volume, because liquids are incompressible. If we plot this relation at some higher temperature. we shall get a similar curve *EFGH* but the horizontal part *GF* will be smaller. If we go on increaseing the temperature. the horizontal part is reduced to a point (*Q*) only. The pressure required to liquify the vapour at this temperature is known as critical pressure and the volume occupied at this temperature and pressure is known as critical volume If the vapour is above this temperature, it cannot be liquefied whatever may be the external pressure applied. We call such a vapour a gas. Therefore, there is no fundamental difference between a vapour and a gas. *Above critical temperature every vapour is a gas, below critical temperature every gas is a vapour.*

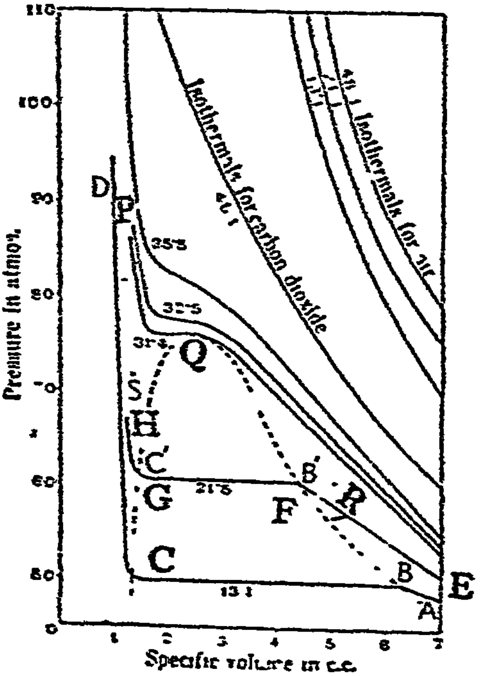


Fig. 38.

Since, the critical temperature of carbon dioxide ammonia, is lower than room temperature they can be liquified by the application of pressure alone. While in case of nitrogen, hydrogen, helium etc. the critical temperature is very low and unless the gas is pre-cooled below that temperature it cannot be liquefied by the application of pressure alone. Therefore, they were called permanent gases by early experimenters.

§ 4. **Methods of measuring Vapour pressure at different temperatures :**—Though the principle and working of these methods is the same as discussed above, yet they vary in minor details in the method of heating

(*i*) **Below 0°C :**—In this case, the barometric tube *B* is made in the form as shown in Fig 39 The liquid is placed in the bulb *C* which is placed in the freezing mixture The difference in level of *A* and *B* gives the vapour pressure of the liquid at the temperature of the mixture

(*ii*) **From 0 to 50°C :**—In this method, both the tubes are surrounded by means of an outer jacket A liquid is maintained at constant temperature in the jacket The given liquid is introduced in *C* till the space above becomes saturated and some liquid remains

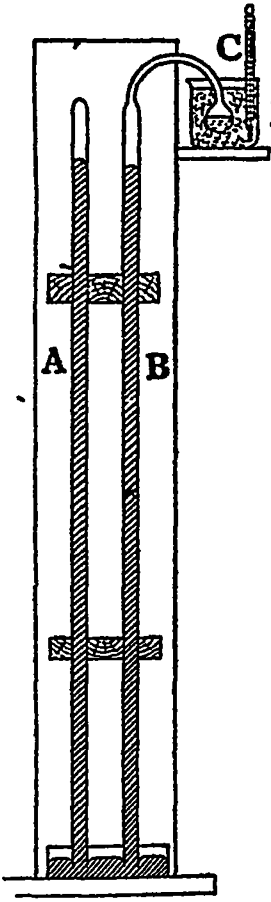


Fig 39

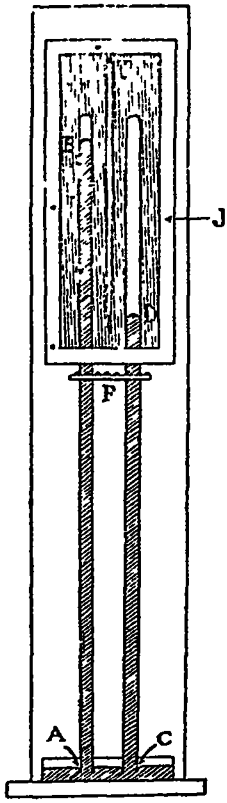


Fig 40

floating on the mercury surface The difference in level in the two tubes gives the vapour pressure of the liquid at the temperature of the bath

For measuring vapour pressures above 50°, these methods are unsuitable For this purpose a different principle is used

§ 5. **Relation between vapour pressure and superincumbent (external) pressure acting in the liquid** —Whenever a liquid begins to boil at any particular temperature, *the saturated vapour pressure of that liquid at that temperature (boiling point) is equal to superincumbent*

pressure We also know that the boiling point of a liquid varies with the external pressure acting in the liquid. By varying this pressure, a liquid can be made to boil at any temperature and then the external pressure acting on the liquid is noted. This will be equal to the vapour pressure of that liquid at that temperature This method is known as Dynamical method

Ramsey and Young's method—Description :—(*See* Fig 41). *A* is a wide boiling tube *H* is a thistle runnel and *T* a thermometer *S* is the stop-cock of the funnel. The experimental liquid is filled in the funnel. The lower end of the funnel is drawn out to a fine point

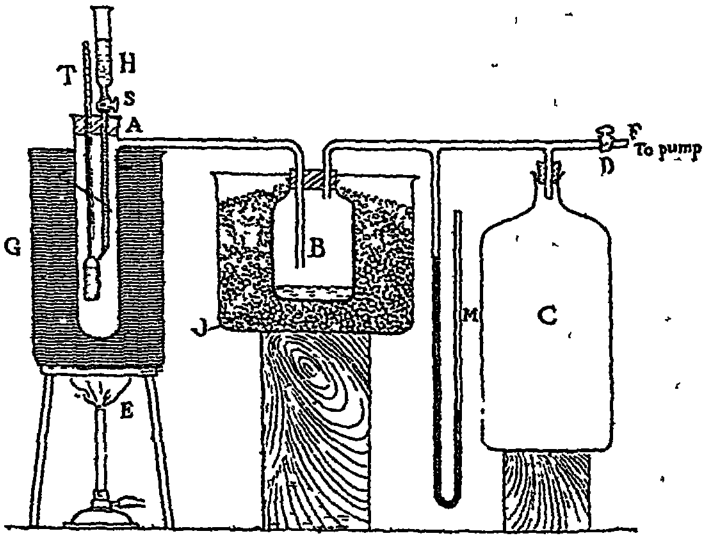


Fig 41.

which is near the bulb of the thermometer The bulb of the thermometer is wrapped round with glass wool or asbestos. *B* is a large bottle immersed in ice and *C* is another large bottle *M* is a manometer *A*, *B* and *C* are connected to a pump through the tube *F* and stop-cock *D* *A* is placed in a bath *G* containing some liquid.

Working :—Heat the liquid bath *G*, a few degrees higher than the boiling point of the experimental liquid at a given pressure Drop the liquid from the funnel slowly As soon as it drops on the bulb it evaporates on account of the large surface When the temperature of the thermometer becomes steady, the pressure from the manometer is noted This gives the vapour pressure of the liquid at the temperature given by *T* A series of readings are taken by varying the external pressure Bottle *B* collects the liquid again and bottle *C* is provided for stabilising the pressure. This method is known as Dynamical method

QUESTIONS

1 Define vapour pressure and saturated vapour pressure. (*See* § 2 and §3)

2 Unsaturated vapours obey gas laws while saturated vapours do not obey Explain (*See* § 2)

3 What is the difference between a vapour and a gas (*See* § 3)

4 Describe a method for measuring S V. P between 50 and 100°C. (*See* § 5)

CHAPTER IX

HYGROMETRY

§1. **Hygrometry:**—We know that on a hot summer day we do not feel as much stuffy as on a rainy day when the temperature is comparatively less This is due to the amount of water vapour present in the atmosphere The study of water vapour present in the atmosphere forms an important part of weather conditions and air conditions This science is known as Hygrometry

§2. **Relative humidity:**—*The amount of moisture present in one cu. foot of space is known as* **humidity**. *But in actual case we are more concerned with relative humidity than with absolute humidity*

Relative humidity:—*It is defined as the ratio of the amount of water vapour actually present at any temperature to the amount of moisture required to saturate it* This is generally expressed in percentage

$$\text{Thus, } R\,H = \frac{\text{Amount of moisture actually present}}{\text{Amount of moisture required to saturate it}} \times 100 \quad . \quad (1)$$

$$= \frac{m}{M} \times 100,$$

where m=Amount of moisture actually present,

and M=Amount of moisture required to saturate it at the same temperature

§3. **To find relative humidity:**—In order to find relative humidity we use chemical hygrometer The apparatus is shown in Fig 42

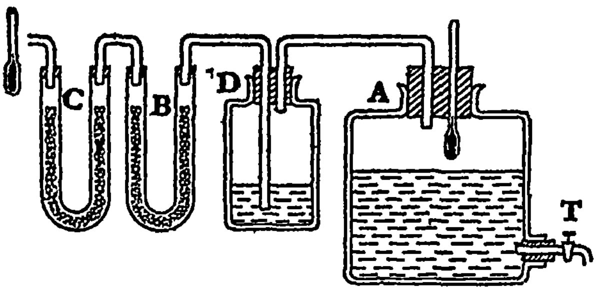


Fig. 42.

A is an aspirator full of water *B* and *C* are two tubes carrying calcium chloride *D* is a catch bottle containing sulphuric acid *B* and *C* are first weighed and then connected to *D* The tap *T* is opened and water is allowed to run out Vacuum will be produced inside *A* and air from outside will rush in through the *U* tubes After some time mark the level of water in *A* and weigh the tubes *B* and *C* again, the difference of the two weights gives the amount of moisture actually present in a certain space Again the aspirator is filled with water and the weighed tubes *B* and *C* are fixed to *D* Again the tap is opened and the water is run out to the same level This time the air from outside will first bubble through water in another bottle before entering *C* and thus will become

saturated with moisture Again we find out the weight of B and the difference of weights, this time, gives M Relative humidity is calculated by the formula

$$R.H. = \frac{m}{M} \times 100.$$

§4. **Relation between Relative humidity and vapour pressure:—** We have read in the previous chapter that the vapours of a liquid exert pressure. therefore, water vapour will also exert pressure. This vapour pressure is proportional to the amount of moisture present in the space Therefore we can also write $R.H.$ as shown below.

$$R.H. = \frac{\text{Amount of moisture actually present}}{\text{Amount of moisture required to saturate}} \times 100$$

$$= \frac{\text{Actual vapour pressure}}{\text{Saturated vapour pressure}} \times 100. \quad \text{.. (2)}$$

§5. **Dew point :—**Generally the moisture present at any temperature is not sufficient to saturate the space. But, if, we go on cooling the surrounding space a stage may be reached when that very amount of moisture may be sufficient to saturate the same space If we cool the space further, condensation will start and drops of moisture will appear. *This temperature at which condensation starts is known as dew point* It follows that the actual vapour pressure at any temperature is equal to the saturated vapour pressure at dew point. Therefore, equation (2) can be modified as below—

$$\therefore R.H. = \frac{\text{Saturated vapour pressure at dew point}}{\text{Saturated vapour pressure at room temperature}} \times 100 \quad \text{...(3)}$$

Standard charts are available which give the values of saturated vapour pressure at different temperatures Therefore in order to find relative humidity we require the determination of dew point.

§6. **Determination of dew point by Daniell's Hygrometer:—**Two bulbs A and B are connected by means of a wide tube as shown in Fig. 43. First, air from the bulbs is removed and some ether is placed in. Bulb A contains ether and bulb B and rest of the tube contain ether vapour. A thermometer is placed in A. The outer portion of A is silvered. Another thermometer is fixed on the stand This will give the room temperature. A muslin cloth is tied on B and ether is dropped over it. When it evaporates, cooling is produced and some of the ether vapour in B condenses Vacuum is created in B and ether from A evaporates Due to this, the sides of the bulb A are cooled down. Go on doing this till some mist appears on bulb A. Note this temperature. Again allow the muslin to dry and mist on A will again evaporate Note this temperature again. The mean of these gives the dew point.

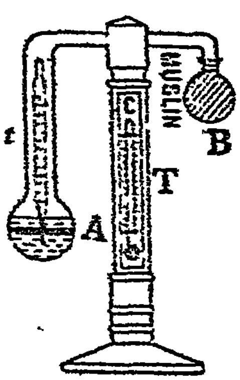
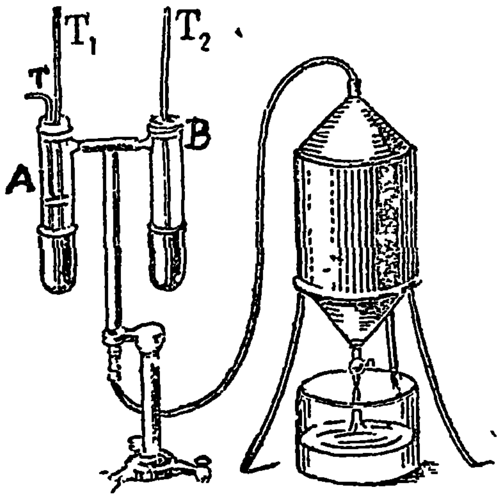


Fig. 43.

Defects of the experiment :—(*i*) Ether, which evaporates outside *B*, contaminates the air and this affects the hygrometric state of air. (*ii*) It is not possible exactly to find when does the mist appear (*iii*) The temperature given by the thermometer may be different from the temperature of the surface of *A* (*vi*) Outside and inside temperatures are not the same (IV)

§7. Regnault's Hygrometer:—The hygrometer is shown in Fig 44 *A* and *B* are two test tubes carrying silver thimbles at their bottom A is the experimental tube and *B* is provided for the purposes of comparison *A* is connected by means of a rubber tubing to an aspirator Some ether is placed in *A* such that the bent tube *T* dips inside ether T_1 and T_2 are two thermometers placed in the tubes to record the temperature



Fig, 44

Working:—When the tap of aspirator is opened, water comes out and vacuum is created inside Air from outside is sucked in through the tube *T* and it bubbles through ether On account of low pressure and bubbling, ether evaporates and takes the necessary latent heat from the rest of the ether and test tube On account of this, the temperature of the tube falls The flow of water is regulated till some mist appears on the outer portion of test tube *A* This can be judged easily by comparing it with the bottom of test tube *B* Note the temperature of T_1 Stop the flow of water and wait till the mist again disappears Note the temperature again The mean of these two temperatures give the dew point Relative humidity can be calculated by the formula,

$$R\,H = \frac{\text{S V P at dew point}}{\text{S V P at room temperature}} \times 100$$

Advantages of Regnault's method.—(*i*) The rate of evaporation of ether can be regulated by regulating the rate of flow of water

(*ii*) Silver being best conductor temperature inside and outside will be the same

(*iii*) Ether is kept well stirred by the bubbling of air through it

(*iv*) The appearance of flow can be judged by comparing the surfaces of *A* and *B*

(v) In order to avoid any disturbances on account of the breathing of the observer, observations can be taken by means of a telescope kept far apart

§8. **Dry and wet bulb hygrometer :**—The hygrometer is shown in Fig 45. *A* and *B* are two thermometers fixed on a stand The bulb of *B* is covered by muslin or cotton wool which dips inside a vessel *D* containing water On account of large exposed surface, water from the wet cloth evaporates and a cooling is produced on the bulb. The reading of *B* is thus always less than reading of *A*. Greater is the rate of evaporation, greater will be the difference in temperature of *A* and *B*. Now the rate of evaporation depends upon the humidity conditions of the atmosphere When moisture is less in the atmosphere, rate of evaporation will be more and *vice versa* Thus, the difference of temperature directly depends upon the humidity conditions This difference in temperature can be calibrated in terms of humidity by comparing the difference of temperature with the humidity found by any of the earlier methods A table is thus prepared and from that table humidity corresponding to any observed difference of temperature can be calculated.

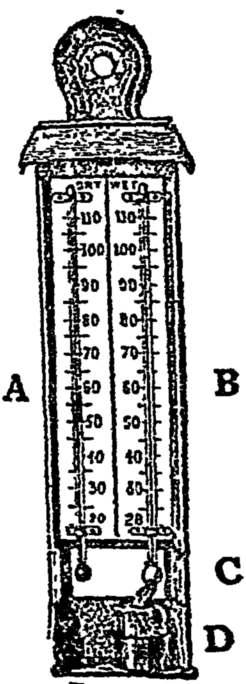


Fig 45

§9. **Hair hygrometer :**—The principle of this hygrometer is that a hair when moist increases in length This increase is proportional to moisture present A hair which is previously treated with caustic soda and washed and dried is stretched between two points *AC* as shown in the Fig 46 At *B* it passes round a grooved wheel and at *A* it is stretched by a spring A pointer is attached to the wheel *B* which moves on a scale *S* which is directly calibrated in terms of relative humidity by comparing its reading with a standard hygrometer When moisture increases the hair increases in length which moves the wheel on account of which the pointer *P* moves on the scale

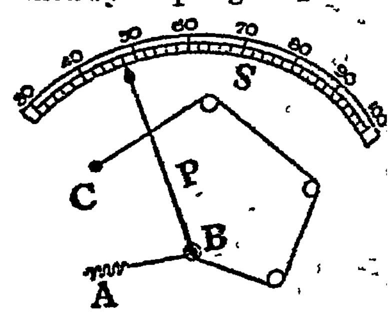


Fig 46

§10. **Condensation of Aqueous vapour :**—All the phenomena like clouds, rain, snow, fog or dew are the results of the condensation of aqueous vapour

When the actual moisture present in atmosphere at any particular temperature is less than what is required to saturate the space, evaporation will take place On the other hand, when the amount of moisture actually present is more than what is needed to saturate the space condensation will take place If at any instant the moisture actually present is less than saturation point and if it cools, either on account of expansion due to low pressure or if it goes to higher levels of low temperature regions or due to any other reason,

it may reach the saturation point and condensation may start This is the reason of the formation of above named phenomena.

(i) Dew —Generally, the temperature during day time is more than that during night. Therefore, the air is not saturated in day time but during night certain things like green leaves, grass, or window panes cool down to such a temperature that the moisture present is more than what is needed for saturation and, therefore, it condenses in the form of small drops at these things This is known as dew

Fog and Mist:—If the temperature of the atmosphere falls to such an extent that not only the things on the surface of the earth but the air surrounding it also cools down so that it becomes saturated, condensation will take place on the dust particle, smoke etc This is more common in cold countries In warm countries, this is seen on a winter day when the nights are clear and the day had been slightly warmer It becomes difficult to see through it on account of diffusion

Clouds:—On account of evaporation from lakes and oceans, air becomes moist Moist air being lighter than dry air, it rises above As it goes up it expands due to low pressure and its temperature falls The fall in temperature also takes place due to cooler regions in the upper layers On account of this cooling, moisture condenses on dust particles or smoke particles These tiny droplets remain suspended in air and float here and there These are known as clouds When these droplets come down they evaporate and when they go up they condense In this way the cycle goes on

Rain —When the clouds go in still cooler regions more water condenses on the droplets, they also coalesce and form bigger drops which cannot remain suspended in air They then fall down in the form of rain While they are falling down moisture from the lower layers also condense on them

Steel:—If before reaching on the surface of the earth, the rain drops freezes, then it is known as steel

Frost:- If the temperature of atmosphere falls below 0°C, moisture is directly converted into ice crystals and they are deposited on the surface of the earth This is known as frost

Snow:—If the formation of ice takes place in surrounding air, it will come down in the form of cotton This is known as snow

QUESTIONS

1 Define humidity and relative humidity. How will you find out R.H by chemical hygrometer? (*See* §1, §2 and §3)

2 What is dew point? How will you make use of it for measuring relative humidity? (*See* §6 and §7)

3 Describe dry and wet bulb hygrometer for measuring relative humidity (*See* §8)

4 Discus the formation of clouds, rain, fog, dew, etc (*See* §10)

CHAPTER X

HEAT ENGINES

§1. **Introduction :**—We have read in Chapter I that mechanical work can be easily converted into heat When we rub our hands, heat is produced, when we bore a hole in anything lot of heat is

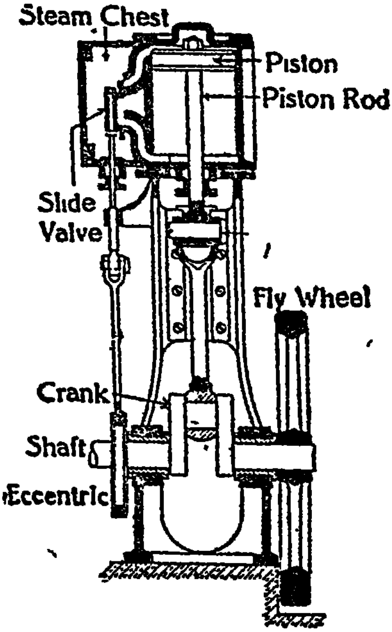


Fig 47

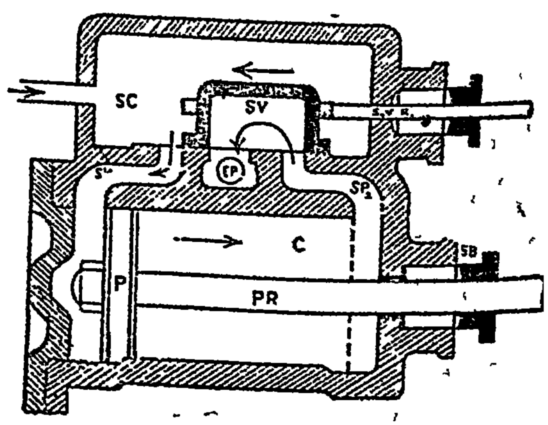


Fig 48

produced, whenever our fan or motor runs for many hours they are heated up In all these cases, some work is done and that work is converted into heat The reverse process, that is the conversion of heat into mechanical work, is more difficult We have to make use of special machine for this purpose This machine is known as heat Engine *Heat Engine is a device for converting heat into mechanical work* Every heat engine essentially consists of the following parts :—

(*a*) **A source of heat :**—This is obtained by burning some fuel like coal, petroleum etc

(*b*) **A working substance :**—It absorbs heat from the source and undergoes some cyclic changes of pressure, volume etc. All these changes are effected in a cylinder fitted with a piston

(*c*) **A condenser or sink :**—This is at lower temperature than the source Sometimes, atmosphere also acts as a condenser

§2. **External and internal combustion engines :**—When the source of heat lies outside the cylinder carrying the working substance, the engine is known as external combustion engine e g steam engine. When the source of heat lies inside the cylinder they are known as internal combustion engines e.g , petrol engines, diesel engine etc

§3. **Steam engine :**—It consists of the following parts .—[*See* Figs 17 and 18].

(*i*) **Boiler :**—In this water runs through steel pipes which are surrounded by flames from the furnace Water is converted into steam at high pressure and temperature

(*ii*) **Steam chest Sc :**—It is a stout metallic chamber which is connected to the boiler through pipes Steam from the boiler enters this chest There are three holes at the bottom of this chest The two holes SP_1 and SP_2 are known as steam ports These holes open in another stout cylinder *C* The middle hole is connected to exhaust pipe *EP*

(*iii*) **Slide valve S.v :**—This is a hollow *D* shaped mass of iron which slides upon the plane base of a cylinder The hollow side is kept downwards It is moved by a rod *S V R* connected to the shorter arm of the crank and shaft arrangement Due to its motion SP_1 and SP_2 are alternately closed and opened and are connected to exhaust valve

(*iv*) **Cylinder C :**—It is a stout cylindrical vessel just in contact with steam chest It communicates with steam chest through SP_1 and SP_2 A piston *P* is fitted in it, which is moved across the cylinder on account of the pressure of the steam The piston is connected by a rod *PR* to a crank and shaft arrangement

(*v*) **Crank and shaft arrangement :**—This is a device by means of which a to-and-fro motion is converted into circular motion The piston rod moves to and fro while the shaft of the crank will rotate

(*vi*) **Flywheel :**—It is a heavy wheel attached to the crank shaft It makes the supply of energy regular and continuous It absorbs the excess energy from the piston during some part of the stroke and gives out during another *The piston rod PR and the slide valve rod S V R are connected in such a way that they move in opposite direction*

Working :—Dry steam from the boiler is admitted in the steam chest Suppose the position of the piston and slide valve is as shown in Fig 48 Here SP_1 is connected to the chest and SP_2 to the exhaust valve Steam from steam chest enters the cylinder and presses the piston in forward direction The piston will move in the forward direction moving the crank The slide valve will move in the backward direction When the piston reaches the dotted line just near SP_2, the slide valve closes SP_1 SP_2 is connected to steam chest and SP_1 to exhaust valve Now the steam enters through SP_2 and moves the piston in backward direction The used steam on the left of the piston passes out through the exhaust valve The slide

valve rod moves in the forward direction. Again when the piston reaches its previous position, SP_2 is closed and steam enters through SP. In this way the process is repeated. The to-and-fro motion of the piston is converted into circular motion by crank and given to wheels

§4. **Efficiency of the engine** :—Out of the total energy obtained by burning the coal, only a portion of it is converted into work. This ratio is known as efficiency

$$\text{Thermal efficiency} = \frac{\text{Heat converted into useful work}}{\text{Heat supplied}} \times 100$$

The efficiency of steam engines varies from 15 to 18 per cent

It must be noted that all heat cannot be converted into work Only when heat is brought from a higher temperature to a lower temperature a part of it is converted into work. The efficiency of an engine can become 100% only if the temperature of the condenser is 0° absolute As this is impossible, the efficiency cannot be 100%

QUESTION

Describe the construction and working of steam engine

SECTION III

LIGHT

CHAPTER I

RECTILINEAR PROPAGATION OF LIGHT

§ 1. **Study of Light.**—The study of light, the other name for which is Optics, is divided into two topics, *viz* (1) *Geometrical Optics* and (2) *Physical Optics.* Geometrical Optics does not concern itself with the nature of light or its production or its propagation It is simply based on certain simple laws which can be experimentally verified From these, now postulates are put forward which can again be verified with the help of geometry This study helps in the construction of optical instruments

In Physical Optics first we try to answer the very question as to what this light is However, this is beyond the scope of this book

§ 2. **What is Light ? :**—The nature of light has puzzled and fascinated scientists since long Without going into the long discussion regarding the nature of light, it is sufficient here to assume that light is that agent which enables us to see objects but is itself invisible. This agent travels from one place to the other in the form of transverse waves As light can travel even through vacuum, we assume that there is an imaginary medium termed ether, which is all pervading and which has such properties that it can transmit light waves within it with a tremendous velocity of 3×10^{10} cm /sec or 1,86,000 miles/sec The wave-length of this light is extremely small—approximately of the order of 10^{-5} cm

§ 3. **Laws of Geometrical Optics :**—There are four fundamental laws on which the whole study of geometrical optics is based The laws are

(i) *The law of reversibility of path of light*
(ii) *The law of rectilinear propagation of light*
(iii) *The laws of reflection*
(iv) *The laws of refraction*

§ 4 **The law of reversibility of path of light :**—Let us suppose that light is travelling along a certain direction say *PQ, QR, RS, ST* See fig 1 If the direction of travelling of light is reversed at *T* so that it travels along *TS*, according to the law it will traverse the whole path exactly but in the reverse direction *i.e* , along *TS, SR, RQ* and *QP*.

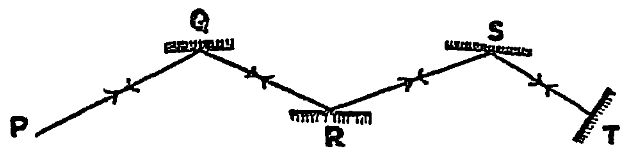


Fig 1

§ 5. **The law of rectilinear propagation of light :**—This law states that light travels between two points in a homogeneous medium along a straight line. By homogeneous medium we mean a medium in which the properties do not change and are uniform every where.

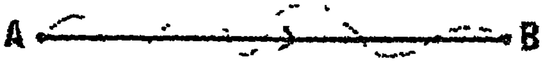
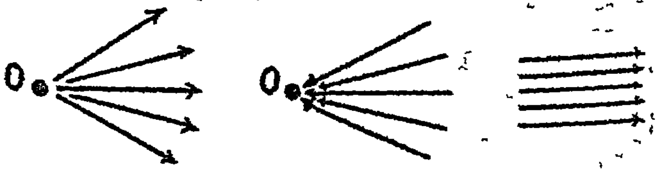


Fig. 2.

If light has to travel between two points *A* and *B* it would travel along *AB* and not along any other zig zag path as shown by dotted line.

§ 6. **A few definitions :**—*The path along which light travels is called a ray of light. A collection of rays is called a beam.*

A beam can be divergent as in Fig 3(*a*) when the rays spread out from a point source of light, convergent as in Fig 3 *b*) when the rays meet at a point and parallel when they go parallel to each other as in Fig. 3(*c*)



ig 3 (*a*) Fig. 3 (*b*). Fig 3 (*c*).

§ 7. **Applications of the law of rectilinear propagation of light :**—

(*a*) **Pin hole Camera :**—It consists of a light proof wooden box one side of which is either covered with a tissue paper or by means of a photographic plate. The side opposite to this has in the centre a hole of the size of a pin. Rays of light starting from *A* after passing through the hole *O* meet at *A'* and from *B* at *B'*.

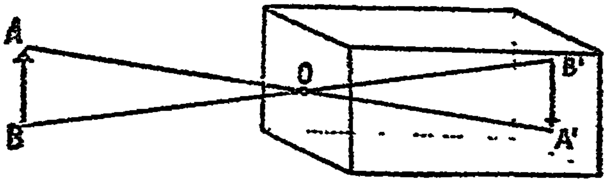


Fig. 4

Thus an inverted image *A'B'* is formed on the paper or on the plate. As the size of the hole is pin hole, the image is not very bright but is well-defined and sharp If the size of the hole is increased which is equivalent to adding other holes to it, it would give rise to other images *A" B"*, *A''' 'B'''* etc. lying close to *A'B'* and we will get a blurred image

The above also explains *why we get circular patches of light in the shadow of a tree* The space between two leaves behaves as a hole and we get blurred images of the sun overlapping each other

(*b*) **Shadow :**—Consider a point source of light at *O* Let *PQ* be any opaque object Rays of light starting from *O* are not able to enter the dotted space and hence it is called shadow of the obstacle. [see Fig. 5 (*a*)]

See Fig. 5(b). OO' is a broad source of light but smaller in size than the obstacle PQ. $O_1 O'_2$ is such a region on the screen SS that rays do not reach here from any portion of the source OO' In

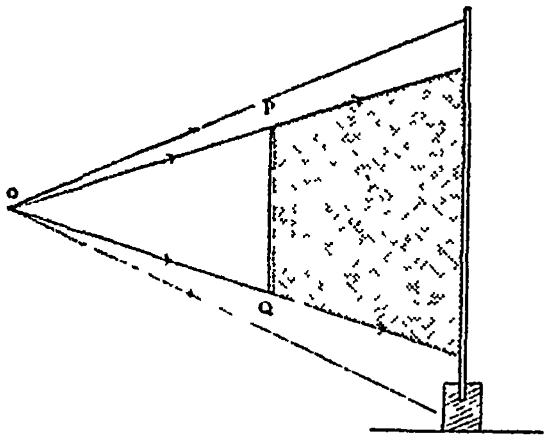


Fig 5 (a)

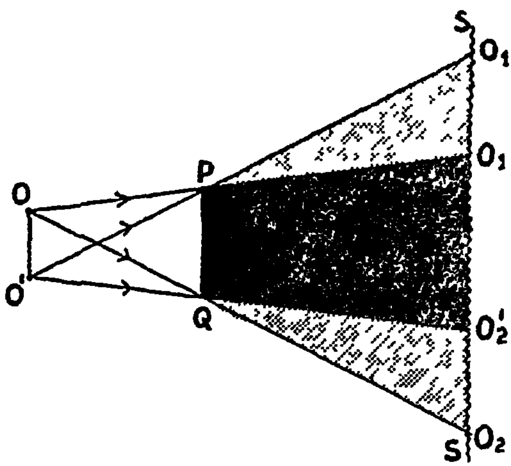


Fig. 5 (b)

the regions $O_1 O'_1$ and $O_2 O'_2$ rays can reach only from part of the source while beyond O_1' and O_2 rays reach from full source. Hence $O_1 O'_2$ where no light reaches is called the full shadow or umbra region while the region $O_1 O'_1$, $O_2 O'_2$ where light reaches from part of the source is partially lighted and hence is called partial shadow or penumbra

If an eye were to be placed in $O_1 O_2'$ region, the source will be completely hidden from sight while it will be partially seen from $O_1 O_1'$ and $O_2 O_2'$ Fig 5c where the source of light is larger than the obstacle is self explanatory If the screen is placed in SS position, the shadow of the obstacle falls at $O_1 O_2'$ on the screen while in the position $S' S'$ there is no shadow on it.

The above explains why a bird or an aeroplane when up in air does not cast its shadow on the ground but forms a shadow when near or on the ground

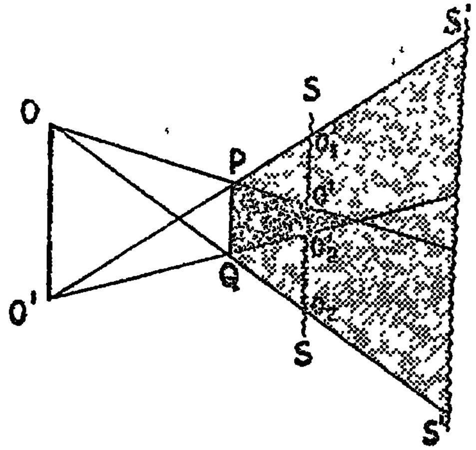


Fig 5 (c).

(c) **Eclipses :**—When moon comes in between the Sun and the earth, it casts its shadow sometimes on the earth For the people on the surface of the earth in the umbra region there would be a total solar eclipse For people lying in the penumbra region, there would be partial solar eclipse as only part of the Sun would be visible at day time See Fig 6 The earth coming in between the Sun and the moon casts its shadow on the Moon Ordinarily it would have been a full moon day but the portion of moon lying in

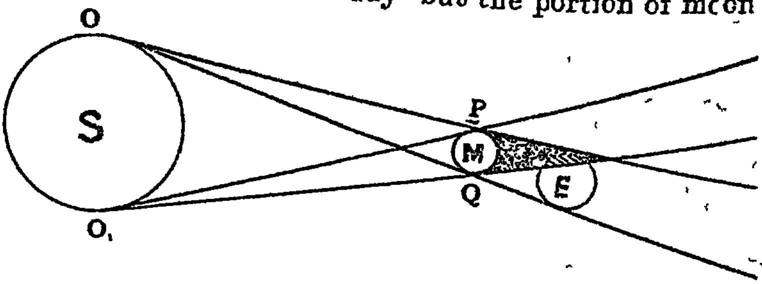


Fig 6 (*a*)

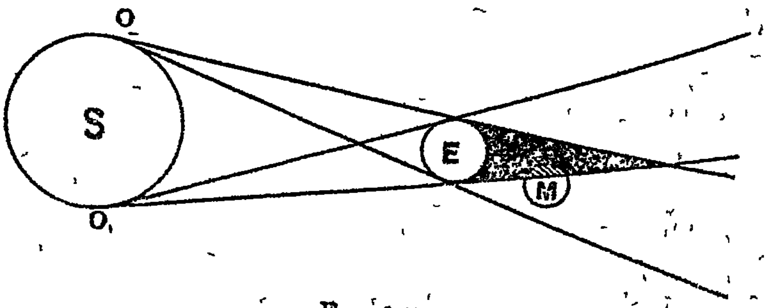


Fig. 6 (*b*).

umbra region will not get any light and hence there would be partial lunar eclipse Had it been fully inside the umbra region, it would have been a total lunar eclipse day.

QUESTIONS

1. State the law of rectilinear propogation of light and discuss its few applications (See § 5 and § 7)

2. Explain the formation of shadows and explain the terms 'umbra' and 'penumbra' [See § 7 (*a*)]

3. Describe, with neat diagram the formation of eclipses [See § 7 (*b*) and 7 (*c*)]

4. Describe a pin hole camera. Explain why we get patches of light in the shadow of a tree ? [See § 7 (*a*)]

will take place in all sorts of directions Such reflection is called diffusion

Objects become visible due to this diffused light. That is why it is extremely difficult to recognise a metal the surface of which is highly polished

§3 **Formation of image in a plane mirror :**—Fig 8 is self-explanatory. When the reflected rays OR, $O'R'$ enter the eye, the eye feels as if they are coming from P', the point obtained by producing RO and $R'O'$ backwards P' therefore is called the image of P The rays are actually coming from P but it only appears as if they are originating at P' and hence P' is not real but is virtual Hence, such an image is called virtual image

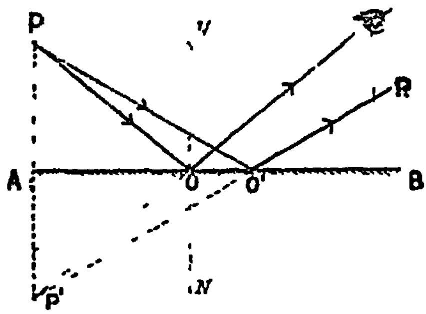


Fig 8.

You already know that the image is formed on the perpendicular dropped from the point source on the mirror and produced It is formed at the same distance behind the mirror as the object is in front of it

Hence, $AP=AP'$

Proof From P drop a perpendicular PA on the surface of the mirror and produce it Produce RO backwards to intersect PA produced at P'. Then we have to prove that $AP=AP'$.

At O, drop a normal NON'

The $\angle$ of incidence $PON=\angle$ of reflection RON

and $\angle RON=\angle P'ON'$, being vertically opposite

Hence, $\angle PON=\angle P'ON'$

But $\angle NOA=\angle N'OA$, being right angles

Hence, $\angle NOA-\angle NOP=\angle N'OA-\angle P'ON'$

or $\angle POA=\angle P'OA$

In $\triangle$s POA and $P'OA$, we have

$\angle POA=\angle P'OA$ as proved above

$\angle PAO=\angle P'AO$ as both are right angles by construction

and OA side is common

Hence, the two $\triangle$s are identical

Therefore, $PA=P'A$

This was to be proved

§4. **Formation of images in two mirrors :**—You already know that if we have two mirrors parallel to each other, as in a barber's shop, we shall get an infinite number of images The number will be limited only by the reflecting power of the mirrors.

Two mirrors at right angles to each other form 3 images.

$\therefore \quad \angle RON' = \angle RON - \angle NON' = (x-\theta)$

Also, in the new position the angle *of* incidence is

$$PON' = \angle PON + \angle NON' = (x+\theta)$$

Hence the new reflected angle $R'ON' = \angle PON' = (x+\theta)$

$$\therefore \quad \angle R'OR = \angle R'ON' - \angle RON'$$
$$= (x+\theta) - (x-\theta) = x+\theta-x+\theta$$
$$= 2\theta$$

Hence proved.

§6. Application of rotation of mirror :—

(*a*) **Lamp and scale methcd for measuring small angular deflectoins :—**

Necessity :—In Physics, there are many instruments in which the small angular deflection of a part are required to be measured *e g* , in galvanometers, deflection magnetometers etc

Requirements :—We know that the measurement of an angle becomes more accurate if the arms of the angles are larger See Fig 10 In position $P'R'$ the angle can be measured more accurately as the distance $P'R'$ is greater than PR Thus the deflecting apparatus must have a long pointer But for sensitiveness it is necessary that the pointer is not heavy Both these requirements cannot be fulfilled by any metallic or non metallic pointer Hence we use a ray of light as the pointer

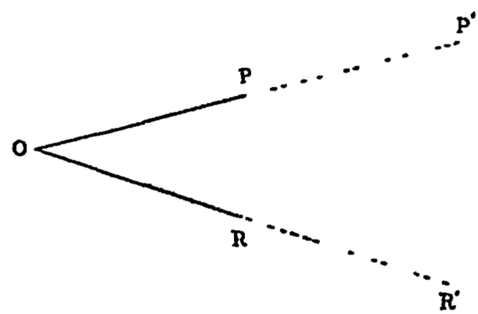


Fig 10

Description :—Let a concave mirror AB be posted to the deflecting apparatus In front of it, at a distance is mounted a scale and a lamp. The lamp is so adjusted that the rays after falling on the mirror are reflected back to the scale forming a point image there.

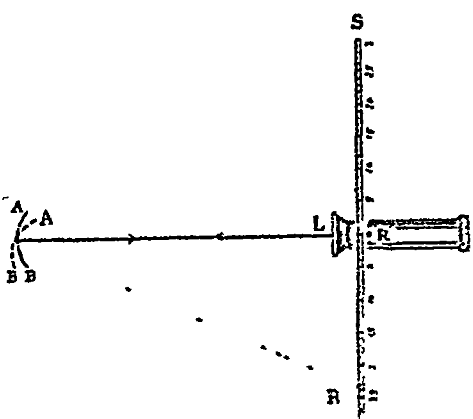


Fig. 11 (*a*).

Working :—Let the lamp L be so adjusted that the rays after reflection form the image at say R When the deflecting part deflects and with it the mirror AB, the new position of the image is at R' And so if the mirror is rotated through $\angle\theta$, $\angle ROR' = 2\theta$ (as proved in §5). See fig. 11

$$\tan ROR' = \tan 2\theta = \frac{RR'}{OR}$$

$$\therefore \quad 2\theta = \tan^{-1}\frac{RR'}{OR}$$

As θ is small and so also 2θ, $\tan 2\theta = 2\theta$.

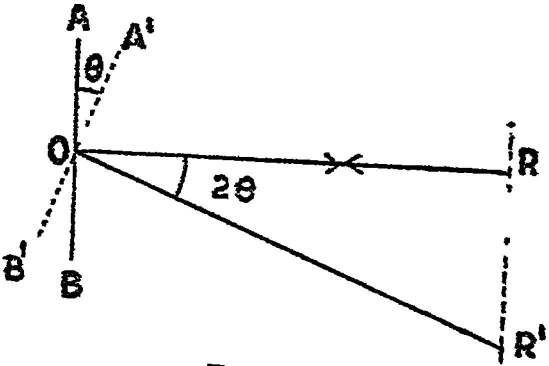


Fig. 11 (b)

$$\therefore \quad 2\theta = \frac{RR'}{OR}$$

or $\theta = \frac{RR'}{2OR} = \frac{d}{2D}$ in radians ... (

where d is the deflection of the image on the scale and D is the distance of the scale from the mirror

$\therefore \quad \theta = \frac{180}{\pi} \times \frac{d}{2D}$ in degrees. ... (2

Method :—The scale is placed at a distance $D=1$ metre or so. The position and height of the lamp is so adjusted that the image of the light spot falls on zero of the scale. When the mirror attached to the deflecting part deflects, the spot of light deflects on the scale to a reading say d cm. Thus knowing d and D, θ is known.

Importance :—1. By increasing D, d is increased and thus measurement of d becomes more accurate with smaller percentage of error.

2 Instead of measuring θ we measure 2θ which further increases the accuracy of the method

Modification :—(a) Instead of using a lamp a telescope can also be used The image of the O of the scale is first focussed in the telescope. After deflection, some other division at a distance d is focussed on the cross wire of the telescope.

(b) **Sextant :**—This instrument is helpful in measuring the height of a distant tower or altitude of Sun etc This is beyond the scope of this book.

§7. **Numerical Example :**—*In a telescope scale method, the scale is placed at a distance of 2 meters and the deflection measured is 10 mm Find the deflection of the mirror. What is the smallest angle which can be measured in this case if the least division on the scale is 1 mm.*

See fig 11(b).

$$\tan 2\theta = \frac{RR'}{OR} = \frac{10}{2000}$$

But $\tan 2\theta = 2\theta$ approximately

$$\therefore \quad 2\theta = \frac{10}{2000}$$

or

$$\theta = \frac{10}{2 \times 2000} = \frac{1}{400} \text{ radian}$$

Now, 3·14 radians is equal to 180°

$$\therefore \quad \frac{1}{400} \text{ radians is } \frac{1}{400} \times \frac{180}{3\,14} = 0\,14^\circ$$

Similarly, because the smallest deflection which could be measured is 1 mm. only, the angle would be $\frac{1}{2} \times \frac{1}{2000} = \frac{1}{4000}$ radians and in degrees $\frac{180}{3{\cdot}14} \times \frac{1}{4000} = 0\,014^\circ$

QUESTIONS

1. State the laws of reflection and explain the difference between reflection and diffusion. Explain why it is difficult to recognise a well polished vessel ? (See § 1 and §2)

2 Show that if a mirror is rotated through an angle, the reflected ray deflects through double the angle provided the direction of incident ray remains fixed. (See § 5)

3 Describe an optical method for measuring small angular deflection. Bring out the merits of the method. (See, § 5, case 1 or 2 and § 6 *a*)

4 Show that the minimum length of a mirror is half the length of a person to see his full length

(Hints. Divide the distance between eye and head in the two equal parts, say at M_4 and between eye and toes say at M_2 The mirror must be of such a length and so placed that one end is in line with M_4 and the other with M_2

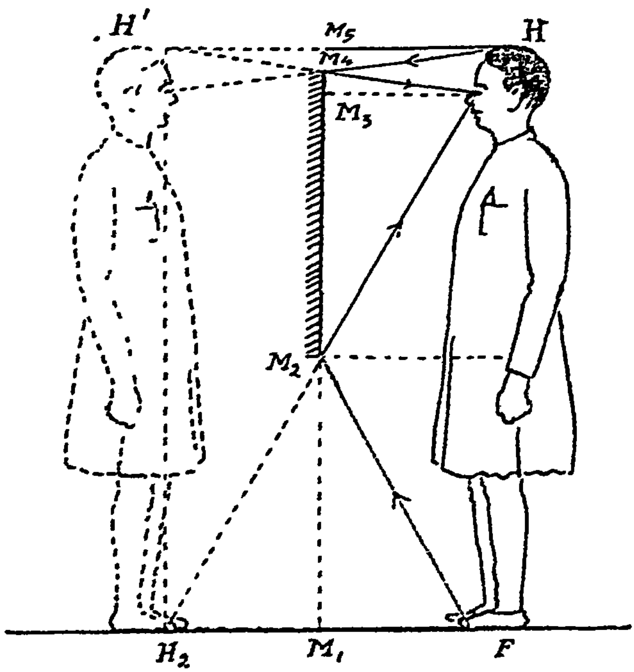


Fig. 12

5. Prove that if a plane mirror is displaced towards the object through a distance x, the image would be displaced through $2x$ towards the object

(Hints See fig 13. $N'P = N'Q' = d$. $N'N = x$

Hence $NP = NQ = d - x$

$\therefore$ $Q'Q = NQ' - NQ = (NN' + N'Q') - NQ$

$= (x+d) - (d-x) = 2x$.

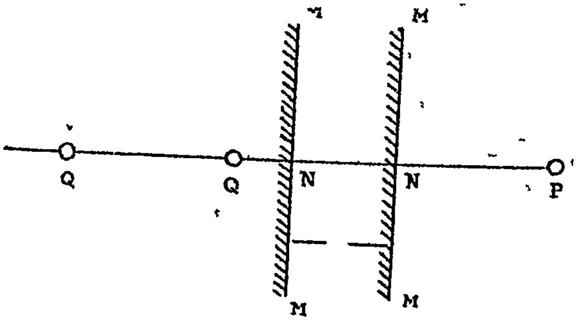


Fig. 13

6 In a telescope scale method, the scale is placed at a distance of 1 metre and the coil is deflected through 1 minute Find the deflection on the scale.

(*Ans* 5·8 mm)

7 The distance between a person and a mirror is 100 ft If the person starts moving towards the mirror with a speed of 5 ft/sec. after how much time the distance between the person and his image would be 100 ft.

(*Ans.* 10 sec)

CHAPTER III

REFLECTION AT CURVED SURFACES

§1. **Spherical Mirror** :—If you consider a part of a spherical surface and silver it, it is called a spherical mirror It is called concave mirror when the hollow side is reflecting and convex when the bulging side is silvered

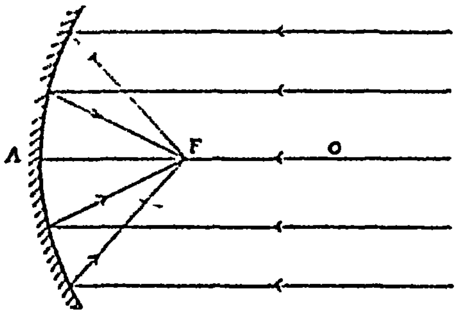


Fig 14

O, the centre of the sphere of which the mirror is a part is called the **centre of curvature.** *The central point A on the surface of the mirror is called* **the pole.** The line joining *the pole A to O the centre of curvature is called the* **principal axis.** *The distance AO is called the* **radius of curvature** *If you join O, the centre of curvature to any point on the surface of the mirror, it is called* **normal** *to the mirror*

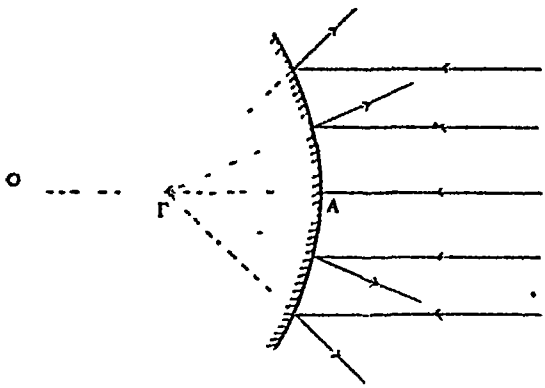


Fig. 14

If you consider *an incident beam parallel to the principal axis, after reflection it meets (as in the case of concave mirror —Cv mirror) or appears to meet (as in the case of convex mirror—Cx mirror) at a point situated on the principle axis This point F is called* **the focus** *of the mirror and the distance AF, as measured from pole to focus is called the* **focal length** *of the mirror* See fig 14

§2. **Sign Convention** :—A particular sign convention is followed while measuring distances in spherical mirrors *All distances are measured from pole of the mirror and are considered positive if measured against the direction of incident rays, and negative if measured in the same direction as that of incident rays*

Thus, the focal length in *Cv* mirror is positive and negative in in *Cx* mirror. Similar is the case for the radius of curvature

Distances measured at right angles to the principal axis are considered positive when measured upwards from the axis and negative when measured downwards

§3. Image formation :—The same laws of reflection, as in the case of plane surfaces are obeyed here.

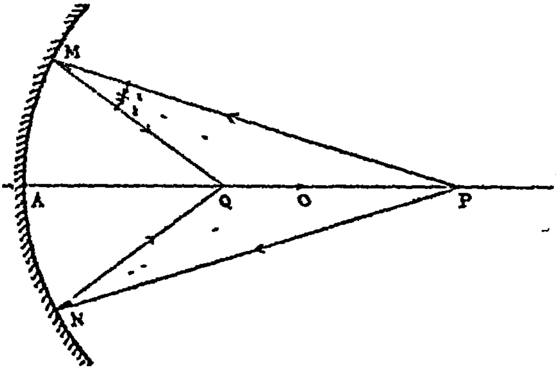


Fig 15 (*a*)

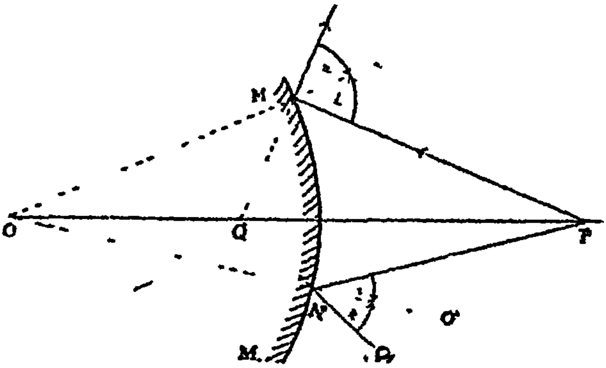


Fig. 15 (*b*).

See figures 15 (*a*) and 15 (*b*). *PM*, *PN* are the incident rays and *OM* and *ON* are the normals at *M* and *N* in *Cv* mirror and *OM* and *ON* produced to *MO'* and *NO'* in *Cx* mirror. Reflection takes place such that $\angle i = \angle r$ and $\angle i' = \angle r'$.

In *Cv* mirror the reflected rays meet actually at *Q* while in *Cx* mirror the reflected rays produced backwards meet at *Q*.

Thus *Q* is the real image in *Cv* mirror while virtual in *Cx* mirror.

§4. Relation between focal length and radius of curvature :—

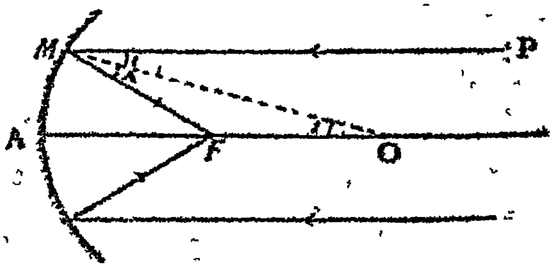


Fig. 16 (*a*).

Let *PM* be the incident ray parallel to the principal axis After

reflection, it meets at F on Cv mirror [Fig. 16 (a) and appears to meet at F in Cv mirror Fig. 16 (b)].

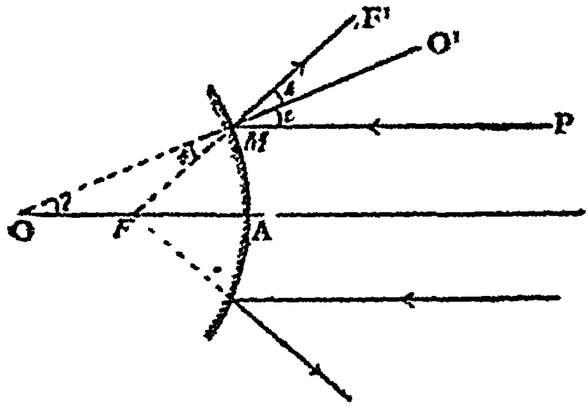


Fig 16 (b)

Here, $\angle i = \angle r$ being the angles of incidence and reflection.

Also $\angle i = \angle MOF$ in fig 16 (a), being alternate angles and in 16 (b) being corresponding angles.

$\therefore \quad \angle i = \angle r = \angle MOF$

Hence, in the $\triangle FMO$, $FM = FO$. (1)

We consider mirrors of small apertures only Aperture of the mirror is defined as the angle subtended by the periphery of mirror at O. Hence the point M is considered as situated very near to the pole A in comparison to the distance AF.

Hence $FM = FA$. (2)

Comparing equation (1) and (2) we get

$$FO = FA$$

i e, F divides the distance AO in two equal parts

$$\therefore \qquad AF = \frac{1}{2} AO \qquad \text{or} \qquad f = \frac{r}{2}.$$

where f is AF is the focal length and r is AO the radius of curvature

Relation :—*Focal length of a mirror = half of its radius of curvature.*

§5. Relation between focal length and object and image distances : For concave mirror :—

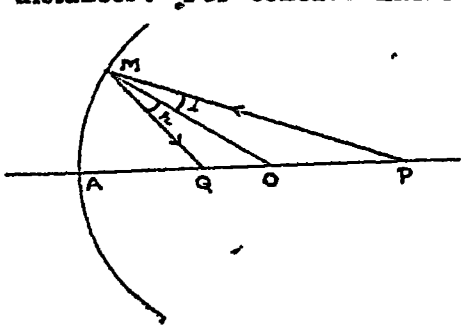


Fig 17 (a).

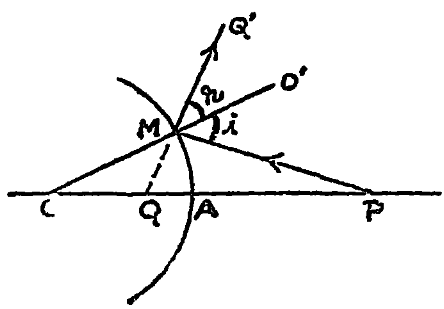


Fig. 17 (b).

See fig 17 (a) PM and MQ are the incident and reflected rays. MO is the normal

In the $\triangle PMQ$, $\angle PMO = \angle QMO$ being $\angle$ of incidence and reflection.

Hence MO becomes the internal bisector of the vertical $\angle QMP$.

Therefore it must divide the base QP internally in the ratio of its adjacent sides

Thus,
$$\frac{MQ}{MP} = \frac{QO}{PO} \qquad ..(1)$$

But, as the aperture of the mirror is small, M is very near A

$$\therefore \qquad MQ = AQ \text{ and } MP = AP$$

so from equation (1) we get

$$\frac{AQ}{AP} = \frac{QO}{PO}$$

Here $\qquad QO = AO - AQ \qquad$ and $\qquad PO = AP - AO$,

Making these substitutions the above becomes

$$\frac{AQ}{AP} = \frac{AO - AQ}{AP - AO} \qquad (2)$$

Let $AP = u$, $AQ = v$ and $AO = r$ where u, v, r are respectively the object distance, image distance and the radius of curvature so equation (2) becomes

$$\frac{v}{u} = \frac{r - v}{u - r} \qquad .. (3)$$

cross multiplying we get

$$v(u - r) = u(r - v)$$

or
$$uv - vr = ur - uv$$

Rearranging i e, taking the terms containing r on one side, we have

$$uv + uv = ur + vr$$

or
$$2\, uv = ur + vr \qquad (4)$$

Dividing both the sides of the above equation by uvr, we get

$$\frac{2uv}{uvr} = \frac{uv}{uvr} + \frac{vr}{uvr}$$

or
$$\frac{2}{r} = \frac{1}{v} + \frac{1}{u} \qquad .(5)$$

But
$$f = \frac{r}{2} \quad \text{(See §4)}$$

$$\therefore \qquad \frac{1}{f} = \frac{1}{\frac{r}{2}} = \frac{2}{r}.$$

Substituting this in equ. (5) we finally get

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \qquad .(6)$$

We know that for a given mirror its radius of curvature and hence focal length is constant. Therefore for a certain value of u,

there will be only one value for v. Hence, if we consider any other ray, say PN, after reflection it will meet at Q only. Therefore, we say, that all rays starting from P after reflection meet at Q and therefore Q is the image of P

For convex mirror :—See fig 17 (b) Here PM and MQ' are the incident and reflected rays and therefore OMO' is the externa bisector of the vertical angle PMQ It will therefore divide the base PQ externally in the ratio of its adjacent side

$$\therefore \quad \frac{MQ}{MP}=\frac{QO}{PO}$$

or as explained already,

$$\frac{AQ}{AP}=\frac{QO}{PO}=\frac{AO-AQ}{AP+AO}$$

or
$$\frac{-v}{u}=\frac{-r-(-v)}{u-r}$$

Here v and r are taken with $-ve$ sign because these are required to be measured in the same direction as the incident rays

or
$$\frac{-v}{u}=\frac{-r+v}{u-r} \text{ or } \frac{v}{u}=\frac{r-v}{u-r}.$$

This is same as equation (3) in the above So we get here also

$$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$$

Thus, in general *for a spherical mirror the relation is*

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

or *the sum of the reciprocals of object and image distances from the pole is equal to the reciprocal of the focal length of a spherical mirror.*

§ 6. **Conjugate Points :**—From the equation $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ it is clear that corresponding to one value of u, we shall have only one value of v Hence, if v is made equal to u, u must be equal to v, *e g*, let f for a mirror be 10 cm If $u=30$ cm in a concave mirror from above formula v would be 15 cm However, if u is made equal to 15 cm, v shall be 30 cm *Thus, it is clear that the position of object and image is interchangeable Such points are called* **conjugate** *to each other* This is also apparent from the law of reversibility of path of light. If QM is the incident ray, MP would be the reflected ray

§ 7. **Cardinal points :**—(a) If you consider a ray parallel to the principal axis, after reflection it passes or appears to pass through the focus of the mirror

(b) From the law of reversibility of path, an incident ray passing through focus will be reflected parallel to the principle axis.

(c) An incident ray passing through centre of curvature would fall normally on the mirror and hence retraces back the same path after reflection.

§ 8. **Image formation in the case of a finite object :**—In view of the above § 7, the Fig 19(a) is self-explanatory. An inverted image is formed at $P'Q'$

In order to draw the position of image in convex mirror a little care is needed. See fig. 18.

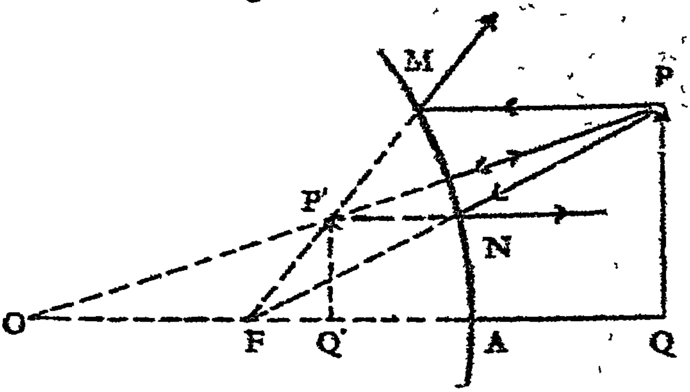


Fig 18.

(i) Draw PM parallel to the principal axis and join FM by means of a dotted line, FM produced gives the corresponding reflected ray

(ii) Similarly try to join FP Let it intersect the mirror at N, From N draw a line parallel to the principal axis. This gives the direction of the reflected ray corresponding to the incident ray PN.

(iii) Join PO.

All the reflected rays appear to meet behind the mirror at P' and thus $P'Q'$ is formed as the virtual image of the object PQ.

Note :—The whole image can be obtained by taking points like P throughout the length of the object PQ and drawing rays as explained.

§ 8. **Magnification** :—The size of the image depends upon the size of an object, its position and the focal length of the mirror. How many times an image is larger or smaller than the object is called the magnification of the image Here we shall only consider the length and hence

$$\text{Linear magnification} = \frac{\textit{Length of the image}}{\textit{Length of the object}} = \frac{I}{O}$$

Formula for magnification :—Let us consider Fig. 19(a) which is self-explanatory.

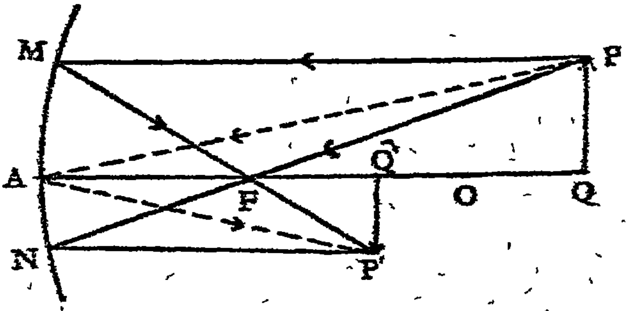


Fig 19 (a).

Join A to P and P'. If PA is the incident ray, AP' would be the corresponding reflected ray.

Consider the $\triangle$s APQ and $AP'Q'$

Here $\angle PAQ = \angle P'AQ'$ according to laws of reflection

$\angle PQA = \angle P'Q'A$ being right angles

and $\therefore$ the remaining angle $APQ = \angle AP'Q'$

Thus the $\triangle$s are similar

Hence $\dfrac{P'Q'}{PQ} = \dfrac{AQ'}{AQ}$ But $P'Q'$ is measured downward and so it must be introduced with a $-ve$ sign

Therefore above becomes $\dfrac{-P'Q'}{PQ} = \dfrac{AQ'}{AQ}$

or $$\frac{-I}{O} = \frac{v}{u}$$

Hence magnification, $$M = \frac{I}{O} = -\frac{v}{u} \quad \ldots (1)$$

As the aperture of the mirror is small, MA and NA may be considered as perpendicular to the axis

Consider now the $\triangle$s MAF and $P'Q'F$

Here $\angle P'Q'F = \angle MAF$ being right angles

$\angle Q'FP' = \angle AFM$ being vertically opposite

and hence the two $\triangle$s are similar

$$\therefore \quad \frac{P'Q'}{MA} = \frac{FQ'}{FA}$$

Here $MA = PQ$ as both of them are perpendicular and lie in between the same two parallel lines

$$\therefore \quad \frac{-P'Q'}{PQ} = \frac{FQ'}{FA} = \frac{AQ' - AF}{AF} = \frac{v-f}{f}$$

or $$\frac{-I}{O} = \frac{v-f}{f}$$

$\therefore$ **Magnification,** $$= \frac{I}{O} = -\frac{v-f}{f} \quad .. (2)$$

Similarly consider the $\triangle$s NAF and PQF

As above it can be shown that they are similar

Hence, $$\frac{NA}{PQ} = \frac{AF}{FQ}$$

or $$\frac{-P'Q'}{PQ} = \frac{AF}{AQ - AF}$$

or $$\frac{-I}{O} = \frac{f}{u-f}$$

$$\therefore \quad \mathrm{M} = \frac{I}{O} = -\frac{f}{u-f} \quad (3)$$

The above three relations can also be proved for convex mirror. Consider the same $\triangle$s here *e.g.*, APQ and $AP'Q'$

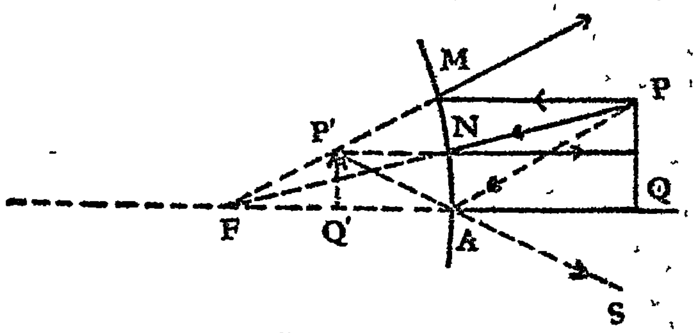


Fig 19 (*b*)

They are similar, because $\angle PAQ = QAS =$ vertically opposite $\angle P'AQ'$

Also $\angle\ PQA = P'Q'A$ being right angles

$$\therefore \qquad \frac{P'Q'}{PQ} = \frac{AQ'}{AQ} \quad \text{or} \quad \frac{I}{O} = \frac{-v}{v}$$

Hence, $$M = \frac{I}{O} = -\frac{v}{v}$$

Thus, taking into consideration that here I is $+ve$ but v and f are negative prove the remaining formulae.

So for spherical mirror, the magnification formulae are

$$\begin{aligned} M &= -\frac{v}{u} \\ &= -\frac{v-f}{f} \\ &= -\frac{f}{u-f} \end{aligned}$$

§ 9. Relation between u, v and f from magnification formulae :—

Equate any two of the three magnification formulae, say

$$-\frac{v-f}{f} = -\frac{f}{u-f}$$

Cross-multiplying, we get

$$(v-f)(u-f) = f \times f$$

Simplifying, we have

$$uv - fv - uf + f^2 = f^2$$

or $$uv = uf + vf$$

Dividing by vuf, we have $\dfrac{1}{f} = \dfrac{1}{v} + \dfrac{1}{u}$

§ 10. Newton's formula and discussion about the relative positions of object and image :—Take focus as the origin and

measure distances of the object and image from it. Let it be x and y respectively such that in Fig 20, $FQ=x$ and $FQ'=y$.

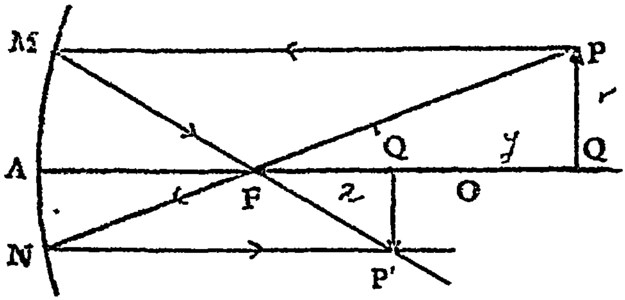


Fig. 20

As in § 8, Δs $P'FQ'$ and MAF are similar.

Hence $$\frac{P'Q'}{AM}=\frac{FQ'}{AF} \quad \text{or} \quad \frac{P'Q'}{PQ}=\frac{y}{f} \qquad (1)$$

Similarly, as ΔNAF and PFQ are similar,

$$\frac{AN}{PQ}=\frac{AF}{FQ} \quad \text{or} \quad \frac{P'Q'}{PQ}=\frac{f}{x} \qquad ..(2)$$

Equating equations (1) and (2) we have

$$\frac{y}{f}=\frac{f}{x} \quad \text{or} \quad xy=f^2 \qquad .\ (3)$$

Eqn (3) is called Newton's formula The same formula holds good for convex mirror also

Discussion :—Study eqn (3) The focal length of a mirror is a finite quantity and whether it is $+ve$ or $-ve$, its square would always be $+ve$ Therefore, the right hand side (r h s) is always $+ve$ Hence the product of x and y must also be $+ve$.

This means that the signs of x and y must be similar, either both of them $+ve$ or both of them $-ve$ *That is, the object and image both will lie on the same side of the focus.*

For concave mirror :—1. We know that $xy=f^2$

$$\therefore \qquad y=\frac{f^2}{x}$$

when $x=\infty$, y will be O

i e , when the object is at ∞, the image will be formed at focus. Naturally it will be real, inverted and diminished

2 When the object approaches the mirror such that it is anywhere *between centre of curvature and infinity, i e* , $x>f$, we have

$$y=\frac{f^2}{x}=\frac{f^2}{>f}=<f$$

i e., the image will be situated between focus and centre of curvature.

Again, it will be real, inverted and diminished.

3. *When object reaches centre of curvature,* $x=f$, and therefore, $y=\frac{f^2}{x}=\frac{f^2}{f}=f$, *i e , the image is also at the centre of curvature*

It is, therefore, *of the same size as image and is real and inverted*

4 When object is in *between focus and centre of curvature,* $x<f$, then $y>f$ *and image lies beyond centre of curvature*

It is, *therefore, real, inverted but magnified*

5 When *object is at focus,* $x=0$ and $y=\infty$ *The image is therefore, at* ∞

It is real, inverted and magnified

6. When the object is on *the left side of focus, i e ,* when x is $-ve$ and is $<f$, *i e., when object lies between pole and focus,*

y will be also $-ve$ and $>f$.

Hence the image will be formed *behind the mirror beyond pole. It would be virtual and magnified.*

7. If *the object is at pole* $x=-f$ and then y is also equal to f. That is the *image is also at the pole.*

It *is virtual and is of the same size as the object.*

Thus, we find that as the object moves from ∞ to pole, the image at first moves from focus to infinity and then from $-ve$ infinity to pole It is sometimes real, sometimes virtual. It is sometimes magnified and sometimes diminished You should try to draw diagrams for all cases mentioned above

For convex mirror :—The above discussion is true for convex mirror but the interpretation is slightly different.

(1) *When object is at* ∞, *i e* when $x=\infty$, $y=0$ and *the image is formed at focus.* The focus is behind the mirror and hence, a virtual, upright and a diminished image is formed.

(2) When $x>f$, *i e , the object is beyond A the pole of the mirror at* any place between pole and infinity, $y<f$ *and the image is formed between focus and pole.*

Thus, for all positions of the object in front of the mirror, the image is always behind the mirror; it is virtual, upright and diminished.

(3) When $x=f$, *i.e , the object is at pole,* $y=f$, *i e , image is also at pole*

It is not possible to make $x<f$ because if we keep object behind the mirror no reflection is possible.

Thus, *with convex mirror we always get a virtual and diminished image which is formed behind the mirror*

It is to be noted that for real images the distance of the image v increases with decrease in distance of the cbject u while for virtual images v decreases with decrease in u.

§ 11. Determination of focal length*: (*a*) **For concave mirror :**—

* The students are recommended the use of "The Text book of Practical Physics" by the authors for details.

1. By one pin:—We know that if an object is placed at the centre of curvature, the image is also formed at the same place

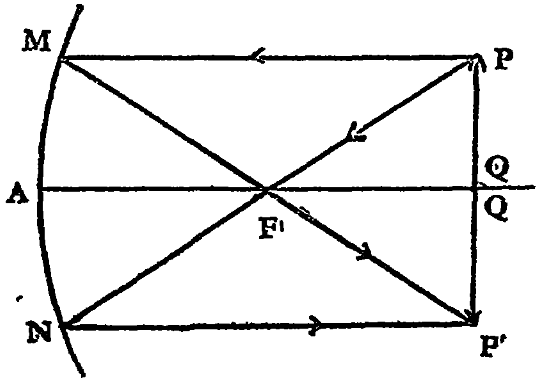


Fig 21

To make use of this, an object in the form of a pin is mounted on an optical bench in front of a mirror See fig 21. The pin *C* is moved forward and backward till the parallex between the pin and its image is removed The distance between the mirror *P* and the pin *C* gives *r*, the radius of curvature Half of it is the focal length of the mirror

2. By two pins:—A pin object *P* is placed on an optical bench in any position to form a real image Sec. fig 22. The

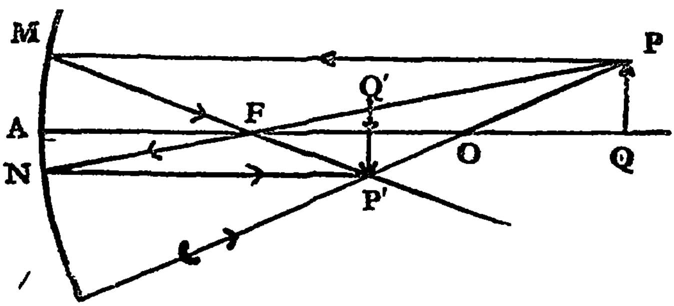


Fig 22

position of the image is found by removing parallax between this image and a second pin *Q* The distance between the mirror *O* and

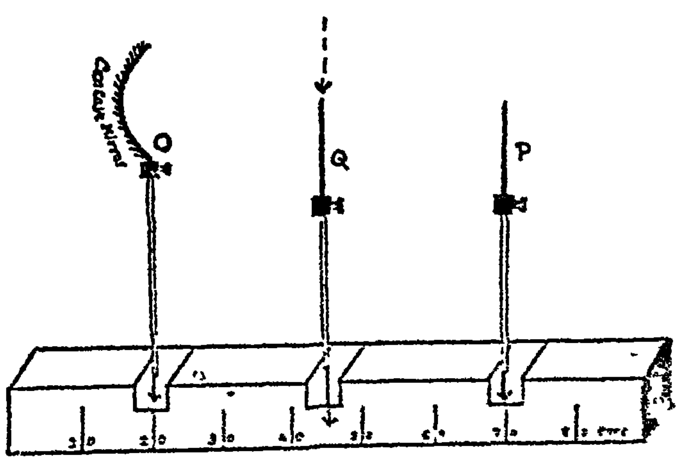


Fig 22 (a)

first pin gives *u* while that between mirror *O* and second pin *Q* is *v*

The use of the relation $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ gives the focal length of the mirror

(*b*) **For convex mirror :**—In this case the image formed is always virtual and is formed behind the mirror. Hence, it is difficult to locate its position with the help of another pin which is required to be placed behind the mirror and hence cannot be seen fully from the front of the mirror. The image and the pin, therefore, do not seem to be in contact and hence the parallax removal is not very accurate.

To make the method accurate use of a plane mirror is recommended.

Mount the given convex mirror, plane mirror and a pin as shown in fig, 23 (*a*). Adjust the heights of these in such a way that

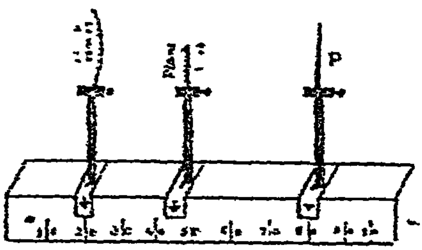


Fig 23 (*a*)

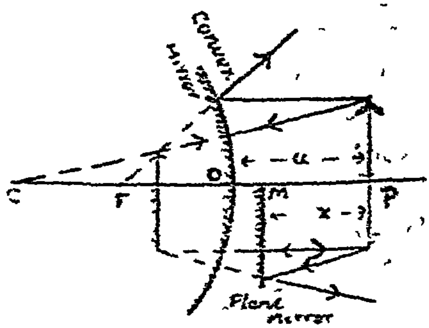


Fig. 23 (*b*).

the image formed in convex mirror and that formed in plane mirror, appear to touch each other By moving the plane mirror forward and backward, the plane mirror image will move accordingly. Adjust the position of the plane mirror in such a way that there is no parallex between the plane mirror image and the convex mirror image

See fig. 23 (*b*) Measure the distance between the convex mirror and the pin. This is $OP=u$. We know that the plane mirror image is formed at the same distance behind it as the object is in front of it. Hence, $PM=QM$, $\therefore\ PQ=PM+QM=2PM=2x$. Therefore, $v=OQ=PQ-PO=2x-u$, So to know v, measure the distance between the plane mirror and the pin. This is x. Double it and from it subtract u to get v.

The image is formed behind and hence substitute the value of v with a $-ve$ sign in the relation $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ and calculate f.

§ 12. **Use of mirrors :**—Spherical mirrior is a very useful optical instrument.

(*i*) Concave mirror of large focal length is used as a shaving glass when a virtual and magnified image of the shaver is formed.

(*ii*) Concave mirror in general and parabolloid mirror in particular is used as a means to produce a parallel beam of light. For this the source of light is placed at its focus. This is used as a search light.

(iii) These are also used as telescopic objectives. They can be easily prepared and hence can be of large aperture. This helps in increasing the resolving power of a telescope.

(iv) Convex mirror by virtue of its forming a diminished image is used for decoration purposes

(v) It is also used as a viewing mirror for a motarist. It gives a full view of the back side in a small space

§ 13. **Distinction between plane, concave and convex mirrors :**—In order to distinguish between these mirrors without touching them, hold an object near them If the image formed is virtual and of the same size it is a plane mirror, if the image is virtual and diminished, it is a convex mirror, and if the image is virtual but magnified or inverted and magnified or diminished it is a concave mirror

§ 14 **A few specimen problems—Example 1 :**—*In a concave mirror an image is formed at a distance which is twice the distance of the object If the focal length of the mirror is 10 cm, find the position of the object and discuss the nature of the image*

Let x be the distance of the object and suppose that it forms a real image So, the distance of image would be $2x$.

Now $u = x, v = 2x$ and $f = 10$ cm.

$\therefore$ According to formula $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ we get

$$\frac{1}{x}+\frac{1}{2x}=\frac{1}{10} \text{ or } \frac{3}{2x}=\frac{1}{10} \text{ or } 2x=30$$

$\therefore$ $x = 15$ cm.

The object is at 15 cm, when the image is real and is at 30 cm For virtual image, let $v = -2x$ and then

$$\frac{1}{x}-\frac{1}{2x}=\frac{1}{10}$$

or $$\frac{1}{2x}=\frac{1}{10} \quad \text{or } 2x=10$$

or $$x=5$$

The object is at 5 cm, when the image is virtual and behind the mirror

Example 2. *Where should an object be placed to get an image three times as large as the object The focal length of the mirror is 15 cm, what kind of mirror is it ?*

Obviously as the image is magnified, the mirror must be concave

Because the magnification is 3, it is possible both for real and virtual images

For virtual image $\frac{v}{u}=-3$. (1)

and for real image $\frac{v}{u}=3$ (2)

Using relation $\frac{v}{u} = -3$, we get $v = -3u$

Using relation $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ and substituting the value of v we get

$$\frac{1}{u} - \frac{1}{3u} = \frac{1}{15}$$

or $$\frac{3-1}{3u} = \frac{1}{15} \quad \text{or} \quad 3u = 30$$

$$\therefore \quad u = 10 \text{ cm.}$$

For real image, $\frac{v}{u} = 3$ or $v = 3u$

and we get $\frac{1}{u} + \frac{1}{3u} = \frac{1}{15}$

or $$\frac{3+1}{3u} = \frac{1}{15} \quad \text{or} \quad 3u = 60$$

$$\therefore \quad u = 20 \text{ cm}$$

Thus the object must be placed at 10 cm. for virtual image and at 20 cm. for real image

Example 3. *An object is placed at a distance of 15 cm from a concave mirror when the distance between it and a convex mirror is 20 cm apart with their reflecting faces facing each other. If the focal length of both the mirrors is 10 cm each and reflection first takes place in concave mirror, find the position of the final image after reflection at convex mirror*

See Fig 24.

As reflection first takes place at concave mirror,

$$u = 15 \text{ cm}$$
$$f = 10 \text{ cm.}$$

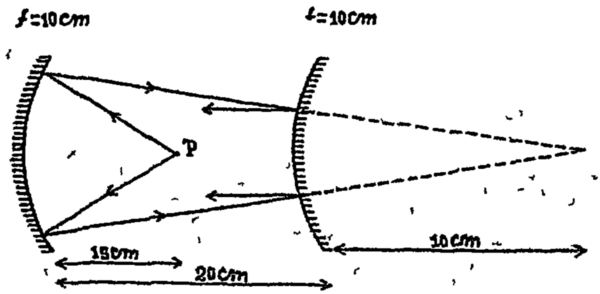


Fig 24

$$\therefore \quad \frac{1}{15} + \frac{1}{v} = \frac{1}{10} \quad \text{or} \quad \frac{1}{v} = \frac{1}{10} - \frac{1}{15} = \frac{3-2}{30} = \frac{1}{30}$$

$$\therefore \quad v = 30 \text{ cm}$$

These reflected rays meet at a distance of 30 cm. from concave mirror, *i.e.*, at a distance of 10 cm. from the convex mirror and behind it.

The object for convex mirror is, therefore, virtual and is at $u = -10$ *cm*

$\therefore$ the formula $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ for it becomes

$$-\frac{1}{10}+\frac{1}{v}=-\frac{1}{10} \text{ as its } f \text{ is also} -ve$$

$$\therefore \qquad \frac{1}{v}=-\frac{1}{10}+\frac{1}{10}=0$$

$$\therefore \qquad v=\infty$$

Or in other words, the reflected beam would be parallel and the image would be at infinity [It is assumed here that these rays would not fall on concave mirror]

Example 4. *In a plane mirror method of determining focal length of a convex mirror, the parallax is removed when the distance between the plane mirror and convex mirror is 5 cm and that between the convex mirror and the pin is 20 cm Find the focal length If the object is moved away by 10 cm, find the new position of the plane mirror for no parallax.*

See Fig. 25 The distance between plane mirror M and the object P is $x = 15$ *cm*

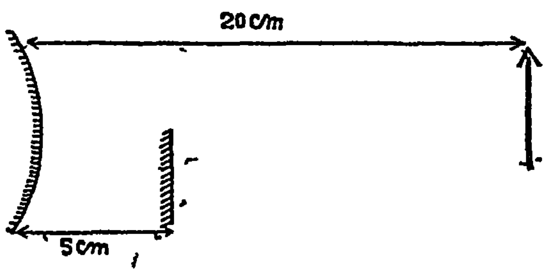


Fig 25

Hence, $$v = 2x - u = 2 \times 15 - 20 = 10 \text{ cm.}$$

$\therefore$ Using the formula we get

$$-\frac{1}{10}+\frac{1}{20}=\frac{1}{f}. \text{ Here, } v \text{ is } -ve$$

or $$\frac{-2+1}{20}=\frac{1}{f}$$

$$\therefore \qquad \frac{-1}{20}=\frac{1}{f}$$

or $$f=-20 \text{ cm.}$$

Now the new position of the object is $u = 30$ cm, as it is moved away by 10 cm.

$$\therefore \quad \frac{1}{30}+\frac{1}{v}=-\frac{1}{20}$$

or $$\frac{1}{v}=-\frac{1}{20}-\frac{1}{30}=\frac{-3-2}{60}=-\frac{5}{60}=-\frac{1}{12}.$$

$$\therefore \quad v=-12 \text{ cm.}$$

Now $v = 2x - u$

or $12 = 2x - 30 \quad \therefore 2x = 12 + 30 = 42 \quad \therefore x = 21$

Hence the distance between plane mirror and pin is 21 cm. or that between convex mirror and plane mirror is $30 - 21 = 9$ cm. i e, the plane mirror is required to be moved by 4 cm. away from convex mirror

QUESTIONS

1 Deduce a relation for a spherical mirror between the distances of the object and image from its pole and its focal length (§ 4 and § 5.)

2. Define magnification Deduce the various formulae for magnification for a spherical mirror and hence prove the formula $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$. (See § 8 and § 9)

3 Deduce Newton's formula and show mathematically that it is possible to get a real or virtual, magnified or diminished concave mirror image but it is impossible to form a real, magnified image with convex mirror (See § 10)

4. Define focal length of a mirror. How will you determine it for a convex mirror? Discuss the importance of the method? What is the use of such mirror? (See § 2, § 11 and § 12)

5 A concave mirror has a radius of curvature 30 cm Determine the two positions of an object placed in front of it such that an image three times as large as the object is formed What is the position of the image in each case? (*Ans* 20 cm, $v = 60$ cm, 10 cm,, $v = 30$ cm. backward).

6 The distance between an object and its image formed by a convex mirror is 36 cm The image is half the size of the object; find the focal length of the mirrior and also its destance from the object. (*Ans*. 24)

7 An object is placed 25 cm from the surface of a convex mirror, when a plane mirror is placed at distance of 20 cm from the object and in between the convex merror and the object, the parallax is removed between the two virtual images Find the focal length of the convex mirror. (*Ans*. 37.5 *cm*)

CHAPTER IV

LAWS OF REFRACTION AT A PLANE SURFACE

§1. **Refraction :**—When light is incident on a few particular media, they, instead of throwing it back into the same medium from which it is coming, allow it to pass through them Just at the boundary of separation of the two media, as their is change of medium, the law of rectilinear propagation is not obeyed and the path of light is altered. This alteration in path is called refraction and takes place according to certain laws

§2. **Laws of refraction :**—See fig 26 PA is the incident ray when the plane boundary of separation of the two media is XY Let NN' be the normal AQ is the path along which light enters into the second medium and is called the refracted ray. The angle $PAN=i$ is the angle of incidence while the angle enclosed between the refracted ray AQ and normal AN' i e $\angle QAN'=r$ is the angle of refraction

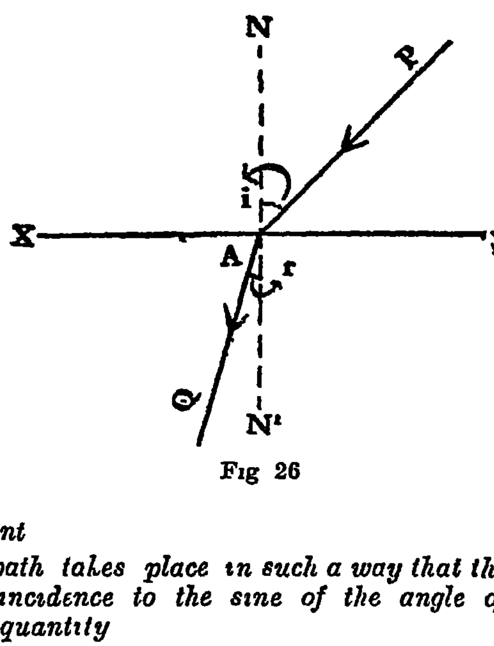


Fig 26

Following are the **Laws of refraction :**—

1 *The incident ray, the normal and the refracted ray lie in the same plane, i e , the plane of incidence and the plane of refraction are coincident*

2 *The alteration in path takes place in such a way that the ratio of the sine of the angle of incidence to the sine of the angle of refraction is always a constant quantity*

Thus, $\dfrac{\sin i}{\sin r}=K$

The value of this constant only depends on (i) *the nature of the two media,* (ii) *the colour or frequency of light and* (iii) *temperature.*

When the first medium is vacuum, the constant is called μ (Mu) the refractive index of the second medium For all practical purposes when a ray of light enters air medium from vacuum, hardly any change of path, i e , refraction takes place and therefore for approximate purposes air is taken as optically similar to vacuum Therefore, instead of vacuum, air may be taken as the first medium.

Refractive index of the medium is, therefore, defined as the constant obtained by taking ratio of the sine of the angle of

incidence and the sine of the angle of refraction, when a ray is going from air to the medium

$$\frac{\sin i}{\sin r}=\mu$$

μ is sometimes denoted as ${}_1\mu_2$ or μ_{12} to denote that rays are going from medium no 1 and entering medium no 2.

If the angle of incidence changes, the angle of refraction will also change but the ratio of their respective sine will always be a constant

§3. **Dependence of refractive index: (a) On medium:**—When a ray of light is going from air to water or from air to glass, other things remaining the same, for water $\frac{\sin i}{\sin r}=\mu_{aw}=1\ 33$ while for glass $\frac{\sin i}{\sin r}=\mu_{ag}=1\ 5$ This shows that if any of the media is changed, the value of μ alters.

Generally, the media are such that for them $\mu>1$ and hence $r\angle i$ Therefore the refracted ray is said to bend towards the normal But if a ray were to go into a medium for which $\mu<1$, it would bend away from the normal making $r>i$.

(b) **On colour of light:**—If the colour of light is changed, say from red to blue, the bending of the ray changes, other factors remaining the same. Thus, $\mu_b>\mu_r$ where μ_b and μ_r stand respectively for blue and red colour

We know that the spectrum colours are in the order—red, orange, yellow, green, blue, indigo and violet As we change the colour from red towards violet, for the same pair of media, μ goes on increasing

Rigourously speaking we should speak in terms of frequency and not colour which is a vague term. *μ increases with frequency of light*

(c) **On temperature:**—With temperature, the density of a medium changes and hence refraction is affected Generally μ decreases with increase of temperature The density of a medium also decreases with temperature According to Gladstone and Dales' law, both these quantities, *i e.*, μ and d *vary with temperature in such a way that* $\frac{\mu-1}{d}$ *remains constant at all temperatures.*

§4. **Relation between μ_{ag} and μ_{ga}:**—When a ray is going from air to glass (See fig 26) $\mu_{ag}=\frac{\sin i}{\sin r}$. If the light path is reversed, according to the reversibility law, *QA* would be the incident ray and *AP* the refracted ray. As ray is going from glass to air $\mu_{ga}=\frac{\sin r}{\sin i}$ because, now the angle of incidence$=r$ and angle of refraction is i.

$$\therefore \quad \mu_{ga}=\frac{\sin r}{\sin i}=\frac{1}{\frac{\sin i}{\sin r}}=\frac{1}{\mu_{ag}}.$$

or $$\mu_{ag}=\frac{1}{\mu_{ga}}$$

The relation is, $\mu_{ag}=\dfrac{1}{\mu_{ga}}$.

§5. Relation between incident and emergent ray when refraction takes place with first and last medium as the same:—Let $WXYZ$ be a parallel slab of glass with WX and ZY surfaces parallel to each other According to fig. 28 (*a*) PA is the incident ray and AQ is the refracted ray inside glass At Q, the ray AQ comes out of glass along QS QS is, therefore, called the emergent ray. Here, $\angle PAN = i$

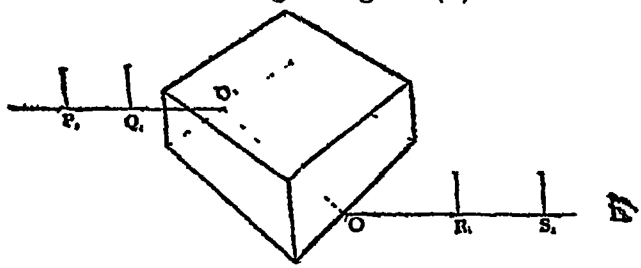


Fig 27.

$\angle QAN' = r = \angle AQM$, being alternate angle, NN'; MM' being parallel and $\angle SQM' = e$, the angle of emergence.

Considering incidence at P from air to glass we have

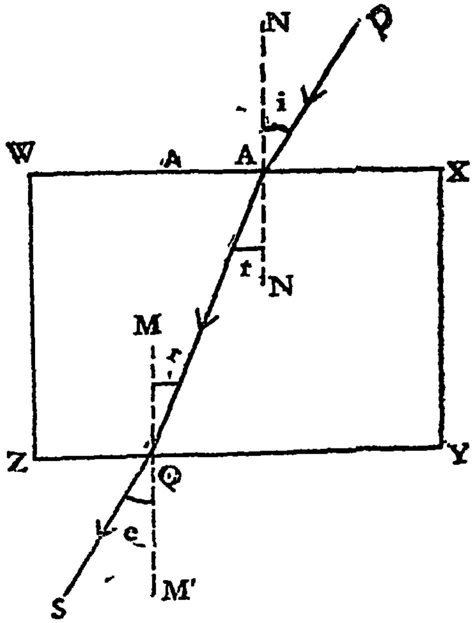


Fig. 28 (*a*)

$$\mu ag=\frac{\sin i}{\sin r} \qquad \text{.. (1)}$$

If you reverse the path of rays i.e., incidence along SQ, the

whole path will be retraced but along reverse direction. Hence, considering incidence along SQ from air to glass at Q we have

$$\mu ag = \frac{\sin e}{\sin r}$$

Here e is the angle of incidence.

As L. H. S. of both equs. (1) and (2) is the same,

$$\therefore \qquad \frac{\sin i}{\sin r} = \frac{\sin e}{\sin r}$$

or $$\sin i = \sin e \qquad \text{or } i = e$$

Therefore PA and QS must be parallel to each other.

Relation :—*Angle of incidence and angle of emergence are equal when refraction takes place through media having parallel boundaries and when the first and the last medium is the same.*

§6. **Refraction through many parallel layers of media or to prove that** $\mu wg = \frac{\mu ag}{\mu wg}$:—Let UV, WX and YZ be the parallel boundaries of separation respectively between air water, water glass

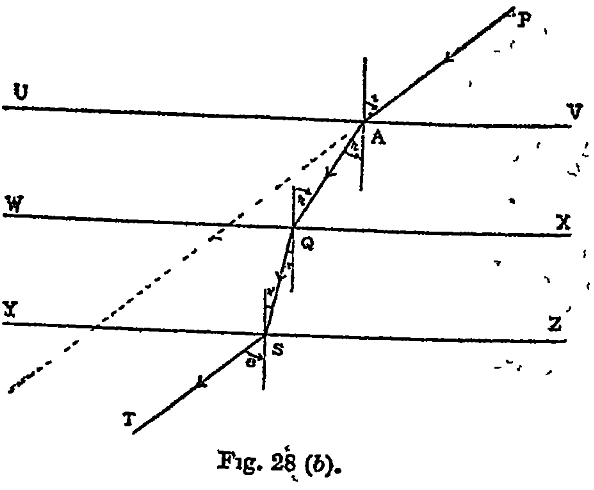


Fig. 28 (b).

and glass air media Fig 28 (b) is self explanatory.

We have $\mu aw = \dfrac{\sin i}{\sin r}$ ... (1) from air to water

$\mu wg = \dfrac{\sin r}{\sin r'}$ ... (2) from water to glass

$\mu ga = \dfrac{\sin r'}{\sin e}$ .. (3) from glass to air

Multiplying the above three equations together, we get

$$\mu aw \quad \mu wg \quad \mu ga = \frac{\sin i}{\sin r} \cdot \frac{\sin r}{\sin r'} \cdot \frac{\sin r'}{\sin e}$$

But according to §5, $i = e$ and hence,

$\mu aw . \quad \mu wg . \quad \mu ga = 1$

or $$\mu wg = \frac{1}{\mu aw.\,\mu ga} = \frac{1}{\mu aw}\;\frac{1}{\mu ga}$$

But according to §4, $\mu ag = \frac{1}{\mu ga}$. Making its use in the above,

$$\mu wg = \frac{\mu ag}{\mu aw}$$

i e., **refractive index of glass with respect to water is equal to the ratio of refractive indices of glass and water.**

§7. Total internal reflection and critical angle :—We know that when a ray is going from rarer to a denser medium, it bends towards the normal making the angle of refraction smaller than the angle of incidence So even when the incidence varies from normal ($i=0°$) to grazing ($i=90°$), refraction is always possible On the other hand, when a ray is going from denser to rarer, it bends away from the normal making angle of refraction greater than the angle of incidence See Fig 29 (*a*) As the angle of incidence increases, angle of refraction increases correspondingly A stage is reached when corresponding to an angle of incidence, say θ, the angle of refraction becomes 90° **This angle of incidence θ is called the critical angle.** *If the angle of incidence is made greater than the critical angle, angle of refraction cannot exceed 90° and hence refraction is not possible The rays, instead of entering the second medium, are reflected back into the same medium according to the ordinary laws of reflection This type of reflection is called* **total internal reflection** It is called total as no portion of light is refracted, all is reflected It is termed internal because the rays are not able to come out of the medium into air.

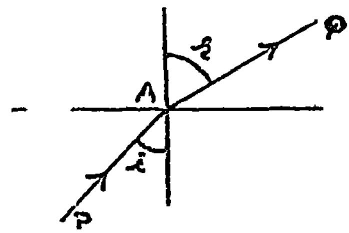


Fig 29 (*a*)

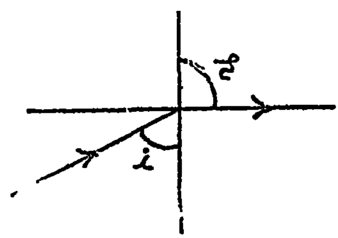


Fig 29 (*b*).

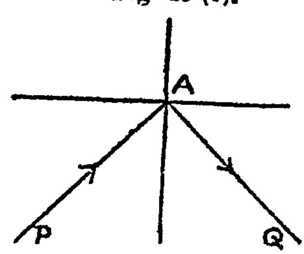


Fig 29 (*c*)

Cracks in glass and drops of air in water appear shining on this account

§8. Distinction between ordinary and total internal reflection :—

Ordinary reflection	Total reflection
1. Takes place when a ray is going from rarer to denser or denser to rarer medium.	1. Takes place only when a ray is going from denser to rarer medium
2. It is possible at all angles of incidence.	2 It is possible only at incidence greater than critical angle.

3. A greater part of light is reflected while a little is refracted also.

3. The whole amount of light is reflected. No portion is refracted.

§9. Relation between critical angle and refractive index of the medium :—Consider Fig 29 (*b*). *PA* is the incident ray in, say glass and *QA* is the refracted ray making the angle of refraction in air equal to 90°.

As the rays are going from glass to air

$$\mu ga = \frac{\sin \theta}{\sin 90} = \sin \theta, \text{ as } \sin 90 = 1$$

$$\therefore \qquad \mu ag = \frac{1}{\mu ga} = \frac{1}{\sin \theta} = \operatorname{cosec} \theta$$

The relation is :—*Refractive index is equal to cosecant of critical angle.*

§10. Determination of critical angles or refractive index of a liquid :—*

Principle :—Just beyond critical angle, no refraction is possible into the second medium.

Apparatus :—Between two thin sheets of glass *AB* and *CD* is enclosed a thin film of air. It is so enclosed that no fluid can enter into it.

Such a film of air is dipped into the experimental liquid which is kept in a rectangular reservoir. To the air film is attached a pointer which moves along a graduated circular scale. See Fig. 30 (*a*).

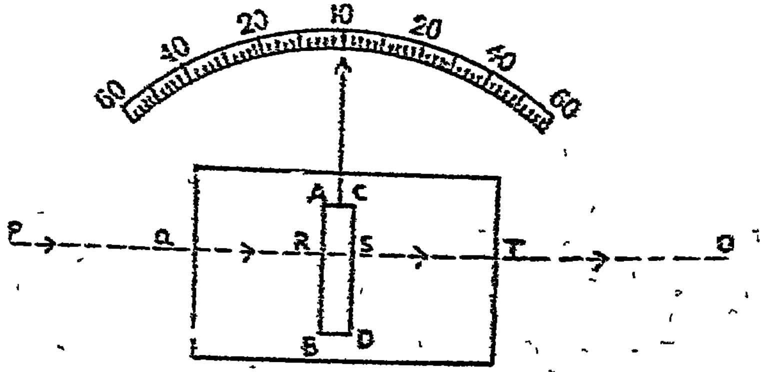


Fig. 30. (*a*)

Method :—Let *P* be a source of light and *O* be the observer on the other side of the reservoir. A ray *PQ* enters the reservoir normally along *QR* and then passing through the air film, re enter the liquid *ST* and then along *TO*. An observer at *O* is able to spot the source of light.

*For practical details—See "A Text Book of Practical Physics" by authors.

In this position QR is the normally incident ray from liquid at the air film. Let the air film be gradually rotated along its vertical axis. As this will happen, QR will make an increasing angle of incidence QRN at liquid air interface where RN is the normal. Obviously the angle QRN is equal to the angle through which the film is rotated from $ABCD$ position to $A'B'C'D'$ position. Refraction through the air film to the other side is possible for incidences upto critical angle and the observer on the other side is able to see the source of light. When the angle QRN becomes equal to the critical angle, the refraction is parallel to the surface $B'A'$ and it is not able to come out of the air film. Consequently the observer is unable to see the source.

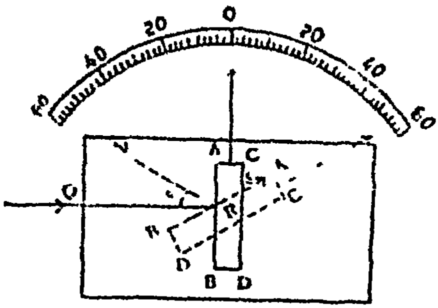


Fig 30 (b)

Therefore, keeping the eye constantly on the source, the film is rotated till the light is just extinguished and the source is cut off from sight. The position of the pointer is noted on the scale. Let it be θ_1.

The film is then rotated in opposite direction towards its initial position when the source becomes visible again. When it has crossed its initial position towards the other side again the same thing happens. When it has been rotated through critical angle, light is again cut off. Let the position of the pointer be θ_2.

The mean of θ_1 and θ_2 gives the critical angle θ for the liquid.

Here we have discussed with the help of a single ray. Actually a point source of light will give out a divergent beam which will make various angles at the film and hence when corresponding to a particular ray it has been turned through critical angle, other rays will not be cut off sharply. Therefore, it is better to have the incident beam parallel and the observations are made through a telescope.

Knowing critical angle, the refractive index of the liquid can be known with the help of the relation $\mu = \operatorname{cosec} \theta$,

§ 11. Effect of refraction on observing the depth of a refracting medium: —

(a) Dip a stick into water. Just at the boundary the stick appears to bend upwards. See fig 31 (a). Its position should have been ABC but instead it appears as ABC'.

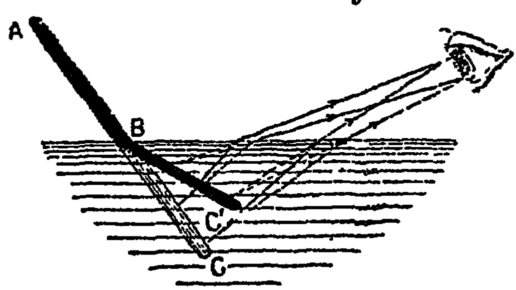


Fig. 31(a).

(b) Try to judge the depth of a river. You find that it is deeper than what it appears to be.

(c) In an opaque vessel, keep a coin and keep the eye in such a position

that it is just not visible Now pour water and you will find that without shifting the position of the eye, you are able to see the coin. See fig. 31 (*b*). The reason for this is that the coin from its original position *C* appears to be raised to *D* position and hence comes in line with the eye

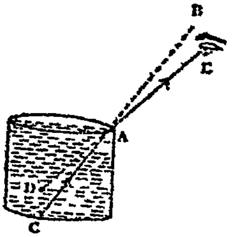


Fig 31 (*b*)

Let us now explain the above experiments.

Let *P* be the point object at the bottom of the vessel and the line of sight is normal, *i.e.*, eye is vertically above *P* When a liquid is poured, the ray *PQR* is normally refracted The ray *PS* which is slightly inclined to the vertical, after refraction at *S*, bends away from the normal along *ST* These refracted rays produced backwards appear to meet at *Q* and hence *Q* appears to be the image of *P* Thus the bottom of the vessel at *P* appears to be raised up to the position *Q*. Consequently *RQ* becomes the apparent depth when *RP* is the actual depth

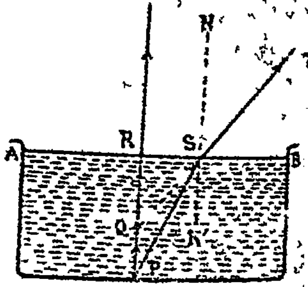


Fig 32

§ 12. Relation between real and apparent depth with the refractive index of the medium :—From liquid to air, *PS* is the incident ray, *ST* is the refracted ray and *NN'* is the normal

Here, $\angle PSN' = i = \angle SPR$ (being alternate angles formed between the two parallel lines *NN'* and *RP* These are parallel being normal to the same surface *ARSB* and $\angle PSN = r = \angle QSN'$ being vertically opposite and is equal to $\angle SQR$ being alternate angle

Hence, as rays are going from liquid to air,

$$\mu_{la} = \frac{\sin i}{\sin r} = \frac{\sin SPR}{\sin SQR} \qquad (1)$$

But in right angled $\triangle$ *SPR*,

$$\sin SPR = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{RS}{SP}$$

and in $\triangle$ *SQR*

$$\sin SQR = \frac{RS}{SQ}$$

Substituting these values in equ (1), we get

$$\mu_{la} = \frac{RS/SP}{RS/SQ} = \frac{RS}{SP} \times \frac{SQ}{RS} = \frac{SQ}{SP} \qquad (2)$$

Here the incidence is almost considered vertical, because only such refracted rays can enter the vertically placed eye. Hence, the ray *PS*, cuts the liquid surface at *S* which is very near to *R*,

Therefore, $SQ = RQ$ and $SP = RP$

Substituting these values in equ. (2), we get

$$\mu_{la} = \frac{RQ}{RP}$$

But $$\mu_{al} = \frac{1}{\mu_{la}}$$

$$\therefore \quad \mu_{al} = 1/\mu_{la} = 1/RQ/RP = \frac{RP}{RQ} = \frac{\text{real depth}}{\text{apparent depth}}$$

Relation :—*The refractive index of a medium is equal to the ratio of its real depth to apparent depth*

§13. Determination of refractive index with a microscope*:— The above relation is made use of in determining μ of a material—say a glass block

Microscope is an instrument which can see small objects at close distance distinctly

Take a microscope and focus it on an ink mark on a paper or on a coin in a beaker—say at P See fig 33 Let the position of the microscope on a scale be a Keep the glass block on the mark or pour water in the beaker The image of P will appear at Q Focus the microscope on it Let this position be b Now sprinkle some lycopodium powder on the surface at R Focus the microscope on it Let this position be C Obviously the real depth would be $RP = c - a$ and the apparent depth would be $RQ = c - b$

Hence, $$\mu = \frac{c-a}{c-b}$$

It is very important here that the microscope is focussed vertically and the quantity of liquid used is neither very large so as to reduce the intensity of the image nor very small so as to reduce the accuracy

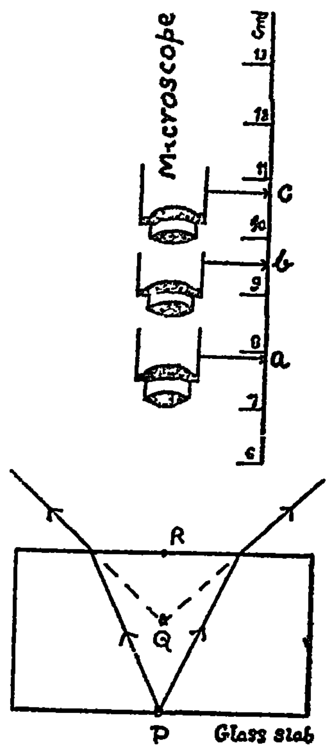


Fig 33.

§ 14 Determination of refractive index of a liquid available only in a few drops :—Both the methods discussed so far are useful only when the liquid is available in large quantity When only a few drops of a liquid are available a concave mirror helps in the determination of μ

Principle :—Let O be the position of the centre of curvature of the concave mirror So that the incident rays OM and OA are

*Authors "A text book of Practical Physics" is recommended for details.

normally incident and hence retrace back their path on reflection.

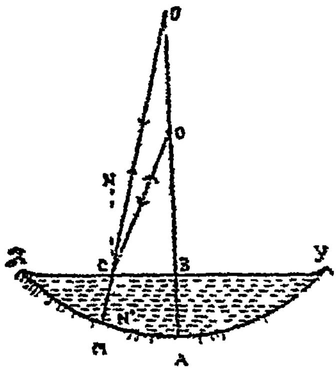


Fig 34

If a few drops of liquid are put on the mirror surface such that XY is its surface, the rays would be refracted at it and will not fall on the mirror normally. If the incident rays however, follow the path $O'C$, such that after refraction at C, the refracted ray is CM, it will normally fall on the mirror and hence will be reflected back along the same path MC and CO', forming the image at O'. Thus, O' will behave as an apparent centre of curvature. See Fig 34

Now $O'C$ is the incident ray at liquid surface CM is the refracted ray and NN' is the normal

Here $\angle O'CN = i = \angle CO'A$ (being alternate angles as NN' and OA are normal to XY and parallel to each other

and $\angle MCN' = r = \angle OCN$ (being vertically opposite and is equal to $\angle COA$ being alternate angles

$$\text{Hence} \quad \mu_{al} = \frac{\sin i}{\sin r} = \frac{\sin CO'A}{\sin COA} \qquad \text{..(1)}$$

$$\text{But} \quad \angle CO'A = \angle CO'B \text{ and } \sin \angle CO'B = \frac{\text{perp.}}{\text{hyp.}} = \frac{CB}{CO'}$$

and $\angle COA = \angle COB$ and $\sin COB = CB/CO$

Substituting these values in equ. (1) we get

$$\mu_{al} = \frac{CB/CO'}{CB/CO} = \frac{CB}{CO'} \times \frac{CO}{CB} = \frac{CO}{CO'} \qquad (2)$$

But as the aperture of the mirror is small, the point C can be regarded as situated near B and hence, $CO = BO$ and $CO' = BO'$.

Also as only a few drops of water are put on the surface of the mirror, the depth BA is negligibly small,

Hence, $CO = BO = AO$ and $CO' = BO' = AO'$

Substituting these values in equ. (2), we get

$$\mu_{al} = AO/AO' \qquad \text{...(3)}$$

Therefore, refractive index of a liquid is equal to the ratio of the actual radius of curvature of the mirror to the apparent radius of curvature when water is poured on the mirror surface.

Method :—Keep the given mirror horizontally on ground and fix a pin on a vertical stand. By moving it up and down remove parallax between it and its image This position is O. Measure its distance from the surface of the mirror. This gives the actual radius of curvature AO.

Now put a few drops of liquid on the mirror surface In order to remove the parallax again, we are required to move the pin to a new position O'. Measure this distance AO' which gives the apparent

radius of curvature. With the help of eqn. (3) determine the refractive index of the liquid

§ 15. **A few optical phenomena :-**

(*a*) **Twinkling of stars :**—The stars due to their infinite distance behave as point objects forming a point images on the retina of the eye Due to the continuous temperature variation of the atmosphere the rays coming from the stars do not follow the same path and consequently the point images formed on the retina shift their position The brain interpretes this continuous shifting as twinkling of the stars

The moon due to its nearness appears as a disc object forming a disc image which occupies a large area on the retina and hence, a little shifting of this image does not affect the apparent position of the moon in the sky That is why the stars twinkle but the moon does not

(*b*) **Setting of Sun :**—Even when the sun has gone below the horizon, it does not appear to set The reason for this is clear from the diagram given below See fig. 35

When the sun is in position *S*, the rays coming from it are refracted by a rarer and rarer medium as it travels from the earth upwards At every refraction as the angle of refraction is greater than the angle of incidence, for second refraction the angle of incidence progressively increases A stage is reached when it reaches the critical stage and is totally reflected downwards These downward rays enter the eyes of the observer in such a direction that for him the sun appears at *S'*.

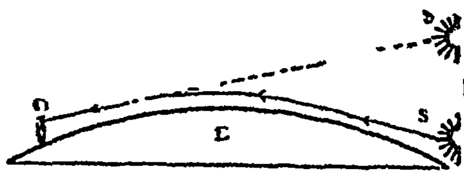


Fig 35

This also explains the sight of an inverted image of a ship hanging up in air at a distance from an observer on sea shore

(*c*) **Mirage :**—A thirsty traveller in deserts gets an illusion of a lake at distance due to the formation of inverted images of trees as

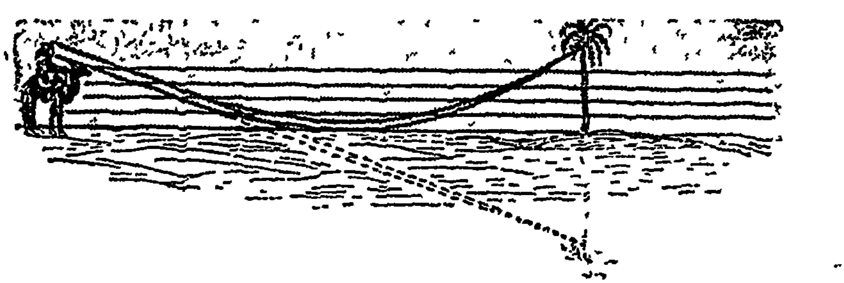


Fig 36

shown in fig 36 This illusion of water is called mirage The illusion of water goes on receding away as the traveller tries to approach it

Due to very high temperature near the ground, the ground layers of air are rarer than the layers of air above. So the rays of light starting from top of the tree downwards are travelling from denser to rarer medium and hence as explained above after a stage total reflection takes places and the downward travelling rays become upward travelling rays. An observer on camel catches these rays and feels that they are coming from an inverted image of a tree. Such an inverted image is formed in water and he gets an illusion of a lake. Being thirsty he sets in its search to get more disillusioned

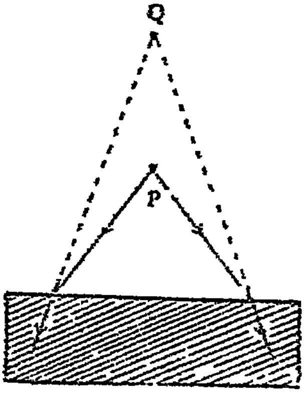


Fig 37.

(*d*) **Apparent depths :**— We have already explained why a river appears to be less deep than its actual depth. For similar reasons if we view an object in air through water or glass, the object appears more distant than its actual position. See fig. 37

An object lying at the bottom of a river is visible from above the river. As the observer is trying to look at increasing inclination the object appears to be raised up more and more. Finally a stage is reached when rays coming from the object are critically reflected inward. Thus eye in e_3 position is not able to see the object. See fig 38

Hence, to an eye under water all objects lying above the surface of water appear to lie inside a cone of semi-vertical angle equal to the critical angle.

Fig 38.

§ 16. **Specimen Examples** **Example 1.** *Find the critical angle of glass with respective to water. Given the refractive indices of glass and water to be 3/2 and 4/3 respectively*

Now $\quad \mu_{wg}=\mu_{ag}/\mu_{aw}$

$\therefore \quad \mu_{wg}=\dfrac{3/2}{4/3}=\frac{3}{2}\times\frac{3}{4}=\frac{9}{8}$

We know that $\quad \mu_{wg}=\text{cosec }\theta$

or $\quad 9/8=\text{cosec }\theta$

$\therefore$ the critical angle $\theta=\text{cosec}^{-1}\ 9/8$.

Example 2. *The refractive index of water is 4/3. Find the apparent depth of a river, if its actual depth is 8 feet.*

$$\mu_{aw}=\frac{\text{Real depth}}{\text{Apparent depth}}$$

$$\therefore \quad \text{Apparent depth}=\frac{\text{Real depth}}{\mu_{aw}}$$

or $\quad$ Apparent depth $=\frac{8}{4/3}=8\times\frac{3}{4}=6$ feet

Example 3. *An observer looking vertically down into a river observes the image of his eye and the image of a stone at the bottom of the river coinciding. If the eye is 6 feet above the surface of water, find the actual depth of the river.* $\mu_{aw}=4/3$.

Obviously the image of the eye is formed due to reflection. Therefore, the eye image and the stone image due to refraction are at a distance of 6 ft from water surface. Therefore, apparent depth is 6 feet.

$\therefore$ $\quad$ Real depth $=\mu_{aw}\times$ apparent depth

$$=4/3\times6=8 \text{ feet.}$$

Example 3. *When a microscope is vertically focussed on the image of an object through a liquid, the microscope position is 'a'. It is b when focussed on the surface of water. More liquid is then added and the former readings are repeated. They are now c and d respectively. Find the refractive index of the liquid.*

See Fig 39, YZ is the original layer of liquid and XY is the new layer added. Therefore, the real depth of the new layer XY is $(d-b)$

Apparent depth of YZ layer is $(b-a)$ and of XZ is $(d-c)$,

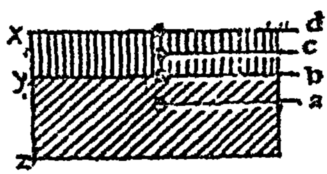


Fig. 39.

Hence apparent depth of XY layer= apparent depth of XZ layer$-$apparent depth of YZ layer

$$=(d-c)-(b-a)=d-c-b+a=a+d-b-c$$

Hence $\quad \mu=\frac{\text{Real depth}}{\text{apparent depth}}=\frac{d-b}{a+d-b-c}$

Example 4 *An object is viewed normally through a plane parallel plate of glass of thickness d cms and having refractive index μ. Prove that the object is apparently displaced towards the observer through a distance* $\frac{(\mu-1)d}{\mu}$.

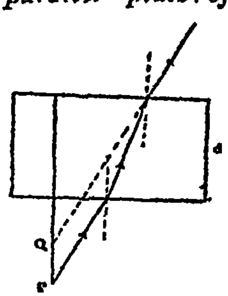


Fig 39(a)

$$\frac{\text{Real depth}}{\text{Apparent depth}}=\mu$$

$$\text{Apparent depth}=\frac{\text{Real depth}}{\mu}$$

$$=\frac{d}{\mu}$$

Hence, displacement towards observer=real depth$-$apparent depth

$$=d-\frac{d}{\mu}=\frac{\mu d-d}{\mu}=\frac{(\mu-1)d}{\mu}.$$

QUESTIONS.

1. Define refractive index. On what factors does it depend and how? Prove that $\mu_{wg}=\mu_{ag}/\mu_{aw}$. (see §2, §3 and §6)

2. What do you understand by critical angle and total internal reflection? How is critical angle related with the refractive index of a medium? Distinguish between total internal reflection and ordinary reflection. (see §7, §8 and §9)

3. How will you determine critical angle of a liquid? Describe the method. (§ 10)

4. Explain why a river appears to be less deep than its actual depth. How are the two related. Describe an experiment which makes use of this relation to determine μ of a liquid. (see §11, §12 and §13)

5. How will you determine the refractive index of a precious liquid? (see §14)

6. Explain why

(a) a crack in glass appears bright. (see §7)

(b) a mirage occurs. (see §15)

(c) a ship appears hanging inverted in air. (see §15)

(d) a river appears less deep than its actual depth. (see §11)

7. If the critical angle for a given liquid with respect to air be 45°, find the refractive index of the liquid. (Ans. $\sqrt{2}$)

8. A transparent cube of 16 cm side contains a small air bubble. Its apparent depth when viewed through one face of the cube is 6 cm and when viewed through the opposite end is 4 cm. Calculate the actual position of the air bubble. Also calculate refractive index of the cube.

(Ans. 9.6 cm. from first surface, μ=1.6)

9. A concave mirror of radius of curvature 32 cm. lies on a table and a pin is moved vertically above it. If it is filled with a liquid of refractive index 4/3, where will the object and its image coincide? (Ans. 24 cm.)

10. A mark is made on the bottom of a beaker and a vertical microscope is focussed on it. The microscope is then raised through a distance of 1.5 cm. What height of water must be poured into the beaker in order to bring the mark again into focus. (Ans. 3.75 cm.)

CHAPTER V

REFRACTION AT PLANE INCLINED SURFACES

§ 1. **Prism :**—We have already considered refraction through a medium bounded by two plane parallel surfaces The emergent ray in this case is parallel to the incident ray but is displaced. This displacement depends on the direction of incidence and the thickness of the refracting medium

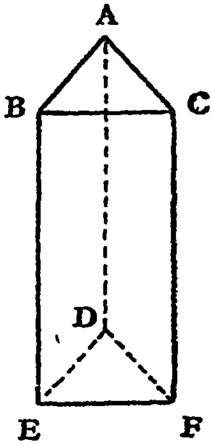


Fig 40 (*a*).

Let us now consider a medium bound between two plane surfaces but inclined at an angle to each other Such a medium is called a *prism* See fig. 40(*a*) *ABED* and *ACFD* are the two refracting surfaces These surfaces meet along a vertical edge *AD* called the refracting edge $\angle BAC$ included between the two surfaces is called the *angle of the prism* *BCFE* is called the base of the prism Diagrammatically, usually it is denoted by a section at right angles through its refracting edge is in fig 40 (*b*).

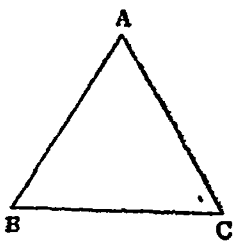


Fig 40 (*b*)

§ 2 **Refraction through a prism:**—*PQ* is the incident ray on the surface *AB* where *MO* is the normal *QR* and *RS* are the refracted and emergent rays respectively. *NO* is the normal at *R*

$\angle PQM$ is the incident angle i
$\angle OQR$ is the refracted angle r
$\angle SRN$ is the emergent angle e.

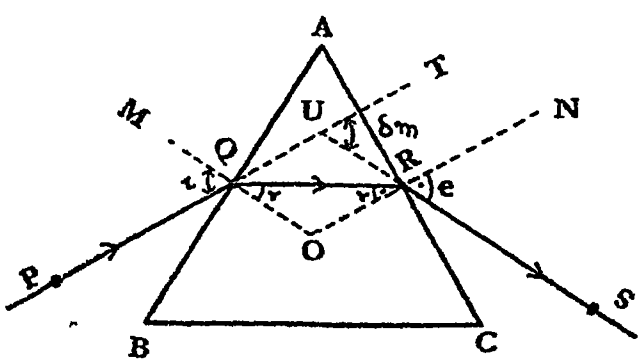


Fig 40.

If *PQ* and *RS* are pro duced forward and backward respectively, they intersect at *U*. The original direction of incidence *PQUT* is changed to *URS* after refraction. The direction, therefore, has changed or deviated through $\angle TUR$. *This angle TUR is called the* **angle of deviation δ.**

§ 3. **Angle of minimum deviation :**—The angle between the incident and emergent ray is called the angle of deviation In the case of parallel plate of glass where the two surfaces *AB* and *AC* are parallel to each other, the angle of deviation is zero.

The value of the angle of deviation depends on the value of the angle of incidence for a certain prism. It is found that, as the incidence is changed from normal to grazing, i.e., from $i=0$ to $i=90$, the angle of deviation at first decreases, gradually becomes minimum and then again starts increasing. This variation is shown graphically in fig 40 (d).

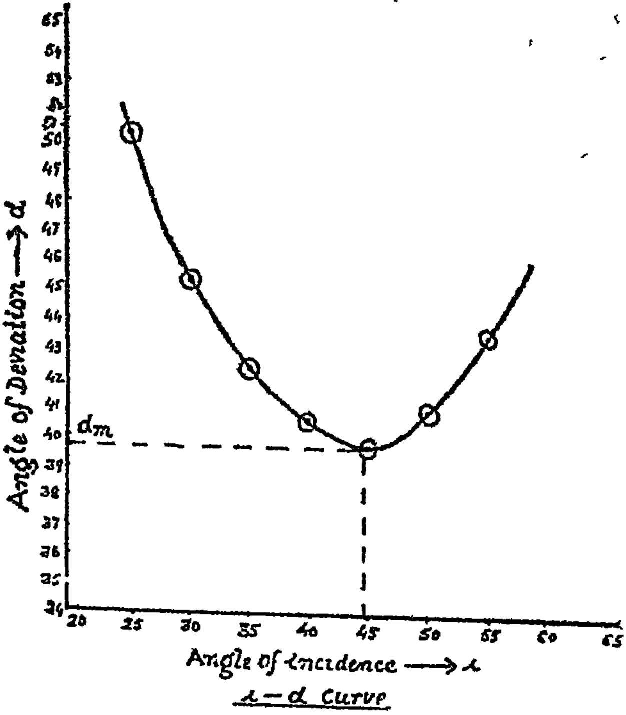


Fig. 40 (d)

The variation in δ at first is very steep (denoted by AB, then it becomes more gradual along BCD and again is steep along DE). At C, the value of δ becomes minimum, i.e., δ_m corresponding to an angle of incidence i_m. *This value of angle of deviation when it becomes minimum is called the* **angle of minimum deviation.**

From the fig 40(d) it is quite clear that if the angle of incidence is even slightly changed from the value of i_m the angle of deviation always increases. Thus, corresponding to the value of angle of minimum deviation, there is only one value of angle of incidence for a particular prism.

§4 **Relation between angle of the prism, its refractive index and angle of minimum deviation :—**

Let the prism be placed in the minimum deviation position such that the angle of incidence for PQ is such that the corresponding angle of deviation is minimum (Note that the prism may not be isosceles).

If the incidence is along PQ, the emergent ray is RS and the angle of minimum deviation is TUR. If the path of rays is reversed such that the incidence is along SR, the emergent ray is QP and the angle of deviation is VUQ.

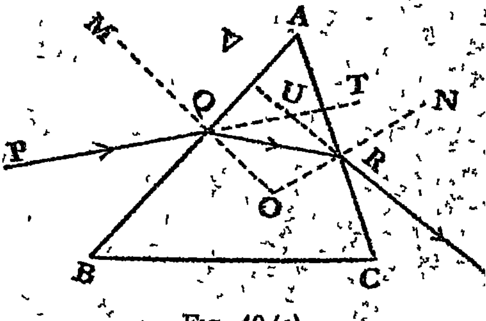


Fig. 40 (e).

But $\angle TUR = \angle VUQ = \delta_m$ (being vertically opposite)

Hence, the corresponding angles of incidence must be equal

i e., $$\angle PQM = i = \angle NRS = e \qquad (1)$$

when incidence is considered at Q,

$$\mu_{ag} = \frac{\sin i}{\sin r_1} = \frac{\sin PQM}{\sin OQR} \qquad . \quad (2)$$

and when incidence is considered at R,

$$\mu_{ag} = \frac{\sin e}{\sin r_2} = \frac{\sin NRS}{\sin ORQ} \qquad . \; . \quad (3)$$

Comparing equations (1) and (2), we get

$$\frac{\sin i}{\sin r_1} = \frac{\sin e}{\sin r_2}$$

But $\qquad i = e$, according to equ (1)

Therefore, the denominators must also be equal

i.e., $\qquad r_1 = r_2 \qquad$ or $\qquad \angle OQR = \angle ORQ \qquad (4)$

If we consider the quadrilateral $QARO$, the sum of the four angles is equal to 4 right angles

$\therefore \quad \angle OQA + \angle QAR + \angle ARO + \angle ROQ = 4$ rt angles

Of these, the angles OQA and ARO each are rt angles being formed with the normals

Hence, the remaining angles, $\angle QAR + \angle ROQ$

$= 2rt$ angles $\qquad . \quad (5)$

If you consider the $\triangle QOR$, the sum of the three angles is equal to 2 rt angles

$\therefore \quad \angle OQR + \angle ORQ + \angle ROQ = 2$ rt angles $\qquad (6)$

Equating equations (5) and (6) as each are equal to 2 rt. angles, we get

$$\angle QAR + \angle ROQ = \angle OQR + \angle ORQ + \angle ROQ$$

or $$\angle QAR = \angle OQR + \angle ORQ$$

or $$A = r_1 + r_2 \qquad (7)$$

Here, $\qquad A = \angle QAR =$ the angle of the prism

But according to equation (4) we know that $\angle r_1 = r_2$, say $= r$.

Then, equ (7) becomes

$$A = r + r = 2r$$

or $$r = \frac{A}{2} \qquad (8)$$

If you consider the $\triangle QUR$, $\angle RUT$ is the external angle and must be equal to the sum of the two opposite internal angles

Hence, $\quad \angle RUT = \angle \delta_m = \angle URQ + \angle UQR \qquad . \; (9)$

But, $\qquad \angle URQ = \angle URO - \angle ORQ \qquad . \; (10)$

As $\quad \angle URO = \angle SRN = e = i$ being vertically opposite

and $\qquad \angle ORQ = r_2 = r$

Therefore, substituting these values in equ (10) we get,

i e $\angle URQ = i - r$

Similarly, $\angle UQR = \angle UQO - \angle OQR = \angle PQM - \angle OQR$

$= i - r$

Substituting these values of $\angle URQ$ and $\angle UQR$ in equ (9) we get

$$\angle \delta_m = i - r + i - r = 2i - 2r$$

But $2r = A$

Hence $\delta_m = 2i - A$ (11)

or $2i = \delta_m + A$

or $$i = \frac{\delta_m + A}{2}$$ (12)

We know that

$\angle AQO = \angle ARO$ being rt. angle

As $\angle AQR = \angle AQO - \angle RQO = 90 - r$

and $\angle ARQ = \angle ARO - \angle QRO = 90 - r$

Hence, $\angle AQR = \angle ARQ$

These being base angles of the $\triangle AQR$,

the adjacent sides are equal,

i e $AQ = AR$ (13)

In other words, the refracted ray in minimum deviation position cuts the refracting surfaces at equal distances from the refracting edge

In addition if the prism is an isosceles prism i e, whose sides AB and AC are equal, we shall have the base angles $\angle ABC$ and $\angle ACB$ also equal $\angle BAC$ being common in both the Δs QAR and BAC, we have $\angle AQR = \angle ABC$ and $\angle ARQ = \angle ACB$.

These being corresponding angles, the refracted ray QR is parallel to the base. This is only true in the case of a prism whose sides are equal

Thus, to summarise, when the prism is placed in the minimum deviation position, we have the following —

(a) $i = e = i$

(b) $r_1 = r_2 = r$

(c) $r = \frac{A}{2}$

(d) $i = \frac{\delta_m + A}{2}$

(e) $AQ = AR$

(f) $QR \parallel BC$, provided the prism is isosceles.

We know that,

$$\mu = \frac{\sin i}{\sin r}$$

Hence, substituting the values of i and r from above, we get

$$\mu=\frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}} \quad \dots (14)$$

If the angles are small, we have approximately

$$\mu=\frac{\frac{A+\delta_m}{2}}{\frac{A}{2}}=\frac{A+\delta_m}{A}$$

Cross multiplying we get,

$$\mu A=A+\delta_m$$

or $$\delta_m=\mu A-A=(\mu-1)\,A \quad (15)$$

From equ (15) it is obvious that the value of angle of minimum deviation depends on

(*a*) μ, *i e*, on material of the prism.

(*b*) A, *i.e*, on the angle of the prism

Greater these are, greater would be the value of δ_m.

§ 5. Importance of minimum deviation position :—If you consider an incident beam coming from a point source almost in minimum deviation position as in fig 41 (*a*) the emergent beam is equally inclined and hence appear to come from a single point Q But when incidence is as shown in fig 41, (*b*) the beam is unequally inclined and some rays appear to come from one point and some from other point The outcome of this is that in first position we get a well-defined image while in the second a blurred one

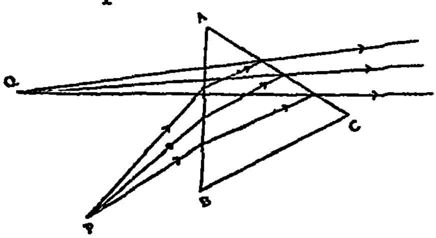


Fig 41 (*a*)

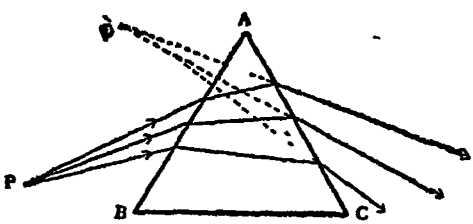


Fig 41 (*b*)

Hence, whenever a well defined sharp image is required as in spectrum, the prism in minimum deviation position is always preferred

§ 6. Determination of refractive index of a prism:—* In order to determine μ of a material in the form of a prism, the use of eqn. 14 is made of.

Determination of A :—The angle of the prism can be determined by drawing its boundary But this is not advised because the boundary cannot be drawn accurately and consequently the measurement would be wrong Therefore to measure A optically, draw two parallel lines and put the prism in between the lines so that the lines touch its surfaces Treating the surface AB as

* For practical details see "A T. B of Practical Physics" by authors.

reflector, fix two pins at E and F on the line drawn See fig. 42 (a). Try to catch the images of E and F formed by reflection and in their line fix two pins at K and J Repeat this procedure on the other side of the prism KJ and SR are the respective reflected rays for the incident rays FE and HG.

As both the incident rays are parallel, the angle between the two reflected rays must be twice the angle between the two reflected surfaces AB and AC.

Therefore, produce KJ and SR backwards to meet at T. The angle $JTR=2A$.

Thus, instead of measuring A we measure $2A$ and this further increases the accuracy

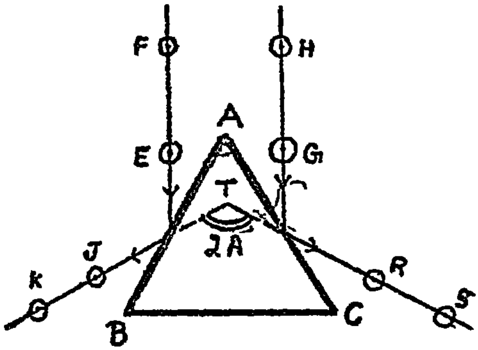


Fig 42 (a)

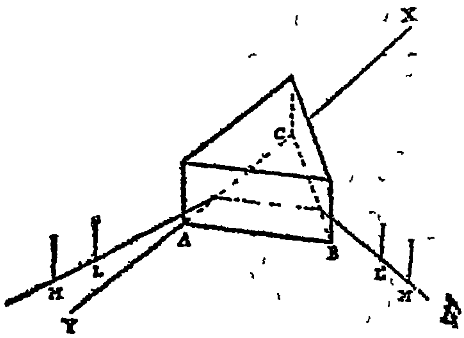


Fig 42 (b)

Determination of δ_m :—The actual arrangement is shown in fig 42(b) and fig 42(c). Draw a line making a certain angle with the normal to the surface AB of the prism. Fix two pins at M and L. See their images through the other side of the prism and in their line fix two pins at L' and M' Then ML is the incident ray and $L'M'$ is the emergent ray. Produce them backwards to meet at O. Measure the $\angle QOL'$ which gives the angle of deviation. See fig 42 (c).

Thus, corresponding to the various angles of incidence, measure the angle of deviation. Draw a graph between i and δ and from it find the angle of minimum deviation. See fig 40 (d)

Knowing A and δ_m, calculate the value of μ with the relation

$$\mu=\frac{\sin\dfrac{A+\delta_m}{2}}{\sin\dfrac{A}{2}}$$

Below is described a simpler method for determination of δ_m. For this, eqn (13) of §4 is made use of.

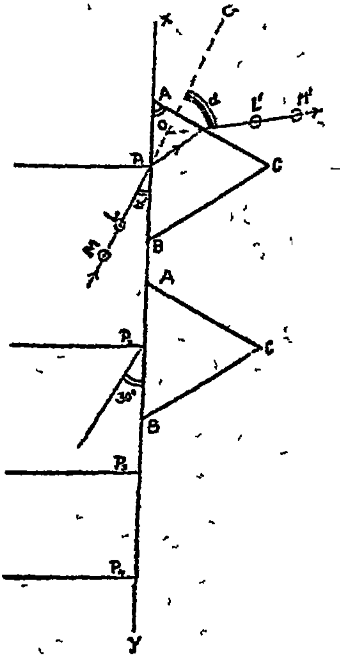


Fig 42 (c)

According to fig 42 (d) fix two pins Q and R on the two sides of the prism at equal distances from the refracting edge A. Now try to fix a pin P on one side and the pin S on the other side in such a way that when seen through AC, all the four pins apppear to be in the same straight line Join PQ and SR and produce them backwards to meet at a point and measure the acute angle between them This directly gives δ_m according to eqn. (13) of §4.

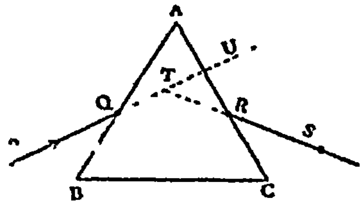


Fig. 42 (d).

§ 7. **Uses of a prism:**—If a whole beam of light is passed through a prism, the different coloured rays bend through different angles and the emergent beam consists of separate colours The separation of white light into its component colours is called dispersion and the bend of colours is called spectrum.

The formation of spectrum and its analysis to understand the nature of matter is an important branch of physics and hence prism forms a very important component of optical instruments.

It is also used to totally reflect a beam of light and to lengthen the path of a beam.

QUESTIONS

1. What do you understand by deviation and minimum diviation. What is the importance of minimum deviation position? How will you determine it experimentally? [(See §2, §3, §5 and §6)

2 Discuss the various properties of the prism in the position of minimum deviation and prove the relation

$$\mu=\frac{\sin\dfrac{A+d}{2}}{\sin\dfrac{A}{2}} \qquad \text{(See §4)}$$

3. How will you determine μ of a material in the form of a prism (See §6)

4. A right angled prism has all its sides equal If a ray enters normally for what smallest refractive index of the material will it be totally reflected? (*Ans.* $\mu=\sqrt{2}$)

CHAPTER VI

REFRACTION AT SPHERICAL SURFACE

§ 1. **A geometrical problem** :—Let us consider a $\triangle ABC$. From any point, say A, drop a perpendicular AD to the opposite side

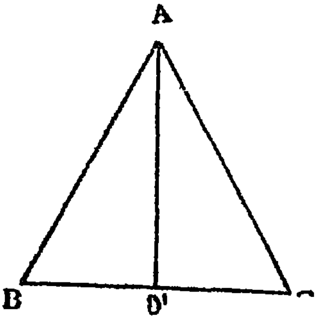


Fig 43

Then, $\sin B = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AD'}{AB}$

and $\sin C = \frac{AD'}{AC}$

Dividing the above two, we have

$$\frac{\sin B}{\sin C} = \frac{AD'}{AB} \Big/ \frac{AD'}{AC} = \frac{AD'}{AB} \times \frac{AC}{AD'}$$

$$= \frac{AC}{AB}$$

or *the ratio of the sines of any two angles is equal to the ratio of their opposite sides* This geometrical relation is made use of in the following

§ 2. **Refraction at a concave spherical surface:**—Let us consider a refracting medium bounded by a concave spherical surface XAY whose centre of curvature is at O and A is the pole. Then AO is the principal axis of the refracting surface, see fig. 44

Let PM be the incident ray and OMO' is the normal As the incident ray is going from the rarer medium to a denser medium instead of continuing along the path MP' bends towards the normal so that MQ' is the refracted ray Another incident ray may be considered normally incident along PA. Both these refracted rays when produced backward appear to meet at Q and thus Q is the corresponding image of P

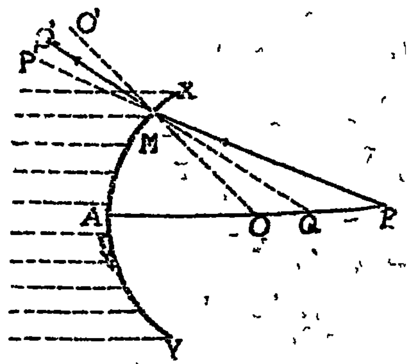


Fig. 44

Here the angle of incidence $= i = \angle PMO$ and the angle of refraction $= r = \angle Q'MO' =$ vertically opposite $\angle QMO$

If we consider the $\triangle PMO$, according to § 1, we have

$$\frac{\sin PMO}{\sin MOP} = \frac{OP}{MP} \quad \cdot \; (1)$$

and in $\triangle QMO$,

$$\frac{\sin QMO}{\sin MOQ} = \frac{OQ}{MQ} \quad \ldots (2)$$ Here, $\angle MOQ = \angle MOP$

Dividing eqn (1) by eqn. (2) we get

$$\frac{\sin PMO}{\sin MOP} \div \frac{\sin QMO}{\sin MOQ} = \frac{OP}{MP} \div \frac{OQ}{MQ}$$

or
$$\frac{\sin PMO}{\sin MOP} \times \frac{\sin MOQ}{\sin QMO} = \frac{OP}{MP} \times \frac{MQ}{OQ}$$

or
$$\frac{\sin PMO}{\sin QMO} = \frac{\sin i}{\sin r} = \frac{OP}{MP} \times \frac{MQ}{OQ} \quad \ldots (3)$$

Here we consider the aperture of the spherical surface to be small. Here M point may be regarded as situated close to A. As a result we may take $MP=AP$ and $MQ=AQ$.

Therefore eqn (3) becomes

$$\mu = \frac{OP}{AP} \times \frac{AQ}{OQ} \quad \ldots (4)$$

But $\quad OP=AP-AO$ and $OQ=AQ-AO$,

Hence
$$\mu = \frac{AP-AO}{AP} \times \frac{AQ}{AQ-AO} \quad \ldots (5)$$

Putting, $AP=u$, the object distance, $AQ=v$, the image distance and $AO=r$, the radius of curvature of the spherical surface, whence

$$\mu = \frac{u-r}{u} \times \frac{v}{v-r} = \frac{v\,(u-r)}{u\,(v-r)} \quad \ldots (5A)$$

Cross-multiplying we get,

$$\mu\, u\,(v-r) = v\,(u-r)$$

Simplifying the above becomes

$$\mu\, u\, v - \mu\, u\, r = vu - vr$$

Separating the terms containing r on r.h.s. we have

$$\mu\, u\, v - uv = \mu ur - vr \quad \ldots (6)$$

Dividing eqn. (6) by uvr, we get

$$\frac{\mu\, u\, v}{uvr} - \frac{vu}{uvr} = \frac{\mu ur}{uvr} - \frac{vr}{uvr}$$

or
$$\frac{\mu}{r} - \frac{1}{r} = \frac{\mu}{v} - \frac{1}{u}$$

or
$$\frac{\mu-1}{r} = \frac{\mu}{v} - \frac{1}{u} \quad \ldots (7)$$

According to eqn. (7), the l.h s. is a constant quantity. Therefore for one value of u there would be only one value for v. Hence, all the rays starting from P after refraction would appear to come from Q making it as the image

§ 3. **Refraction at a convex spherical surface :**—The same notations are used as in the above

Here, $\quad i = \angle PMO'$ .. (a)

and $r = \angle Q'MO =$ vertically opposite $\angle QMO'$ ... (b)

If we consider the $\triangle PMO$, according to § 1 we have,

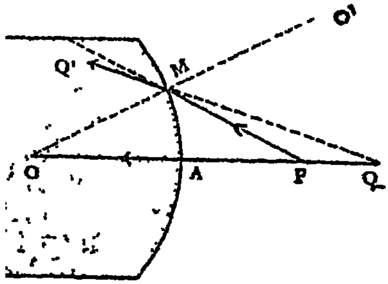


Fig 45

$$\frac{\sin PMO}{\sin MOP}=\frac{OP}{MP} \quad \ldots (1)$$

and in $\triangle QMO$,

$$\frac{\sin QMO}{\sin MOQ}=\frac{OQ}{MQ} \quad . (2)$$

Dividing eqn. (1) by eqn. (2) we get as in above article §2,

$$\frac{\sin PMO}{\sin QMO}=\frac{OP}{MP}\times\frac{MQ}{OQ} \quad \ldots (3)$$

Note that so far the procedure is identical with § 2.

Here, $\angle PMO=\angle O'MO-\angle PMO'=180-i$

and $\angle QMO=\angle O'MO-\angle QMO'=180-r$ according to eqn (a) and (b)

Also $MP=AP$ and $MQ=AQ$ for reason explained in § 2.

Therefore eqn. (3) is altered to

$$\frac{\sin(180-i)}{\sin(180-r)}=\frac{OP}{AP}\times\frac{AQ}{OQ}$$

But we know that $\sin(180-i)=\sin i$

$$\sin(180-r)=\sin r$$

and

$$\frac{\sin i}{\sin r}=\mu$$

Substituting these values in the above

$$\frac{\sin i}{\sin r}=\mu=\frac{OP}{AP}\times\frac{AQ}{OQ} \quad . (4)$$

From fig. 45 we see that

$$OP=AP+OA \text{ and } OQ=AQ+OA$$

Hence eqn. (4) becomes

$$\mu=\frac{AP+OA}{AP}\times\frac{AQ}{AQ+OA} \quad . (5)$$

Put $AP=u$, $AQ=v$ and $AO=-r$. Here r is substituted with $-ve$ sign as the surface is convex

Hence, $$\mu=\frac{u-r}{u}\times\frac{v}{v-r}=\frac{v(u-r)}{u(v-r)} \quad \ldots (5\text{A})$$

This eqn. (5A) is the same as in § 2 and hence proceeding exactly as is explained in § 2 we get

$$\frac{\mu-1}{r}=\frac{\mu}{v}-\frac{1}{u}$$

Thus, in general, for a spherical surface we have the relation

$$\frac{\mu}{v}-\frac{1}{u}=\frac{\mu-1}{r}.$$

§ 4 Focal lengths of a refracting spherical surface :—

In the above relation $\frac{\mu}{v}-\frac{1}{u}=\frac{\mu-1}{r}$, if we put $\mu=\infty$, *i e*, if we assume the incident beam to be parallel to the principle axis of the surface, after refraction we get

$$\frac{\mu}{v}-\frac{1}{\infty}=\frac{\mu-1}{r}$$

But $\frac{1}{\infty}=0$, hence

$$\frac{\mu}{v}=\frac{\mu-1}{r} \text{ or } v(\mu-1)=\mu r.$$

or

$$v=\frac{\mu r}{\mu-1} \qquad \dots (1)$$

That is the image is formed at a distance $\frac{\mu r}{\mu-1}$ This, therefore, is called the image focal length.

On the other hand, if $v=\infty$, *i e.*, the refracted beam is parallel we have

$$\frac{\mu}{\infty}-\frac{1}{u}=\frac{\mu-1}{r},$$

or

$$-\frac{1}{u}=\frac{\mu-1}{r}, \text{ as } \frac{\mu}{\infty}=0$$

or

$$u(\mu-1)=-r$$

or

$$u=-\frac{r}{\mu-1} \qquad . (2)$$

For this condition object has to be placed inside the material at a distance $\frac{r}{\mu-1}$ This, therefore, is called the object focal length

§ 5. A few specimen examples :—1. *A small object is enclosed in a solid sphere of glass of radius 5 cm The object is situated 1 cm from the centre and is viewed from the side to which it is nearest. If $\mu_{ag}=1\ 5$, where will it appear.*

Also calculate its apparent position if it is viewed along the diameter through the greatest thickness of glass.

In first problem, according to fig 46,
A is the pole so that

$u=AP=AO-OP=5-1=4$ cm

Rays are going from glass into air, so in place of μ we will have to consider μ_{ga}.

Hence eqn $\frac{\mu}{v}-\frac{1}{u}=\frac{\mu-1}{r}$ for this problem becomes

$$\frac{\mu_{ga}}{v}-\frac{1}{4}=\frac{\mu_{ga}-1}{5}$$

Fig 46

or
$$\frac{1}{\mu_{ag}/v}-\frac{1}{4}=\frac{\frac{1}{\mu ag}-1}{5}$$

Multiplying the above by μ_{ag} we have,
$$\frac{1}{v}-\frac{\mu ag}{4}=\frac{1-\mu ag}{5},$$

Substituting the value of $\mu ag=1{\cdot}5=\frac{3}{2}$ we have
$$\frac{1}{v}-\frac{\frac{3}{2}}{4}=\frac{1-\frac{3}{2}}{5} \qquad \ldots (A)$$

or
$$\frac{1}{v}-\frac{3}{8}=\frac{-\frac{1}{2}}{5}=-\frac{1}{10}$$

or
$$\frac{1}{v}=\frac{3}{8}-\frac{1}{10}=\frac{15-4}{40}=\frac{11}{40}$$

∴
$$v=\frac{40}{11}=3{\cdot}636\ldots=3\;64 \text{ cm.}$$

The image is, therefore, formed on the same side as the object. Remember here u and r are both taken $+ve$ quantities.

In second case, the line of sight is along POA' and hence $u=5+1=6$ cm. The rest of the procedure is the same.

In eqn. (A) substitute 6 in place of 4 and get the answer $v=6\;67$ cm

Example 2. *A small air bubble inside a solid glass sphere of diameter 6 cm. appears to be 1 cm. from the surface when looked at along a diameter Find the actual position of the bubble. Given* $\mu_{ag}=1{\cdot}5$.

Here $r=\frac{6}{2}=3$ cm and $v=1$ cm. of course in place of μ_{as} we have to put μ_{ga}.

Hence
$$\frac{\mu_{ga}}{v}-\frac{1}{u}=\frac{\mu ag-1}{r}$$

or
$$\frac{1/\mu_{ag}}{v}-\frac{1}{u}=\frac{1/\mu_{ag}-1}{r}$$

Substituting the values of v, r and μ_{ag} we get
$$\frac{1/\frac{3}{2}}{1}-\frac{1}{u}=\frac{1/\frac{3}{2}-1}{3}$$

or
$$\frac{2}{3}-\frac{1}{u}=\frac{\frac{2}{3}-1}{3}=-\frac{\frac{1}{3}}{3} \text{ or } =-\frac{1}{9}$$

or
$$-\frac{1}{u}=-\frac{2}{3}-\frac{1}{9}=\frac{-6-1}{9}=-\frac{7}{9}$$

∴
$$u=\frac{9}{7}=1\;28$$

The bubble is situated at a distance of 1·28 cm approximately from the surface.

QUESTIONS

1. Deduce the relation between μ, u, v and r for a spherical surface (see § 2 and § 3)

2. Explain why an air bubble inside a solid glass sphere appears at various distances when viewed from various sides

3 What do you mean by focal length of a spherical surface. Determine its focal length if $\mu=1.5$ and $r=3$ cm. (see § 4)

4 A small object is embedded in a solid glass sphere of 14 cm diameter. It is situated at a distance of 1 cm from the centre Where will the object appear if it is viewed from the side nearest to it. Refractive index of glass is given to be 1·4 (*Ans.* 5 676 behind the side from which it is viewed)

5. There is a glass sphere of 10 cm. diameter and its refractive index is 1·4. Find its principal focus. (*Ans* 2 5 cm behind the second surface)

CHAPTER VII

REFRACTION THROUGH A LENS

§ 1. A Lens :—A lens is defined as the refracting medium bounded between two spherical surfaces as shown in fig. 47. The

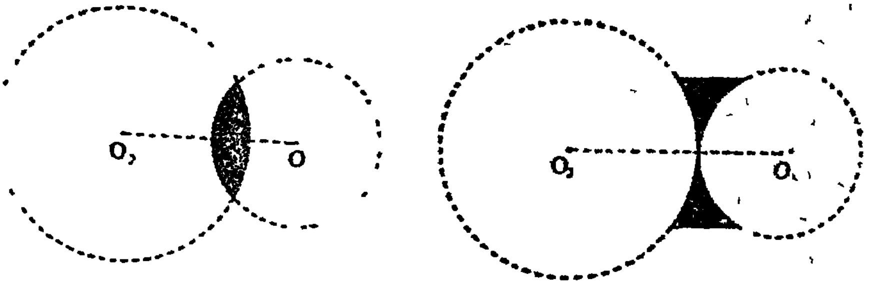


Fig. 47.

shaded portion is the lens while the dotted ones are the imaginary spheres of which the two spherical surfaces are parts.

§ 2 Types of Lens :—Spherical lenses are divided into two types—(1) convex and (2) concave.

Convex lens is thicker in the centre while goes on becoming thinner at the edges Reverse is the case in concave lens. The properties of these lenses are also quite distinct.

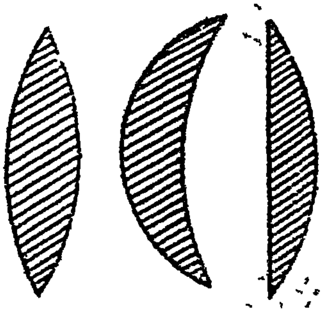


Fig. 48 (*a*) (*b*) (*c*)

Each type is further sub-divided into three types :

1. (*a*) Double convex or *bi* convex. See fig. 48 (*a*)

(*b*) Concavo convex or Convex meniscus. See fig. 48 (*b*)

(*c*) plano convex See fig 48 (*c*).

2 (*a*) Double concave or *bi* concave See fig. 49 (*a*).

(*b*) Convexo concave or Concave meniscus See fig 49 (*b*)

(*c*) Plano concave See fig. 49 (*c*)

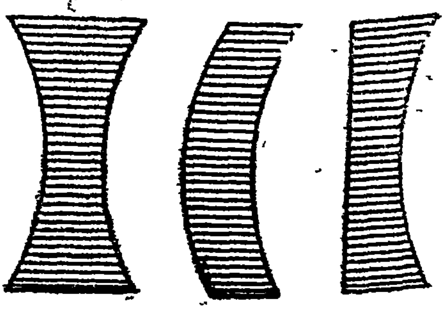


Fig 49 (*a*) (*b*) (*c*)

§ 3. Similarity between the action of a lens and a combination of truncated prism :—

Take a prism of a small angle and truncate it by means of removing its prism angle portion Keep on both sides of it such

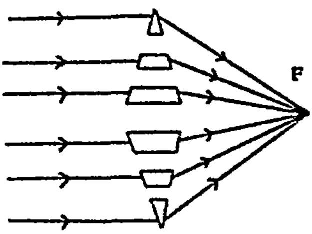


Fig. 50 (a)

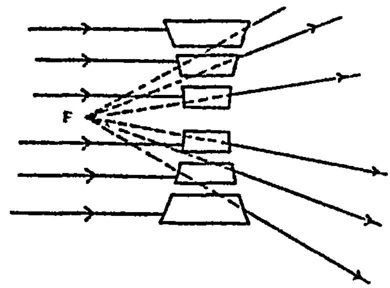


Fig 50 (b).

truncated prisms but of gradually increasing angle of prism as shown in Fig 50 (a) and 50 (b)

We know that the deviation suffered by ray is directly proportional to the angle of the prism Hence a parallel beam incident as shown in figs will go on suffering increasing deviation as they are more removed from the central portion. The deviation takes place towards the base of the prism Thus the converging action of the convex lens and the diverging action of the concave lens becomes apparent.

§ 4. **Optical Centre :**—Let PQ be an incident ray on a les.o O_1 and O_2 are respectively the centres of curvature of the ntw surfaces of the lens respectively Join O_1 to Q and from O_2 draw a line RO_2 parallel to O_1Q and let it intersect the second surface of the lens at R. If we draw tangents at Q and R respectively, they would be parallel to each other and hence if we were to consider surfaces just near Q and R, it would constitute a parallel slab of glass Hence, corresponding to an incident ray PQ we shall get the emmergent ray RS which would be parallel to PQ

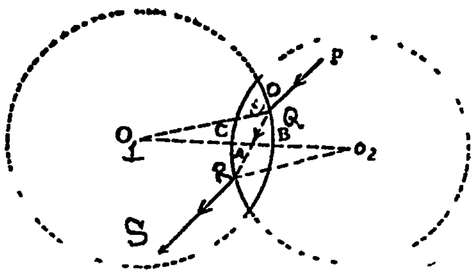


Fig. 51.

Join QR This would give the refracted ray inside the lens, in tersecting the line joining Q and O_2 at A

In $\triangle$s O_1AQ and O_2AR, we have

$\angle O_1QA = \angle O_2RA$ being alternate angles formed in between the parallel lines O_1Q and O_2R

$\angle O_1AQ = \angle O_2AR$ (being vertically opposite,

and hence, the remaining angles are also equal

Therefore, $\triangle$s are similar and we have

$$\frac{O_1A}{O_2A} = \frac{O_1Q}{O_2R} \qquad (1)$$

But, $O_1Q = O_1B$ and $O_2R = O_2C$ (being radii of the same spheres)

Making these substitutions, we have

$$\frac{O_1A}{O_2A} = \frac{O_1B}{O_2C} \qquad (2)$$

We already know that if

$$\frac{a}{b}=\frac{c}{d},\quad \frac{a}{b}=\frac{c}{d}=\frac{a-c}{b-d}$$

Making use of this in eqn. (2), we get

$$\frac{O_1A}{O_2A}=\frac{O_1B}{O_2C}=\frac{O_1B-O_1A}{O_2C-O_2A}=\frac{AB}{AC}$$

Thus $$\frac{AB}{AC}=\frac{O_1B}{O_2C}=\frac{r_1}{r_2}$$

$$=\frac{\text{radius of curvature of first surface}}{\text{radius of curvature of second surface}} \quad \ldots (3)$$

Thus, we find that A divides the thickness of the lens internally in the ratio of its radii of curvature. This point A is called the optical centre.

If we were to consider the case of a meniscus, we can prove the property of the optical centre.

In general, we define an optical centre as a point so situated along its principle axis that it divides the thickness of the lens internally or externally in the ratio of its radii of curvature.

We know that in a parallel slab of glass the emergent ray is parallel to the incident ray and the displacement depends on the thickness of the slab. If it is zero, the displacement is also zero

If we, therefore, consider a thin lens and consider an incident ray directed towards the optical centre, it will emerge without suffering any displacement or deviation

§5. **Cardinal points:**—If we consider an incident beam parallel to the principle axis, after refraction through a lens (i e, after being refracted through both the surfaces of the lens) converges to (as in convex lens) or appears to diverge from (as in concave lens) a point situated on the principle axis. This point F_2 is called the image focal point of the

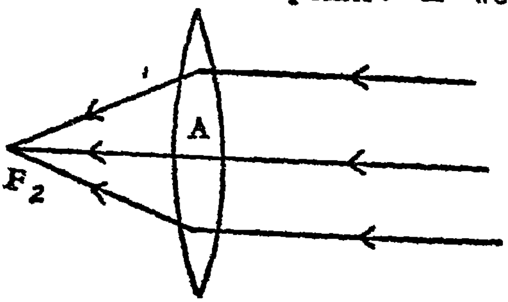


Fig 52 (a)

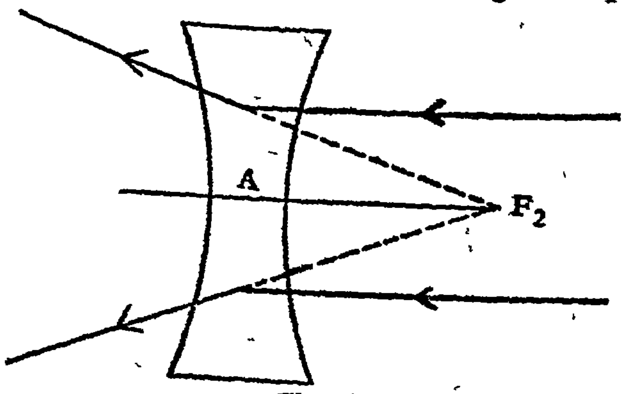


Fig 52 (b).

lens In convex lens F_2 is real and the distance AF_2 from the surface of the lens upto the focal point is called the focal length and is $-$ve While in concave lens, F_2 is virtual and the focal length is $+$ve See figs 52 (*a*) and 52 (*b*).

If we consider a beam coming from a point (fig. 53 *a*) or directed to a point on the principle axis (fig. 53 *b*) such that after refraction the emergent beam is parallel to the principle axis, the point F_1 is called the object focal point and the distance

$$AF_1 = AF_2$$

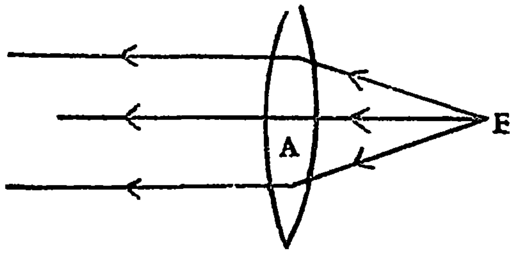


Fig 53 (*a*)

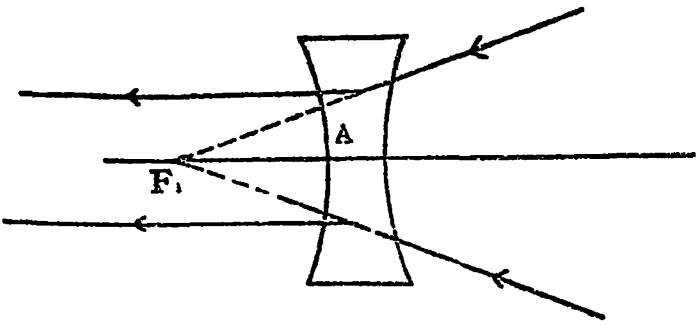


Fig. 53 (*b*)

Here we shall consider only thin lenses of small aperture The lens is thin that the distance can be measured from any surface Normally the optical centre is taken coincident with the pole of the lens.

These two focal points together with the optical centre are called the Cardinal points and are useful in image formation

§6 **Image Formation :**—Let PQ be the object Draw PM ray parallel to the principle axis After refraction it must pass

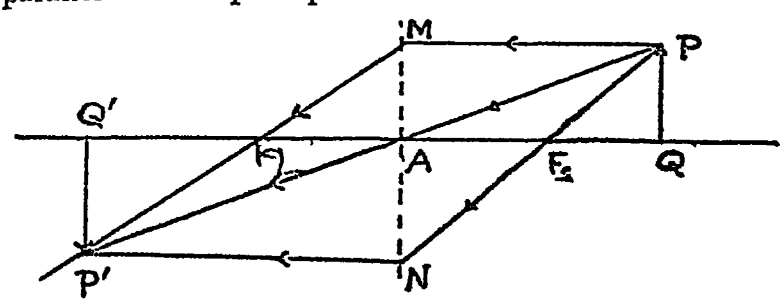


Fig. 54 (*a*)

(fig. 54 (*a*) through image focal point F_2 PN ray passing through object focal point F_1, after refraction goes parallel to the principle axis A ray passing hrough the optical centre A goes undeviated. All these three rays meet (fig 54 *a*) at P' and therefore P' is the image of P. The same procedure can be followed for all points in between PQ and we get the corresponding image $P'Q'$.

In the case of convex lens the image is real and inverted while for concave lens, it is virtual and upright

Note —For practice, take PQ at various distances from the lens and find the position of the image $P'Q'$.

§7. **Magnification :**—As in mirrors, the magnification of an image is defined as the ratio of the linear size of the image to the linear size of the object.

Thus $M=I/O$, where M is magnification, I and O are the lengths of the image and object respectively.

See Fig 54 *a* Consider $\triangle$s NAF_1 and PQF_1 MAN is considered as a straight line at right angles to the principle axis.

The $\triangle$s are similar as already explained in the chapter of mirrors

Hence $$\frac{AN}{PQ}=\frac{AF_1}{F_1Q}.$$

Here $AN=P'Q'$ being between the same parallel lines

Therefore $$\frac{P'Q'}{PQ}=\frac{AF_1}{F_1Q}=\frac{AF_1}{AQ-AF_1}$$

Substituting $P'Q'=-I$, $PQ=O$, $AF_1=-f$ and $AQ=u$, we have

$$\frac{-I}{O}=\frac{-f}{u-(-f)}=\frac{-f}{u+f}$$

$$\therefore \qquad M=\frac{I}{O}=\frac{f}{u+f} \qquad \text{.. (1)}$$

Similarly, the $\triangle$s $P'Q'F_2$ and MAF_2 are similar

Hence $$\frac{P'Q'}{AM}=\frac{Q'F_2}{AF_2}$$ [F_2 is the point of intersection of AM' and AQ']

or $$\frac{P'Q'}{PQ}=\frac{Q'A-AF_2}{AF_2}$$

or $$\frac{-I}{O}=\frac{-v-(-f)}{-f}=\frac{f-v}{-f}$$

$$\therefore \qquad M=\frac{I}{O}=\frac{f-v}{f}$$

Also in similar $\triangle$s $AP'Q'$ and APQ, we have

$$P'Q'/PQ=AQ'/AQ$$

or $$-I/O=-v/u$$

or $$M=v/u$$

Thus the magnification formulae are :

$$M=\frac{f}{u+f}$$
$$=\frac{f-v}{f}$$
$$=v/u$$

§8. **Relation between u, v and f :—**

Equating any two of the above formulae we get,

$$\frac{v}{u}=\frac{f}{u+f}$$

Cross multiplying, we have

$$v(u+f)=uf$$

or
$$vu+vf=uf$$

Taking terms containing f on one side,

$$vu=uf-vf$$

Dividing the above equation by uvf, we get

$$\frac{vu}{uvf}=\frac{uf}{uvf}-\frac{vf}{uvf}$$

or
$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u} \qquad (1)$$

Note —Students are advised to prove magnification formulae and thence the relation between u, v and f for a concave lens themselves The relations are the same

Thus, for a spherical lens,

$$\mathbf{\frac{1}{v}-\frac{1}{u}=\frac{1}{f}}.$$

§9. **Relation between u, v and f for a spherical lens considering it as made up of two spherical surfaces :—**

Let us consider a lens with two spherical surfaces XYZ having radius of curvature $YO_1=r_1$ and XUZ having radius of curvature $UO_2=r_2$

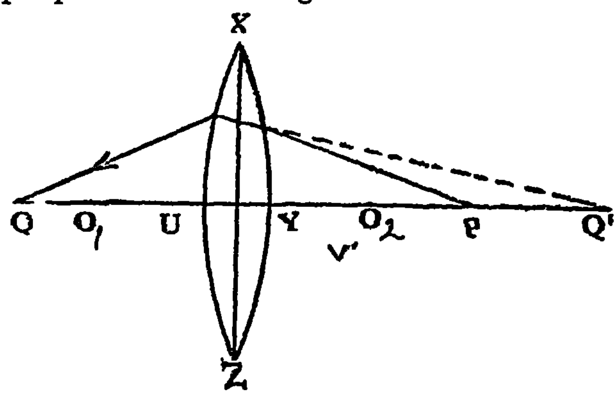


Fig. 55

Let us consider an object at P such that $YP=u$. Refraction takes place at XYZ at air glass surface and the image Q' is formed at a distance v' from Y This refracted ray falls on the second surface XUZ and refraction takes place again at this glass air surface For this Q' behaves as the object and $u=UY+YQ'=t+v'$ where t is the thickness of the lens. But we are considering very thin lenses for which $t=O$ and hence here object distance$=v'$. Let the final image be formed at Q at a distance v from the surface of the lens.

Thus when refraction takes place at first spherical surface XYZ,

(*i*) rays are going from air to glass

(*ii*) object distance is v

(*iii*) image distance is v'

(*iv*) radius of curvature of the spherical surface is r_1

Therefore, according to the formula $\frac{\mu}{v}-\frac{1}{u}=\frac{\mu-1}{r}$ for re-

fraction at a spherical surface (see §s 2 and 3) chapter 5, we get

$$\frac{\mu_{ag}}{v'}-\frac{1}{u}=\frac{\mu_{ag}-1}{r_1} \quad \ldots (1)$$

The refraction at the second surface XUZ,

(i) rays are going from glass to air
(ii) object distance is v'
(iii) image distance is v
(iv) radius of curvature is r_2.

Therefore according to the relation for refraction at spherical surface, we get

$$\frac{\mu_{ga}}{v}-\frac{1}{v'}=\frac{\mu_{ga}-1}{r_2} \quad \ldots (2)$$

But $\mu_{ga}=1/\mu_{ag}$, hence eqn (2) becomes

$$\frac{1/\mu_{ag}}{v}-\frac{1}{v_1}=\frac{1/\mu_{ag}}{r_2} \quad \ldots (3)$$

Multiplying both the sides of equation (3) by μ_{ag}, we get

$$\frac{\mu_{ag}\cdot 1/\mu_{ag}}{v}-\frac{\mu_{ag}}{v'}=\frac{\mu_{ag}\cdot 1/\mu_{ag}-\mu_{ag}}{r_2}$$

or
$$\frac{1}{v}-\frac{\mu_{ag}}{v'}=\frac{1-\mu_{ag}}{r_2} \quad \ldots (4)$$

Adding eqn, (1) and (4), we have

$$\frac{\mu_{ag}}{v'}-\frac{1}{u}+\frac{1}{v}-\frac{\mu ag}{v'}=\frac{\mu_{ag}-1}{r_1}+\frac{1-\mu_{ag}}{r_2}$$

or
$$\frac{1}{v}-\frac{1}{u}=\frac{\mu_{ag}-1}{r_1}-\frac{\mu_{ag}-1}{r_2}$$

$$=(\mu_{ag}-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right) \quad . (5)$$

Putting μ for μ_{ag} above becomes

$$\frac{1}{v}-\frac{1}{u}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right) \quad .. (6)$$

If we consider incidence from ∞, *i e* if $u=\infty$ such that the incident beam is parallel to the principle axis, according to definition $v=f$ and we get from (6),

$$\frac{1}{f}-\frac{1}{\infty}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

or
$$\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right) \quad . (7)$$

The value of *r. h s* depends only on the material of the lens and form of the surfaces and hence for a given lens it is a constant. Therefore, the value of f is constant.

Making the substitution of eqn. (7) in eqn. (6), we get

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \quad (8)$$

The above treatment and relations are true for any form of a lens and in general for a spherical lens, we get

$$\left\{\begin{aligned} \frac{1}{v}-\frac{1}{u}&=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right), \\ \frac{1}{f}&=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right) \text{ and} \\ \frac{1}{v}-\frac{1}{u}&=\frac{1}{f}. \end{aligned}\right.$$

§ 10. Dependence of f on radii of curvature :—

For a double convex lens, the first surface has $-$ve radius while the second radius is $+$ve and hence the relation is

$$\frac{1}{f}=(\mu-1)\left(-\frac{1}{r_1}-\frac{1}{r_2}\right)=-(\mu-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

For double concave lens, r_1 is $+$ve while r_2 is $-$ve, and we have

$$\frac{1}{f}=(\mu-1)\left\{\frac{1}{r_1}-\left(-\frac{1}{r_2}\right)\right\}=(\mu-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

For meniscus, both r_1 and r_2 have similar signs and we have

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

Here for convex lens, $r_1 > r_2$ and for concave lens $r_2 > r_1$.

For plano convex or concave lens as one surface is plane.

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{r}\right)$$

As r_2 is ∞ For convex surface r is $-$ve and $+$ve for concave surface.

§ 11 Relative positions of object and image :—One method of discussing this is the same as discussed in 3rd Chapter for mirrors Here, we discuss another method.

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

For a convex lens, f is $-$ve, and hence

$$\frac{1}{v}=-\frac{1}{f}+\frac{1}{u}=\frac{f-u}{uf}$$

$$\therefore \qquad v=\frac{uf}{f-u} \qquad \ldots (1)$$

(1) Eqn (1) can be written as

$$v=\frac{uf}{u(f/u-1)}=\frac{f}{f/x-1}$$

Here, if $u=\infty$, $v=f$ and is $-$ve

(2) If $u > 2f$, $v < 2f$ and is $-$ve

(3) If $u=2f$, $v=\dfrac{f}{f/2f-1}=-2f$

(4) If $u < 2f$ but $> f$, $v > 2f$ and is $-$ve

(5) If $u=f$, $v=\infty$ and is $-$ve

(6) If u is less than f, v would be $+$ve and greater than f

Thus, we find that for all positions of object beyond focus, w get real and inverted image. It is magnified if the object is i between focus and $2f$. For the position of the object between foc and pole, the image is on the same side as the object and is re virtual and magnified.

All these positions are denoted in the tabular column below.

Position of object	Position of image	Nature and magnification of image
1 At pole	At pole	Virtual, same size, upright
2 Between pole and focus	Beyond $2f$ on the same side	Virtual, magnified, upright
3 At focus	At ∞, on the other side	Real, magnified, inverte
4. Between focus and $2f$	Beyond $2f$ on the other side	Real, magnified, inverte
5 At $2f$	At $2f$ on the other side	Real, same size inverte
6 Beyond $2f$	Between f and $2f$ on the other side	Real, diminished, inverted
7. At ∞	At focus on the other side	Real, diminished, inverted

The convex lens is called a magnifying glass by virtue of it forming a magnified virtual image of an object when held close t an object.

In the case of concave lens, f is +ve and hence the eqn (1 becomes

$$v=\frac{uf}{u+f}=\frac{f}{1+f/u}$$

For all value of u from O to ∞, denominator is greater o equal to 1 and hence, v is always less than or at the most equal to f The sign of v is also +ve and hence the image is formed on th same side as the object The image is, therefore, always virtual diminished upright and is formed between pole and focus

Thus we find that the behaviour of convex lens is similar t that of concave mirror and of concave lens to convex mirror.

§ 12 **Power of a Lens :**—Power of a lens is defined as the reciprocal of its focal length. Hence, if, P would be the power of a lens of focal length f, we have, $P=\frac{1}{f}$. Smaller the focal length of lens more will it converge or diverge Thus, the capacity to converge or diverge depends inversely on the value of its focal length

The unit in which the power of a lens is measured is called *dioptre* **If the focal length of a lens is 1 metre or 100 cm., its power is called 1 dioptre** Obviously, a lens of focal length 10 cm *i e*, $\frac{1}{10}$ metre will have a power of 10 dioptres.

Usually the power of a convex lens is said to be +ve with −ve focal length and *vice versa* for concave lens. This is the practice followed by opticians *But in this book we shall consider the same sign for power as for focal length.*

§ 13. Power of the combination of two lenses :—

Let us consider two lenses of focal lengths f_1 and f_2 respectively kept in secure contact with each other An object will first form image after refraction through the first lens These refracted rays would then pass through the second lens In other words, image formed by first lens will behave as an object for the second lens and then the final image will be formed

Let for first lens of focal length f_1, the object distance be u and the image distance be v' Then, according to the general equation $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, we have

$$\frac{1}{v'}-\frac{1}{u}=\frac{1}{f_1} \tag{1}$$

For the second lens of focal length f_2 the object distance would be v'. Note here that the thickness of the lenses is negligible and hence, the distances measured from any surfaces of the lens would be the same

Let the image finally be formed at v. Then

$$\frac{1}{v}-\frac{1}{v^1}=\frac{1}{f_2} \quad \ldots (2)$$

Adding eqns (1) and (2), we get,

$$\frac{1}{v^1}-\frac{1}{u}+\frac{1}{v}-\frac{1}{v^1}=\frac{1}{f_1}+\frac{1}{f_2}$$

or

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f_1}+\frac{1}{f_2} \quad \ldots (3)$$

If we treat both the lenses as behaving like one lens of focal length F and if u is the distance of the object from such a lens, the image is formed at v Hence,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{F} \quad \ldots (4)$$

Comparing equs (3) and (4) the *l h. s* of which are equal, we get

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2} \quad \ldots (5)$$

If P denotes the power of the combined lens and p_1 and p_2 respectively the powers of the component lenses, per definition, we have

$$P=p_1+p_2 \tag{6}$$

Thus, we prove that the power of the combination of two lenses kept in contact is equal to the sum of the powers of the component lenses.

The focal length F or power P of the combination of two lenses is called the focal length or power of an *equivalent lens i e*, a lens which can replace for all purposes two lenses kept together.

As an example, a convex lens of focal length 10 cm. when combined with a concave lens of focal length 20 cm. would behave as a convex lens of focal length 20 cm.

Because, $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}=-\frac{1}{10}+\frac{1}{20}$.

Hence, $F=-20$ cm.

§ 14. Determination of focal length :—For convex lens :—

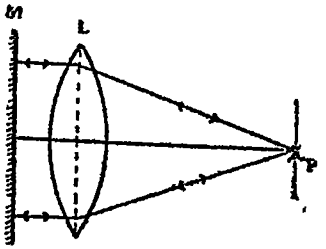


Fig 56

1 **By one pin.**—As shown in fig 56 mount a convex lens and a pin P on optical bench Mount a plane mirror behind the lens and close to it. Observe from the pin side Adjust its distance in such a way that there is no parallax between it and its inverted image

This will happen when the pin is at the focus of the lens Rays starting from it would then be refracted from lens parallel to the principal axis. A plane mirror placed behind would normally reflect these back and the rays would retrace their path forming the image at the pin itself. The position of P is independent of the position of the plane mirror But it is advisable to keep the plane mirror as near to the lens as possible This will make the loss of light less and if the mirror is not properly put, the beam would be much deviated.

Measure the distance between the pin and lens This is focal length

Parallel beam coming from sun can be focussed sharply on a screen and the distance between the screen and the lens would then give the focal length of the lens.

2 **By two pins.**—Mount two pins P and Q on either side of the lens as shown in fig. 57. Preferably adjust the pin P at such a

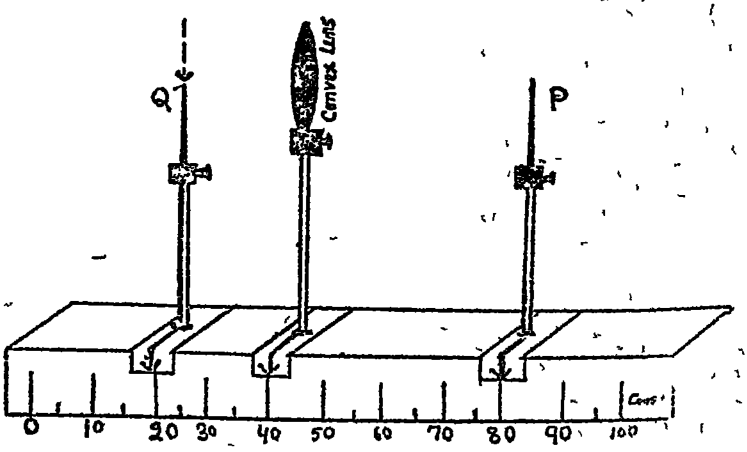


Fig 56

*Refer to "A. T B of Practical Physics" by authors for details.

distance that when viewed from the other side we get an inverted image of it Remove parallax between it and another pin Q The distance of P and Q respectively from the lens gives u and v With the help of the relation $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ Calculate f remembering that v is to be substituted with a negative sign

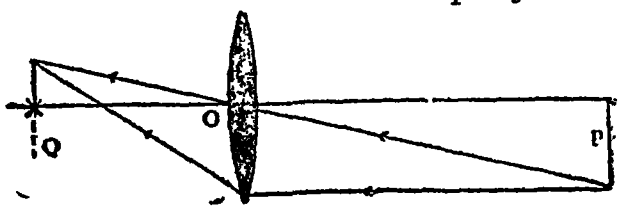


Fig. 57.

Here we find that, like mirrors, Q can be made an object when the image would be at P The positions of object and image are therefore interchangeable. Such two points (true for real image) are called conjugate points and hence the above method is also called conjugate foci method.

(3) **Displacement method :**—As explained in method 1, find the focal length of the given lens approximately and fix two pins P and Q at a distance greater than $4f$ from each other.

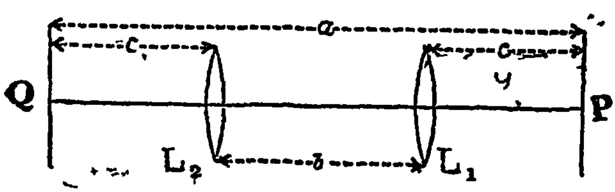


Fig 58.

Introduce the given lens in between the two pins in such a position say L_1 that there is no parallax between one of the pin say Q and the image of the second pin P. P is the object and Q is the position of the image.

Hence $u=L_1P$ and $v=L_2Q$.

Now, we know that P and Q are conjugate points and hence, what is u in this case can be v in other case and v can be u Therefore, if, the lens is displaced to the position L_2 such that

$L_2P=L_1Q$ and $L_2Q=L_1P$, parallex will again be removed between the image of P and the pin Q

Note —here instead of viewing from the other side and interchanging the roles of P and Q, we have changed the distances to suit the condition of conjugate foci by shifting the lens

Let $L_1P=L_2Q$ be equal to c, $L_1L_2=b$ and $PQ=a$

Thus, $$a=c+b+c=b+2c$$

or $$c=\frac{a-b}{2}$$

Considering the lens in position L_1,

$$u=L_1P=c=\frac{a-b}{2}$$

and $$v=L_1Q=b+c=b+\frac{a-b}{2}=\frac{2b+a-b}{2}=\frac{a+b}{2}$$
and remember that v is $-$ve,

Substituting these values in the relation $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$ we get

$$\frac{1}{f}=\frac{1}{-\frac{a+b}{2}}-\frac{1}{\frac{a-b}{2}}$$

$$=-\left\{\frac{2}{a+b}+\frac{2}{a-b}\right\}=-2\left\{\frac{(a-b)+(a+b)}{(a+b)(a-b)}\right\}$$

$$=-2\left\{\frac{a-b+a+b}{(a+b)(a-b)}\right\}=\frac{-4a}{a^2-b^2}$$

$$\therefore \quad f=-\frac{a^2-b^2}{4a} \quad (1)$$

Thus, to determine f, the distance between the two pins a and the displacement of the lens b is measured.

Eqn (1) can be rewritten as

$a^2-b^2=4af$, the $-$ve sign is neglected as it only denotes the focal length of the convex lens

or $$b^2=a^2-4af=a(a-4f) \quad (2)$$

If the distance between two pins is $a=4f$, according to eqn. (2)

$$b^2=0 \quad \text{or} \quad b=0$$

and hence, there would be only one position possible for the lens For b to be a real quantity, the r h s must be +ve. This is only possible if a is greater than $4f$ If a is less than $4f$, $a-4f$ would be negative and then the square root of a $-$ve quantity is imaginary. In other words, for $a < f$, there would be no position of the lens for which parallax would be removed

Thus, we say that the least distance between an object and its real image is 4f where f is the focal length of the convex lens.

Merit of the method :— In the first and second method we are required to measure the distances from the surface of the lens and hence, if, a lens is thick, it would be a great source of error. In the method discussed above, no measurement is made from the surface of the lens Only the distance between the pins and the displacement of the lens is measured As such thickness of the lens does not cause error in the measurement of any quantity. The method, therefore, is particularly suitable for thick lenses

However, it is to be remembered that as the distance between the two pins has to be $> 4f$, the method is suitable only for lenses of short focal length

The plane mirror method is suitable when the lens has large focal length.

For concave lens :—

1 **By combining it with a convex lens :—** A concave lens forms a virtual image and therefore, as in the case of convex mirror

it is more difficult to locate their positions A concave lens, therefore, is combined with such a convex lens (of shorter focal length) that the combination behaves as a convex lens. The focal length of this convex compound lens F can be determined by any of the methods described above. Similarly, the focal length f_1 of the convex lens can be determined Then, with the help of the relation, $(1/F = 1/f_1 + 1/f_2)$ f_2, the focal length of the concave lens can be determined

2 **By keeping a convex lens apart** :—As shown in fig 59, a real image Q of the object P is formed by a convex lens. The position of Q is noted by use of a pin. In between this pin Q and the convex lens A is introduced the given concave lens. It has the diverging properties The incident rays directed towards Q now diverge and meet at R. Thus, with concave lens in between the real image of P is now formed at R. This position is located by parallax method

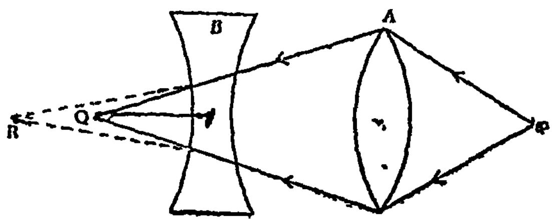


Fig 59

Now for the concave lens in position B, Q behaves as the virtual object and hence $u = -BQ$ and R is the image making

$$v = -BR$$

Substituting these values in the relation $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ we get

$$-\frac{1}{BR} - \left(-\frac{1}{BQ}\right) = \frac{1}{f}$$

or

$$\frac{1}{BQ} - \frac{1}{BR} = \frac{1}{f}$$

Thus, knowing BQ and BR distances, f is known

3 **By using a concave mirror** :—The centre of curvature I of a concave mirror can be determined by removing parallax between the pin and its image If the given concave lens is introduced in between I and A, the parallax between the pin at I and its image would be produced In order to remove the parallax, the pin will have to be shifted to I' In this case, the rays starting from I' after refraction through concave lens would fall on the mirror normally and hence would retrace back their path to I' If the refracted rays at concave lens were to be produced backwards, they would appear to meet at I. Thus, corresponding to the object at I', I would behave as the virtual image. Hence.

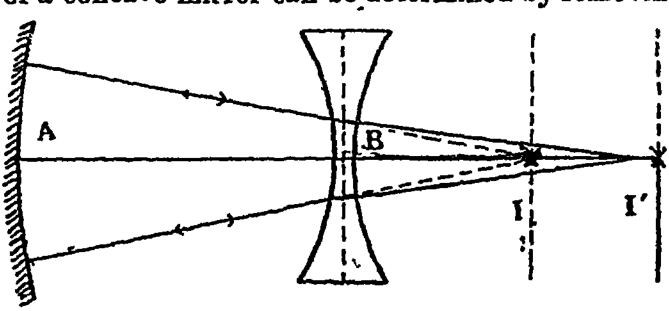


Fig 60

$$u = BI' \qquad \text{and} \qquad v = BI$$

Therefore, $$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{BI} - \frac{1}{BI'}$$

Knowing BI and BI', f is calculated.

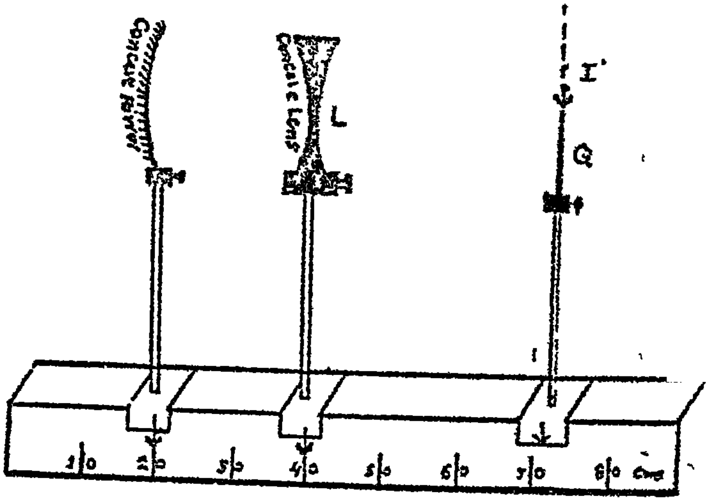
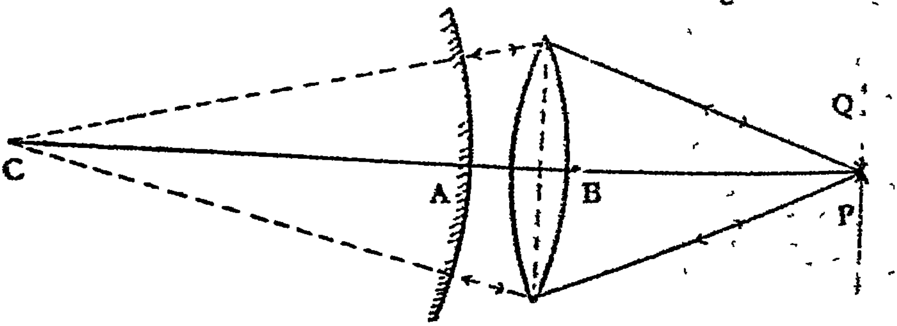


Fig. 60 (a).

§15. Determination of radius of curvature (a) Of a convex surface*.—Mount a pin P, a convex lens B and the given convex



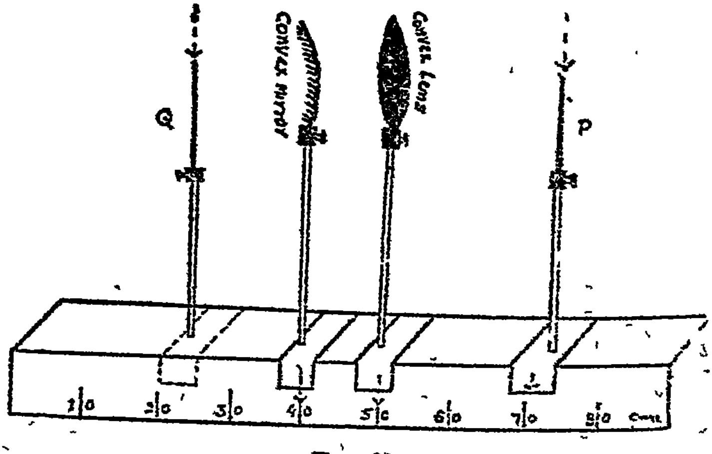


Fig. 61.

*For details see "T. B. of Practical Physics" by authors

surface A in such a way that when seen from the pin side there is no parallax between the pin P and its inverted image See fig 61. This happens when the rays starting from P after refraction at lens B fall on the convex surface normally They are, therefore, reflected back along the same path and hence retrace back their path to P. If the rays at convex surface are allowed to proceed they would meet at C which would obviously be the centre of curvature of the convex surface. The convex surface is, therefore, removed the position C is determined by removing parallax between the image of pin P and a second pin placed at C The distance BC obviously gives the radius of curvature of the convex surface If it is a mirror, half, of it would give its focal length

(b) **Of second surface of a lens :**—This method is to be performed in a dark room Let us suppose we are to determine radius of curvature of the second surface of the lens as in fig 62 Mount a self luminous object at P in such a way that an image is formed on the screen placed at the same place as P This means that ray PM after refraction at M fall on the second surface along the normal MN Apart of it is reflected back along the same path forming image at P If NM is produced backwards it would meet at O which would be the centre of curvature of the second surface of the lens O would also behave as the virtual image of the real object P formed due to refraction through the lens Note here that the ray MN is incident normally on the second surface and, therefore, would be refracted as such

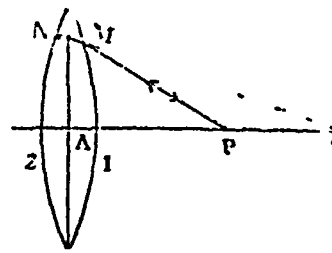


Fig. 62

If f be the focal length of the lens determined by other method, $u=AP$ and $v=r_2=AO$, the radius of curvature of second surface, we have

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}=\frac{1}{r_2}-\frac{1}{u}$$

But f of convex lens is $-$ve

hence
$$-\frac{1}{f}=\frac{1}{r_2}-\frac{1}{u}$$

or
$$\frac{1}{r_2}=\frac{1}{u}-\frac{1}{f}=\frac{f-u}{uf} \qquad (1)$$

Thus knowing u and f, r_2 is determined

§16. **Determination of refractive index of a precious liquid available in a few drops :**—As shown in fig 63, mount a plane mirror horizontally and place on it a convex lens of small focal length Mounting a pin P find the focal length of the lens by removing parallax between the pin and its image AP distance gives f_1, the focal length of the lens

Now in between the lens and the mirror are introduced a few drops of a liquid This forms a plano concave liquid lens in

between. Thus, we get a combination of convex glass lens and a liquid plano concave lens. Obviously the focal length F of this combination would be greater than f_1. Hence, now the parallax would be removed at Q Measure AQ which is equal to F Now

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$$

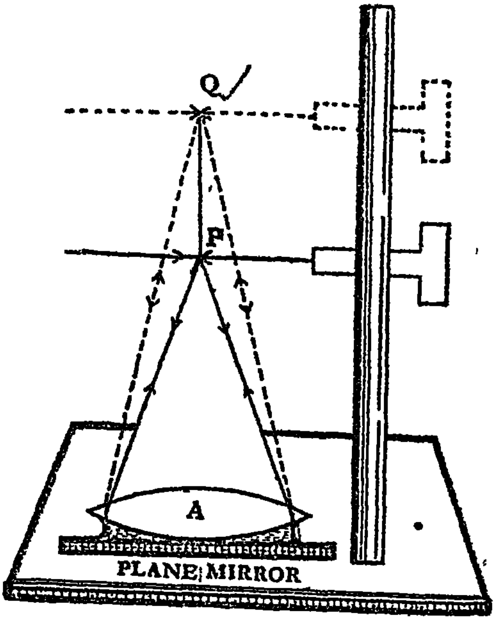


FIG 63.

where f_2 is the focal length of the liquid lens whose refractive index is μ_{al}, knowing f_1 and F, f_2 is calculated

Now according to the relation,

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right), \text{ we have}$$

$$\frac{1}{f_2}=(\mu_{al}-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

for the plano concave liquid lens

Being the plano concave lens, $r_2=\infty$. Hence,

$$\frac{1}{f_2}=(\mu_{al}-1)\ \frac{1}{r_1}=\frac{\mu_{al}-1}{r_1}. \qquad (1)$$

In the above expression r_1 is the radius of curvature of the concave surface of the liquid lens which is same as the surface of the glass lens in contact with the liquid r_1, therefore, is either determined optically or with the help of a spherometer

Knowing f_2 and r_1, with the help of eqn (1) μ_{al}, the refractive index of the liquid is known

§17. Distinction between a convex and a concave lens :—

Convex lens	*Concave lens*
1. It is bulging from the centre	1 It has depression in the centre.
2. It is thicker at the centre than at the rim	2. It is thinner at the centre and thicker at the rim.
3 When held near an object forms a magnified virtual image	3 It forms a virtual but diminished image.
4 When the lens is slightly, displaced to and fro, the image appears to move in the reverse direction to the movement of the lens	4, With the movement of the lens, the image moves in same direction.

§18. Uses of lenses:— 1 Both types of lenses find a variety of uses in the construction of optical instruments like telescope, microscope, binoculars, photo camera, etc

2 To remove defects of vision, they are used as spectacle glasses

3. Convex lens is used as a magnifier and a combination of both types as condenser of light.

§18-*a*. The optical Lantern :—It is an apparatus used for projecting a magnified image of a transparent object Its main parts are (*a*) a bright source of light, (*b*) a condensing lens, (*c*) slide for introducing the object to be projected and (*d*) a projecting lens

(*a*) *Bright source A.*—It is an arc lamp or any other bright source of light

(*b*) *Condencer C*,—It consists of two plano convex lenses kept apart The convex sides face each other Its function is to concentrate light on the object to be projected

(*c*) *Slide*—This is an arrangement in which the film to be projected is placed It is so placed that it is well illuminated by the condenser

(*d*) *The projecting lens P L*—It consists of a combination of two convex lenses of short focal length kept apart so that it behaves as a lens of very short focal length

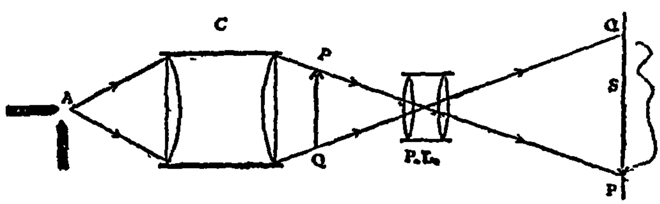


Fig 64

Consequently it forms a magnified image $P'Q'$ of an object PQ on the screen S

§18-b. Episcope.—It is an arrangement to project magnified image of an opaque object like the printed picture, printed material, etc The arrangement is clear from fig. 65

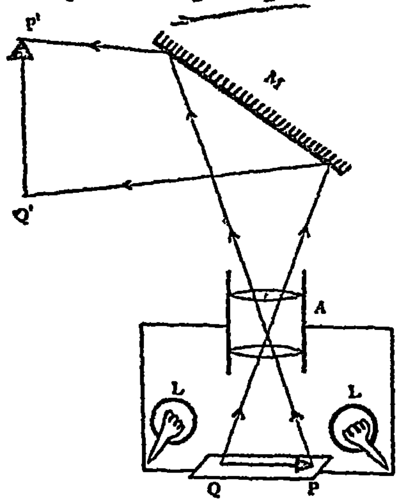


Fig. 65

L, L are bright sources of light. They illuminate an object PQ covered with a glass sheet to avoid heat radiations falling on it. The object scatters light in all directions A part of it is condensed and projected by a lens A. After reflection on a mirror M, the image falls on a screens. Naturally, as only a part of the scattered light is projected, the brightness of the image is very low as compared with the image in magic lantern

A combination of magic lantern and episcope is called Epidiascope.

A few specimen examples :— 1. *An arrow 5 cm long is placed longitudinally in such a way that its nearest arrow head is at a distance of 15 cm. from a convex lens of focal length 10 cm. Find the position, nature, and magnification of the image.*

See fig 66 A point is situated at a distance 15 cm while B point at a distance of 20 cm from the lens.

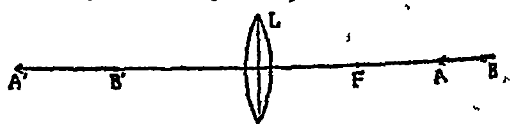


Fig 66

If v_1 and v_2 denote the distances of the corresponding images, we get.

$$\frac{1}{v_1}-\frac{1}{15}=-\frac{1}{10} \qquad (1)$$

and
$$\frac{1}{v_2}-\frac{1}{20}=-\frac{1}{10} \qquad (2)$$

Solving eqn. (1), we get

$$\frac{1}{v_1}=-\frac{1}{10}+\frac{1}{15}=\frac{-3+2}{30}=-\frac{1}{30}$$

$\therefore \quad v_1=-30$ cm

Solving eqn (2), we get,

$$\frac{1}{v_2}=-\frac{1}{10}+\frac{1}{20}=\frac{-2+1}{20}=-\frac{1}{20}$$

$\therefore \quad v_2=-20$ cm

Therefore, the image is $30-20=10$ cm long, with A head at a distance of 30 cm and B end at a distance of 20 cm. from the lens. It is real and magnified to double the size.

Example 2. *A convex lens of focal length 10 cm. forms a real image three times as large as the object through what distance should it be moved to make the magnification only two times as large*

Here $\quad v/u=+3$

$\therefore \quad v=+3u$ But as it is real, it is with $-$ve sign.

Then $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ becomes $-\frac{1}{3u}-\frac{1}{u}=-\frac{1}{10}$

or $\frac{-1-3}{3u}=-\frac{4}{3u}=-\frac{1}{10}$

$\therefore$ $u=40/3$ cm.

Now $v/u=2$

Hence $-\frac{1}{2u}-\frac{1}{u}=\frac{-2-1}{2u}=-\frac{3}{2u}=-\frac{1}{10}$

or $u=15$ cm.

Initial position of the object is 40/3 cm It has to be shifted through $15-\frac{40}{3}=\frac{5}{3}$ cm away from the lens

Example 3. *In a displacement method for determining the focal length of a lens, the sizes of the images for two positions of the lens are respectively 2 and 8 cms Calculate the size of the object If the distance between the two screens is 9 cms, calculate the focal length of the lens*

See fig 58 If d is the size of the object, and d_1, d_2 the sizes of the images in the two positions respectively

in position L_1, $\frac{v}{u}=\frac{I}{O}$

or $\frac{b+c}{c}=\frac{d_1}{d}$ ... (1)

and in position L_2, $\frac{c}{b+e}=\frac{d_2}{d}$ ✓ ... (2)

Multiplying equations (1) and (2), we get

$$\frac{b+c}{c}\cdot\frac{c}{b+c}=\frac{d_1}{d}\quad\frac{d_2}{d}$$

or $1=\frac{d_1\,d_2}{d_2}$ or $d^2=d_1d_2$

$\therefore$ $d=\sqrt{d_1\,d_2}$.

Hence $d=\sqrt{2\times 8}=4$ cm. ✓

Now $a=9$ cm $=u+v$ ✓ .. (3)

and $v/u=\frac{L_1Q}{L_1P}=\frac{2}{4}$

or $4v=2u$

or $v=\frac{u}{2}$ ✓ . (4)

Substituting value of eqn (4) in eqn (3), we get

$u+\frac{u}{2}=9$ or $\frac{3u}{2}=9$

or $u=6$ cm ✓ ✓

$\therefore$ $v=3$ cm.

Hence, $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$

$$=-\frac{1}{3}-\frac{1}{6}=\frac{-2-1}{6}=\frac{-3}{6}=-\frac{1}{2}$$

$\therefore$ $f=2$ cm.

Example 4. *A lens of focal length 10 cm. is held completely immersed in a tank of water If a parallel beam is incident on it, where will it be focussed. Refractive index of glass and water is respectively 1 5 and 1·3.*

In air glass medium, $\frac{1}{f}=(\mu_{ag}-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$

or $$-\frac{1}{10}=(1{\cdot}5-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)={\cdot}5\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

or $$\left(\frac{1}{r_1}-\frac{1}{r_2}\right)=-\frac{1}{10}\times\frac{1}{{\cdot}5}=-\frac{1}{5} \quad \ldots (1)$$

In water glass medium, $\frac{1}{f}=(\mu_{wg}-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$

or $$\frac{1}{f}=\left(\frac{\mu_{ag}}{\mu_{aw}}-1\right)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

$$=\left(\frac{1{\cdot}5}{1{\cdot}3}-1\right)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

Substituting the value of $\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$ from eqn. (1), we get

$$\frac{1}{f}=\left(\frac{1{\cdot}5}{1{\cdot}3}-1\right)\times-\frac{1}{5}$$

$$=-\frac{1{\cdot}5-1{\cdot}3}{1{\cdot}3}\times\frac{1}{5}=-\frac{{\cdot}2}{6{\cdot}5}$$

$\therefore$ $f=\frac{65}{2}=-32{\cdot}5$ cm

The beam will be focussed at a distance of 32·5 cm in water.

Note—If μ of the liquid is greater than 1·5, this lens would behave like a diverging lens and not like a converging one.

Example 5. *The distance between a convex and a concave lens is 10 cm If the focal length of each is 20 cm, find where would a parallel beam incident on them meet ?*

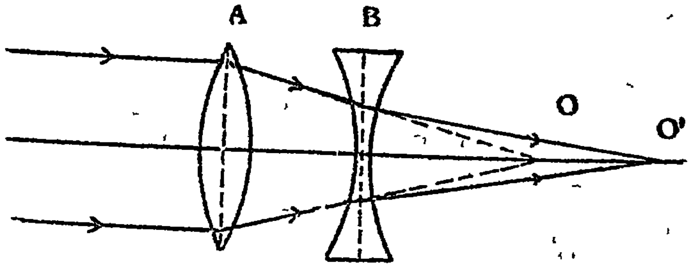


Fig 67

(a) If the incidence is first on A, the beam would after refraction at A meet at O at a distance of 20 cm from A. See fig. 67 But as the concave lens is in

between, it would now meet at O' Therefore,
for concave lens of $f=20$ cms $u=BO=AO-AB=20-10=10$ cms

Here, v is $-$ve

Hence, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

or $\frac{1}{v}-\left(-\frac{1}{10}\right)=\frac{1}{10}$

or $$\frac{1}{v}=\frac{1}{10}-\frac{1}{10}=0$$

$\therefore$ $$v=\infty$$

Hence, the final image is at ∞.

If the incidence is from concave lens side, the beam appears to diverge from O at a distance of 20 cm from the concave lens B.

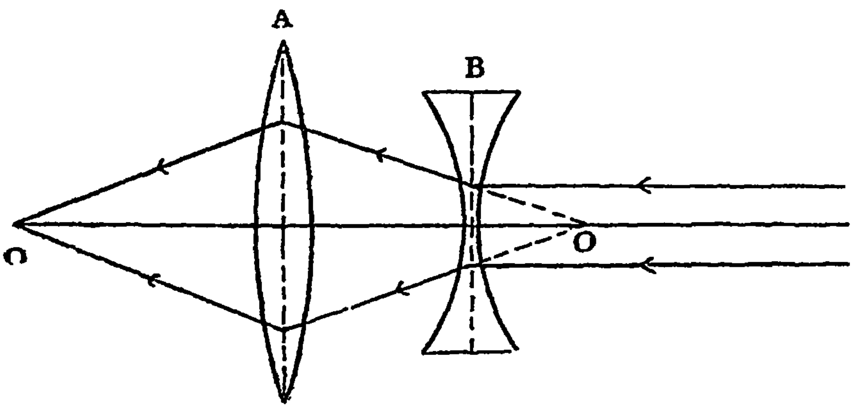


Fig 68

For convex lens, therefore, the object is at a distance

$$u=AO=AB+BO=10+20=30 \text{ cms}$$

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

or $$\frac{1}{v}-\frac{1}{30}=-\frac{1}{20}$$

or $$\frac{1}{v}=-\frac{1}{20}+\frac{1}{30}=\frac{-3+2}{60}=-\frac{1}{60}$$

$\because$ $$v=-60 \text{ cms}$$

The image would be real and at a distance of 60 cms from the convex lens

Example 6 *A plano convex lens is silvered on its plane side. It then acts like a concave mirror of 25 cm focal length If it is silvered on its convex side it acts like a concave mirror of 9 cm focal length Find the refractive index of the lens*

When the plane side is silvered, it is equivalent to a convex lens on a plane mirror. It behaves as concave mirror of $f=25$ cms.

i e, of radius of curvature 50 cm That is an object placed at 50 cms. from such a silvered lens forms image also at the same place Therefore, focal length of the lens is 50 cm

When convex side is silvered, an object placed at 18 cms from it is imaged at the same place. Hence if r is the radius of curvature of the convex surface, we have (See §15(b).

$$\frac{1}{r_2}-\frac{1}{u}=\frac{1}{f}$$

or
$$\frac{1}{r_2}-\frac{1}{18}=-\frac{1}{50}$$

or
$$\frac{1}{r_2}=\frac{1}{18}-\frac{1}{50}=\frac{25-9}{450}=\frac{16}{450}$$

$$\therefore \quad r_2=\frac{450}{16}=28\frac{1}{8}\text{ cms.}$$

Now
$$\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

$$=-(\mu-1)\frac{1}{r_2},\ r_1 \text{ is } \infty \text{ being plane.}$$

Hence
$$-\frac{1}{50}=-(\mu-1)\frac{16}{450}$$

or
$$(\mu-1)=\frac{450}{16}\times\frac{1}{50}=\frac{9}{16}$$

$$\therefore \quad \mu=1+\frac{9}{16}=1\ 5625.$$

QUESTIONS

1. What is optical centre? What is its importance? Deduce its position in a lens. (see § 4)

2. With the help of magnification formulae, get a relation between u, v and f for a convex lens, (see § 7)

3 Considering refraction at a spherical surface, deduce a relation between focal length and radii of curvature of a lens Hence, get a relation between u, v and f (see § 6)

4. Show that the power of a compound lens is equal to the sum of the powers of the component lenses (see § 11)

5 Describe a method for determining focal length of a thick lens. Discuss why a particular method is chosen (see § 13)

6 Describe how you will determine μ of a liquid with convex lens plane mirror method (see § 15)

7. How will you distinguish between a convex and a concave lens? (see § 16)

8. Write a short note on projection of pictures (see § 17)

9 Calculate focal length of a plano convex lens Given $\mu=1\ 5$ and $r=10$ cm. (*Ans.* 20 cm)

10 A convergent beam of light passes through a divergent lens of focal length 20 cm and is brought to a focus at a point 15 cm behind the lens In absence of the lens where would this beam have met? (*Ans.* 8·57 cm)

11 The image formed by a convex lens is 1 5 larger than the object. The distance between the object and the screen is kept fixed If, now the lens is displaced through 25 cm , an image is again focussed on the screen It is diminished. Find the focal length of the lens (*Ans* 30 cm)

12 For the two positions of the lens in the displacement method the size of the image respectively is 2 mm. and 8 mm The distance between these two positions of the lens is 25 cm Find the focal length of the lens and the length of the object (*Ans* f=16 66 , 4 mm)

13 A plano convex lens behaves like a concave mirror of radius of curvature 50 cm when its plane side is silvered On silvering its convex side, it behaves like a concave mirror of 18 cm radius Find refractive index of the material of the lens (*Ans* § 1 5625)

14 A concave lens of 2 dioptres power is kept in contact with a convex lens of 1 dioptre Find the focal length of the combination of the lens so formed (*Ans* 100 cm behaves like concave lens)

CHAPTER VIII

PHOTOMETRY

§1. **What is Photometry :**—The science of measurement of light or comparing two sources of light is called photometry.

The apparent brightness of a screen as it appears to the eye depends not only on the actual amount of light falling on the screen but, it also depends on the colour of light. Human eyes are not equally sensitive to all colours Photometry, therefore, does not apply to the absolute measurement but, it depends on the sense of sight also

§2. **Unit of light :**—In order to measure light, we have to define a unit of light. For this it is necessary to choose a standard source of light A standard source of light is a standard candle. This is made of sperm wax and weighs 1/6 lbs. It burns at the rate of 120 grains per hour. Now a days it is replaced by more reliable standards

The amount of light contained in a unit solid angle per second from such a source is taken as a unit of light. One such unit is lux or foot candle

Lux .—*It is the amount of light falling normally on 1 sq cm. area held at a distance of 1 cm in 1 second from a standard candle*

Foot-candle —*It is the amount of light falling normally on 1 sq. ft. area held at a distance of 1 ft in a second from a standard candle*

§3. **Intensity of illumination:**—*If the amount of light falling normally on a screen of area A uniformly be Q, the intensity of illumination I of the screen is defined as the amount of light falling* perpendicularly on unit area, *i.e.*, $I=Q/A$ If light is not falling uniformly, we have to define intensity of illumination at a point Let the amount of light falling on a small area '*a*' round the required point be q Then, I at that point$=q/a$

Dependence of intensity of illumination on inclination :—As the inclination of the rays falling on a screen increases from normal, the intensity of illumination proportionately decreases. If θ is the angle which the rays make with the normal to the screen,

$$I \propto \cos \theta$$

This is the reason why we try to hold a book normally to the rays if we find that light falling on it is not sufficient.

§4 **Inverse Square Law :**—Like inverse square law in gravitation, magnetism or electrostatics, the law here also states that the intensity of illumination of a screen varies inversely as the square of its distance from a point source.

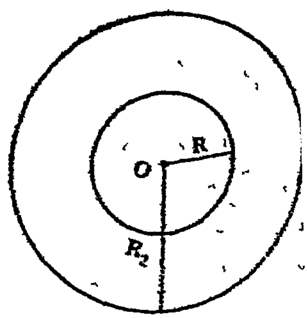


Fig. 69.

i.e., $I \propto 1/d^2$, where d is the distance between the point source and the screen

Imagine a point source O giving Q amount of light per second If we were to imagine a sphere of radius R_1 surrounding O, the intensity of illumination at any point on this imaginary sphere would be

$$I_1 = Q/4\pi R_1^2 \quad \text{... (1)}$$

If we were to imagine a sphere of radius R_2, it would be

$$I_2 = Q/4\pi R_2^2 \quad \text{. (2)}$$

Dividing eqn. (1) by eqn. (2), we get

$$\frac{I_1}{I_2} = \frac{Q/4\pi R_1^2}{Q/4\pi R_2^2} = \frac{R_2^2}{R_1^2}$$

or

$$I_1 R_1^2 = I_2 R_2^2 \quad \text{.. (3)}$$

or

$$I \propto \frac{1}{R^2} \quad \text{... (4)}$$

i e, intensity of illumination varies inversely as the square of its distance

§5. **Illuminating power** —The amount of light given by a source depends on a quantity which is called the illuminating power of a source. The illuminating power of a source is defined in terms of a standard candle The illuminating power of a standard candle is taken as one Hence, *the illuminating power of a source is defined as how many times a source is more powerful than a standard candle or it is also defined as the ratio of the amount of light given by a source per unit solid angle per unit time to the amount of light given by a standard candle under identical conditions, i e,*

Illuminating power

$$= \frac{\textbf{Amount of light given by a source}}{\textbf{Amount of light given by a standard candle under identical conditions}}$$

We also know that if from a standard candle at a unit distance we keep a screen, the intensity of illumination of it would be

$$I = \frac{Q}{A} = \frac{1}{1} = 1,$$

because a standard candle gives $Q=1$ (unit) light at a distance of 1 cm on a 1 sq. cm area

If instead of 1 standard candle, we consider 10 standard candles, the amount of light would be 10 times larger and thence, the intensity of illumination of the screen at unit distance would be 10 Obviously, the ten standard candles together are ten times more powerful than a standard candle and hence, their illuminating power is ten

Illuminating power of a source, then, can also be defined as numerically equal to the intensity of illumination of a screen placed at a unit distance from the source.

It is to be noted here that if the illuminating power of a source is S, the intensity of illumination of a screen at a unit distance would be S At a distance R from it, according to inverse square law, it would be S/R^2

§6. **Comparison of illuminating powers of two sources by Photometers :**—Photometer is an optical device which helps us in comparison or determination of illuminating powers of optical source. The simple principle on which their working is based is the equalisation of intensity of illumination of two patches produced by the two sources respectively.

(*a*) **Simple photometer.**—It consists of a light proof box as shown in fig 70, with an aperture on one side and a white paper on opposite side

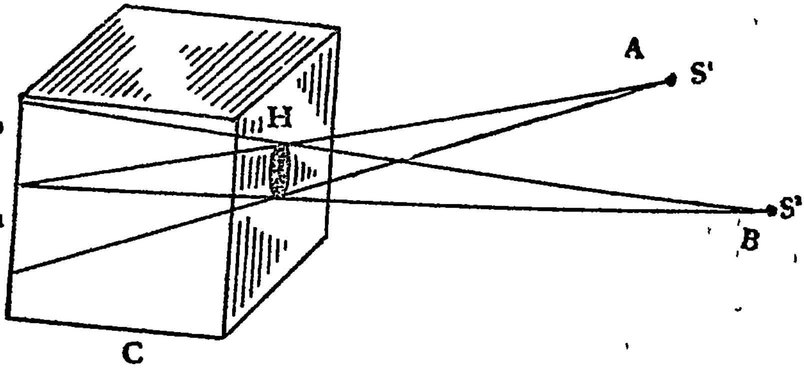


Fig 70

A and *B* are the given two sources of light with illuminating powers as S^1 and S^2 respectively They form two patches of light *a* and *b* respectively side by side as shown in the fig. The intensity of illumination of the patches is equalised visually by adjusting the distances of the sources *A* and *B* from the aperture When the patches become equally bright and appear as one, let the distances of *A* and *B* respectively from *a* and *b* be R_1 and R_2.

Then as already explained in §4,

$$I_1=\frac{S^1}{R_1^{\,2}}=I_2=S^2/R_2^{\,2}$$

$$\therefore \quad \frac{S^1}{S^2}=R_1^{\,2}/R_2^{\,2} \qquad \ldots (I)$$

Knowing R_1 and R_2, the illuminating powers are compared. If one of the source is a standard candle, the illuminating power of the other is known

(*b*) **Rumford's Photometer**.—It is a modification of simple photometer Here, instead of patches of light, the shadows are compared For this purpose, aperture is replaced by an obstacle.

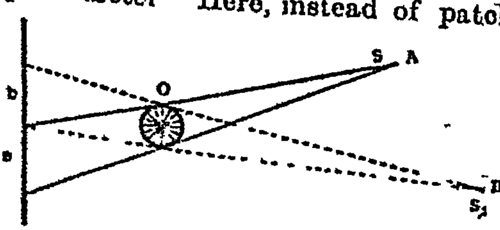


Fig. 71

Here *O* is the obstacle and as shown it forms shadows of *O* due to *A* and *B* respectively at *a* and *b*. The shadow *a* does not receive light from *A* but receives from *B* while *b* receives from *A* and not from *B*. *a* and *b* portions

In order to make the spot disappear, when seen by transmitted or reflected light, the loss of light should be made good by keeping a source of light in the opposite direction.

Let A and B be the two sources of light with illuminating powers S_1 and S_2 respectively. Let G be the grease spot placed in between the two sources such that Q_1 and Q_2 per unit area are the respective amounts of light falling on G from the two sources. Let a and b be the coefficient of reflection for the grease spot and the rest portion respectively, i.e., if unit light is incident on it, a would be reflected from the grease spot and b from the remaining portion. In other words, $(1-a)$ and $(1-b)$ would be transmitted respectively through the grease spot and paper. As Q_1 is the amount of light incident from A, aQ_1 and bQ_1 will be reflected from the spot and paper respectively, i.e., towards B will pass $(Q_1-aQ_1)=Q_1(1-a)$ from grease spot and $Q_1(1-b)$ from the paper.

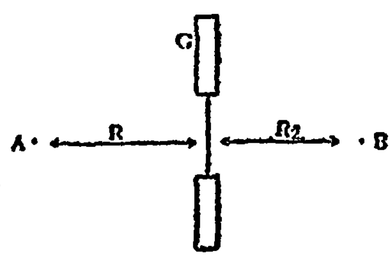


Fig 72

The light Q_2 is incident from B. Out of these aQ_2 and bQ_2 would be reflected back towards B from grease spot and paper respectively and $Q_2(1-a)$ and $Q_2(1-b)$ would be transmitted

Thus, the total amount of light travelling towards A would be

from grease spot	aQ_1	by reflection from A	.. (1)
and	$Q_2(1-a)$	by transmission from B	.. (2)
from paper	bQ_1	by reflection from A	.. (3)
and	$Q_2(1-b)$	by transmission from B	.. (4)

Similarly, towards B would be

from grease spot	aQ_2	by reflection from B	... (5)
and	$Q_1(1-a)$	by transmission from A	... (6)
from paper	bQ_2	by reflection from B	.. (7)
and	$Q_1(1-b)$	by transmission from B	.. (8)

Hence, when the grease spot is seen from B side,

total light coming from grease spot is $aQ_1+Q_2(1-a)$ .. (9)

and from paper is $bQ_1+Q_2(1-b)$ .. (10)

If the grease spot is to disappear, term (9) must be equal to term (10).

i e,
$$aQ_1+Q_2(1-a)=bQ_1+Q_2(1-b)$$
or
$$aQ_1-aQ_2+Q_2=bQ_1-bQ_2+Q_2$$
or
$$aQ_1-bQ_1=aQ_2-bQ_2$$
or
$$(a-b)Q_1=(a-b)Q_2$$

As $(a-b)$ is not zero, hence
$$Q_1=Q_2 \qquad .. (11)$$

The same can be proved, if we view from B side.

Thus, the essential condition for the grease spot to disappear i e, for it to lose its distinctive feature is that equal amount of light per unit area should fall from A and B on the paper with the spot

Let R_1 and R_2 be the respective distances of A and B from G when the above condition (11) is satisfied.

If I_1 and I_2 are the intensities of illumination, i.e., amounts of light per unit area Then

$$I_1=Q_1=S_1/R_1^2$$
and
$$I_2=Q_2=S_2/R_2^2$$
Hence,
$$S_1/R_1^2=S_2/R_2^2 \qquad .(12)$$
or
$$S_1/S_2=R_2^2/R_1^2$$

Apparatus and Method :—On an optical bench are mounted (See Fig 73) the given two sources A and B and the grease spot as shown. The grease spot is mounted in a box with two apertures on opposite sides The grease spot is backed by two mirrors at rt $\angle$s in such a way that each mirror is inclined to an $\angle$ of 45° with the grease spot The grease spot now can be viewed at right angles to the optical bench as both the sides of the spot

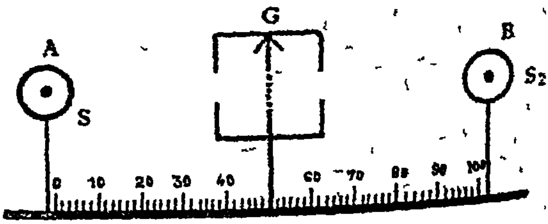


Fig. 73

are simultaneously imaged on the mirrors This arrangement has the advantage that we need not see from any particular side

The distances are so adjusted that the spot just disappears. Using eqn (12), the illuminating powers are compared.

§7. A few specimen examples :—1. *Two lamps of 8 and 32 candle powers respectively are separated by a distance of 120 cm Where should a grease spot be placed in between so that it disappears.*

Let the distance from 8 candle power lamp be x. From 32 c. p lamp it will be at $(120-x)$

Then per relation (12) of §5, we have

$$\frac{8}{x^2}=\frac{32}{(120-x)^2}$$

or

$$\frac{1}{x^2}=\frac{4}{(120-x)^2}$$

Taking square roots, we get

$$\frac{1}{x}=\frac{2}{120-x}$$

or $$120-x=2x$$
or $$3x=120$$
or $$x=40$$

The distance of the spot would be 40 cm. from 8 c p lamp

It is to be noted here that while taking square root, the $-$ve sign is not considered as it would give $x=-120$ cm which is absurd

2. *An electric lamp hangs 4 ft above the centre of a circular table of 6 ft diameter. Find out how many times the intensity of illumination at the centre would be greater than the intensity of illumination at the edge of the table.*

Let the centre of the table be O and A be the edge Let L be the Lamp See fig. 74

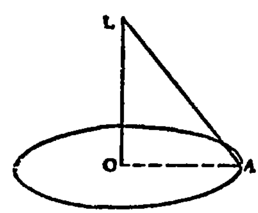


Fig 74

Then $LO=4$ ft and $OA=3$ ft.

$\therefore \quad LA^2=LO^2+OA^2=4^2+3^2=25$

$\therefore \quad LA=5$ ft

Hence, intensity at the edge

$$I_e=\frac{Q}{LA^2}=\frac{Q}{5^2}$$

and intensity at the centre

$$I_c=\frac{Q}{LA^2}=\frac{Q}{4^2}$$

Thus $$\frac{I_c}{I_e}=\frac{Q/4^2}{Q/5^2}=\frac{5^2}{4^2}=1.56 \text{ approx}$$

The intensity of illumination at the centre would be 1.56 approx. times greater than than at the edge

3. *A lamp with a sheet of glass when placed at a distance of 40 cm from a screen produces same intensity of illumination*

which it produced at a distance of 50 cm. from it without the sheet of glass. Find the percentage of light stopped by the sheet of glass.

If S and S' are the illuminating powers with and without sheet of glass, we have

$$\frac{S}{40^2}=\frac{S'}{50^2}$$

$$\therefore \qquad S=\frac{S'\times 40^2}{50^2}=\frac{16}{25}S'$$

Hence, amount of light absorbed

$$=S'-16/25S'=9/25S'$$

In S', the amount of light absorbed is $9/25S'$

Hence, in 100, the amount of light absorbed is $\frac{9}{25}\frac{S'}{S'}100=36$,

Light absorbed by sheet of glass is 36%.

QUESTIONS

1 What is photometry ? Define a standard candle, lux, intensity of illumination and illuminating power. (See § 1, 2, 8 & 5)

2 Explain the principle of Grease spot photometer, Describe it in detail and explain now it can be used to determine the illuminating power of a source (see § 6)

3, The distance between two lamps respectively of 25 c p and 100 c p is 3 ft. A grease spot placed in between them disappears The 25 c p lamp is moved 2 ft farther off Find the distance through which the grease spot be moved to disappear it again. (*Ans.* 1 333 ft)

4 A grease spot disappears when two lamps are respectively placed at distances of 20 cm and 30 cm from it. Now a sheet of glass is introduced between the spot and the brighter lamp To restore equilibrium, the weaker source is required to be moved by 10 cm Find the percentage of light absorbed by the sheet (*Ans* 55 55%)

CHAPTER IX

AIDS TO VISION

§1. **Size of an object :**—Knowledge about physical objects is gained by the aid of human eye When the image of an object is formed on the retina, the sensations are carried to the brain and we say that we are seeing an object.

The apparent size of an object as seen by the eye depends upon the angle which the object subtends at it However, in the interpretation of the size, the human experience also counts. Ordinarily, greater and greater is the angle α subtended by the object QP at the eye, greater and greater would its size appear The angle α obviously depends *on the actual size PQ and its distance D* Therefore, for a partiticular object, when its distance D is reduced, its apparent size goes on increasing Therefore, for clearer observation we always want to bring the object closer and closer But, there is a limit beyond which the object to be viewed cannot be brought closer. This limit is called the least distance of distinct vision For a normal eye, this distance is approximately 25 cm If this limit is crossed, the eye will be able to view the object but there would be a great strain upon it That is why, generally, we are advised to hold a book at a distance of 25 cm for reading

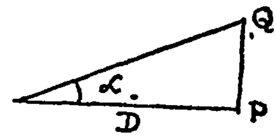


Fig 75

§2. **Microscope :**—As seen above the maximum apparent size of an object is as when seen at least distance of distinct vision If, we want to further increase this size we need some aid to vision This aid to vision is called a microscope **Microscope,** *therefore, is an optical instrument which is used to magnify the size of an object which can be brought close at hand* When the magnification is achieved in one stage, it is called a simple microscope When the magnification is in two stages, it is called a compound microscope

§3. **Magnifying power :**—When we want to view an object through a naked eye, the object is always to be placed at least distance of distinct vision D Hence, if the size of the object is PQ, the angle which it would subtend at the eye at a distance D would be α Here, $\alpha = PQ/D$ But, when we assume the object to be small, $\angle\alpha$ is also small As such, we can assume that $\tan\alpha = \alpha$

Hence, $\alpha = PQ/D$.

When the object is viewed with the help of a microscope, the image as seen by it is say $P'Q'$ formed at a distance v from the eye Then the angle subtended by this image would be $\beta = \dfrac{P'Q'}{v}$ at the eye. We would like to make β as large as we could possibly make The number of times β is greater than α would be the magnifying power of the microscope Hence, the **magnifying power of a microscope** *is defined as the ratio of the angle subtended by an image*

as seen through the microscope at the eye to the angle subtended by the object at the eye when placed at least distance of distinct vision,

or Magnifying Power of a microscope $= \dfrac{\beta}{\alpha}$

$$= \frac{\text{the angle subtended by the image as seen through the microscope at the eye}}{\text{the angle subtended by the object at the eye when seen naked at least distance of distinct vision}}$$

§4. **Simple Microscope :**—A simple microscope is nothing but an ordinary convex lens used as a magnifier To view an object, the lens is so placed that the object lies in between the pole and focus When the eye is placed closed to the lens, a virtual and magnified image is seen. The path of the rays is shown in Fig 76.

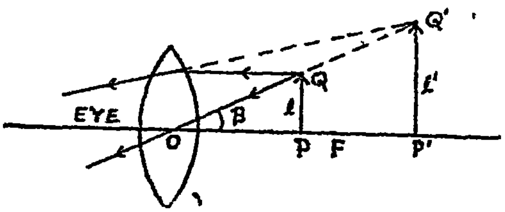


Fig. 76

Here, $\alpha = \dfrac{PQ}{D} = \dfrac{l}{D}$

and $\beta = \dfrac{P'Q'}{v} = \dfrac{l'}{v}$

From diagram it is clear that $PQ = l$, $P'Q' = l'$ and $OP' = v$

M. P. of Simple microscope $= \dfrac{\beta}{\alpha}$

$$= \frac{l'/v}{l/D}$$

$$= \frac{l'}{v} \times \frac{D}{l} = \frac{l'}{l} \times \frac{D}{v} \quad \ldots (1)$$

But, from simple theory we know that for a convex lens its linear magnification

$$M = \frac{\text{size of an image}}{\text{size of an object}} = \frac{l'}{l}$$

$$= \frac{\text{distance of the image from the pole}}{\text{distance of the object from the pole}}$$

$$= v/u.$$

(Remember that the eye is placed closed to the lens and hence, it can be assumed that the eye coincides with the pole of the lens)

Making this substitution, $l'/l = v/u$ in eqn (1) we get

M. P. of a simple microscope $= \dfrac{\mathbf{v}}{\mathbf{u}} \cdot \dfrac{\mathbf{D}}{\mathbf{v}}$

$$= \frac{\mathbf{D}}{\mathbf{u}}$$

For a convex lens, we know that

$$\frac{1}{\ } - \frac{1}{\ } = \frac{1}{\ }$$

Multiplying both the sides by D we get

$$\frac{D}{v}-\frac{D}{u}=\frac{D}{f}$$

or

$$-\frac{D}{u}=\frac{D}{f}-\frac{D}{v}$$

But, the focal length f of a convex lens is negative

Hence,

$$-\frac{D}{u}=-\frac{D}{f}-\frac{D}{v}$$

or

$$\frac{D}{u}=\frac{D}{v}+\frac{D}{f}$$

Substituting this value of D/u in eqn 2, we get,

M. P. of a simple microscope $=\dfrac{D}{u}=\dfrac{D}{v}+\dfrac{D}{f}$

When the distance u is so adjusted that the final image is formed at least distance of distinct vision, *i e.*, $v=D$, we get by substituting $v=D$ in the above.

M P. of a simple microscope $=\dfrac{D}{u}=1+\dfrac{D}{f}$

When the distance u is adjusted that the image is formed at ∞, we have $v=\infty$ and we get

M P of a simple microscope $= D/f$ $= \dfrac{D}{u}$

Thus, we get

General expression for M P. of a simple microscope is

$$\mathbf{M.\ P.}=\frac{\mathbf{D}}{\mathbf{u}}$$

When image at D ; M. P.=1+D/f

When image at ∞ ; M. P.=D/f

§5. Compound Microscope :—With a simple microscope magnification is done only in one stage. This is not sufficient Moreover, the image so formed suffers from usual defects of images. With a view to remedy this, compound microscope is used

Construction :—It consists of the following main parts .

(*a*) *objective*

(*b*) *eye piece*

(*c*) *cross wire*

(*d*) *Rack and Pinion arrangement*

All these parts are housed in a metal tube

(*a*) **Objective** :—It, generally, consists of a convex lens of short focal length and is fixed at one end of the brass tube nearer to the object to be viewed In costlier type, the objective consists of a battery of convex and concave lenses so placed that together it behaves like a convex lens of short focal length free from all defects of images It is represented in figure by L_o

Thus, we find that both f_o and f_e occur in the denominator. Hence, smaller and smaller all these focal lengths, greater and greater will be the magnifying power of a microscope

Use.—Microscope is used to magnify very minute objects which cannot be seen very clearly. Its use in biology is very wide spread. Microscopes of very large magnifying power have been built now-a-days.

§6. **Telescope :**—When an object lies at a very great distance from us, even if its size is very large, due to distance it subtends a very small angle at the eye Hence, its apparent size is very small. *To magnify such objects we use an instrument which is called a* **telescope** Mainly telescopes are of two kinds :—

(1) *Astronomical* to view astronomical objects. These form inverted images , and (2) *Terristrial* to view earthly objects. These form upright images

§7. **Magnifying power of a telescope :**—Because the object viewed is at a very great distance (unapproachable for all practical purposes) the angle which it subtends at the eye, say θ, is the maximum angle and cannot be increased unless we use some other means. Now $\theta = PQ/x$ where PQ is the size of the object and x is its distance from the observer's eye. How many times this angle can be magnified by the telescope will decide its magnifying power. If the final image viewed in telescope be $P''Q''$ and if it is formed at a distance v from the eye, the angle subtended by it would be $\beta = P''Q''/v$. Then, **the magnifying power of a telescope** *is defined as the ratio of the angle subtended by the final image at the eye to the angle subtended by the object at the eye.*

or Magnifying power of a telescope

$$= \frac{\text{the angle subtended by the image as seen in telescope at the eye}}{\text{the angle subtended by the object at the eye}}$$

$$= \frac{\beta}{\theta} = \frac{P''Q''/v}{\dfrac{PQ}{x}}.$$

§8. **Astronomical Telescope :—Construction.**—Like microscope it has also the following main parts

(a) *objective*

(b) *eye piece*

(b) *cross wire*

(d) *Rack and Pinion arrangement*

All these are housed in a metal tube.

(a) **Objective.**—As in microscope, it is a convex lens fitted at that end of the brass tube which is away from the eye Its focal length and aperture are as large as possible. It is usually a single lens.

(b) **Eye piece**—Same as in microscope

(c) & (d) **Cross wires and Rack and Pinion arrangement.**—Same as in microscope. Here, the objective is kept fixed while only

the eye piece with the cross wires is moved forward and backward to focus the image.

Working.—The telescope is first pointed towards a white wall and by moving the eye piece the cross wire is well focussed. The telescope is then pointed towards the object PQ to be viewed By the rack and pinion arrangement, the eye piece with cross wire is so moved that a clear image $P''Q''$ is formed without parallax with the cross wires.

In this position PQ forms its real, inverted and diminished image $P'Q'$ due to the objective lens L_o This image $P'Q'$ is formed in between the focus and pole of the eye piece L_e and hence, we get the final image $P''Q''$ which is virtual, inverted and magnified.

Magnifying power —See fig 78 PQ is the distant object situated at a distance U from the objective Actually, the length of

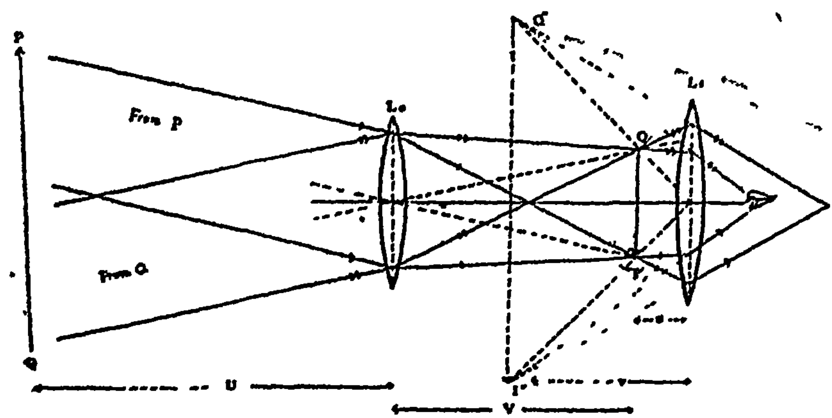


Fig 78

the tube is so small, that the angle subtended by the object at t objective, i e, PQ/U can be regarded as the angle subtended by t object at the eye which is placed at the other end of the tube Hence, $\theta = PQ/U$ Parallel beam (because object is at a very great distance off) coming from respectively P and Q after refraction meet $P'Q'$ forming the inverted image at a distance V from L_o Thus, at $P'Q'$ lies in the focal plane of L_o $P'Q'$ lying in between focus and pole of the eye lens L_e forms a virtual, magnified image at $P''Q''$. Let the distances of $P'Q'$ and $P''Q''$ from L_e be respectively u and v. Then, $\beta = P''Q''/v$ Hence,

$$\text{Magnifying power of a telescope} = \beta/\theta$$

$$= \frac{P''Q''/v}{PQ/U}$$

$$= \frac{P''Q''}{PQ} \times \frac{U}{v} \quad \ldots (1)$$

Now as already explained in compound microscope, with respect to L_o, PQ and $P'Q'$ are object and image and hence,

$$\frac{P'Q'}{PQ} = \frac{V}{U} \quad \ldots (2)$$

Similarly, with respect to L_e, we get

$$\frac{P''Q''}{P'Q'} = \frac{v}{u} \quad .. \quad (3)$$

Therefore, on multiplying equations (2) and (3), we get

$$\frac{P'Q'}{PQ} \times \frac{P'Q''}{P'Q'} = \frac{V}{U} \times \frac{v}{u}$$

or

$$P''Q''/PQ = V/U \times v/u \quad .. \quad (4)$$

Substituting the value of $P''Q''/PQ$ from eqn. (4) in eqn (1), we get

$$\textbf{M P. of a telescope} = \frac{\mathbf{V}}{\mathbf{U}} \times \frac{\mathbf{v}}{\mathbf{u}} \times \frac{\mathbf{U}}{\mathbf{v}}$$

$$= \frac{\mathbf{V}}{\mathbf{u}} \quad .. \quad (5)$$

To get the final image at least distance of distinct vision, u should be equal to say u_o and for the image to be formed at ∞, it should be f_e where f_e is the focal length of the eye piece. Then, we get,

General expression for $\quad \mathbf{M\ P} = \frac{\mathbf{V}}{\mathbf{u}}$

When image at D ; $\quad \mathbf{M\ P} = \mathbf{V/u_o}$

When image at ∞ ; $\quad \mathbf{M\ P} = \mathbf{V/f_e}$

Now, generally, the object is situated almost at ∞ and hence, the image $P'Q'$ is formed in the focal plane of the objective Hence, $V = f_o$ Hence, in the above expression f_o can be substituted for V. Thus, because f_o is in the numerator and f_e is in the denominator, for large M P, we need an objective of large focal length and the eye piece of short focal length.

Use —It is used to view distant objects clearly and distinctly.

§8 **Types of telescopes** —Because the astronomical telescope forms the final image which is inverted it cannot be used for earthly things, *viz*, for observing a cricket match Galileo, therefore, constructed his terristrial telescope In this, the convex lens of the eye piece was replaced by a concave lens This modification, in addition to making the image upright, reduced the length of the tube By reducing the length, the instrument becomes quite handy

In good telescopes we need large apertures Now, it is difficult to construct a very huge convex lens free from all defects. Newton, therefore, constructed his reflecting telescope In this telescope, he replaced the convex lens objective by a concave or a paraboloid mirror objective. This modification helped in the construction of huge objectives World's largest telescopes are of this type.

QUESTIONS

1 What do you understand by a simple microscope. Define its M P. and deduce it (§ see 3 and § 4)

2 Describe the construction and working of a compound microscope with a neat diagram. Deduce its M P (§ see 5)

3 Define M P of a telescope. Describe the construction and working of an astronomical telescope Deduce its M. P. (§ see 6, 7)

4 Write a short note on "Types of Telescopes" (§ see 8)

Section IV

MAGNETISM

CHAPTER I

MAGNETIC PROPERTIES

§1 **Magnetism :**—Long back a shepherd was grazing his sheep in a jungle. He was wearing shoes which had iron nails fixed at the bottom Once he found that his shoes were attracted by a dark coloured stone. He picked up that stone and found that it had the property of attracting iron nails As this stone was found in Magnesia, a province in Asia Minor it, was given the name of magnetite Later on, near about 200 B C., Chinese discovered that when this stone was suspended by a thread it came to rest in north-south direction The property by virtue of which it attracts *iron nails and comes to rest in north south direction is called Magnetism*

§2. **Natural magnets :**—Magnetite as described in the previous article exists in nature It is a composition of iron and oxygen (Fe_2O_3) It is found in Norway, Sweden, Asia minor, Urals, and in some parts of America Ancient sailors used this magnetite in finding direction when other methods failed On account of this use, it was also called lode-stone or simply leading stone, *i e*, which leads the way

§3 **Types of magnets :**—(*i*) **Natural magnets** —As already described they are nothing but pieces of magnetite which occur in nature.

(*ii*) **Artificial magnets :**—Natural magnets are irregular in shape and weak in magnetic properties Hence they cannot be put to many practical uses So these days mostly we use artificial magnets Artificial magnets are those magnets in which these magnetic properties are artificially developed They can be prepared in number of shapes Generally, they are of the following types —

(*a*) **A bar magnet** This is in the form of a magnetic steel bar of rectangular or circular cross section as shown in figs (1) and (2).

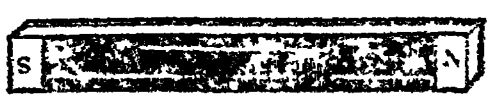


Fig 1.

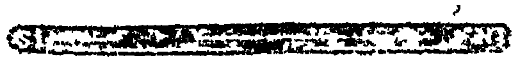


Fig 2

(*b*) **A horse shoe magnet** —It is of the shape U resembling a horse shoe as shown in fig (3)

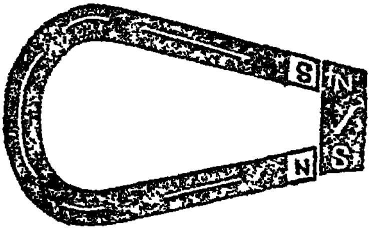


Fig 3

(c) **A magnetic needle** :—It is a thin strip of magnetised steel It has pointed ends. It is pivoted on a vertical axis passing through its centre of gravity The pivot should be a diamond point or an agate stone in less costly instruments. It has been shown n fig. (4).

(d) **Ring magnet** .—It is in the form of a ring.

(e) **Sheet magnet** .—It is in the form of a magnetised sheet.

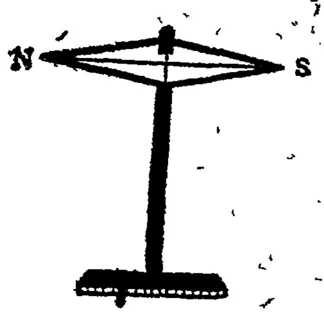


Fig. 4

§4 **Properties of magnets** :—
(1) **Attractive** :—Take a piece of magnetite or a small bar magnet and put it in iron filings as shown in fig (5) You will see that most of the filings stick to the two ends As you move towards the centre practically no filings are attracted. You can easily infer from this experiment that the force of attraction is not the same all along the length of the magnet It has the greatest value at the two ends and almost nil in the middle.

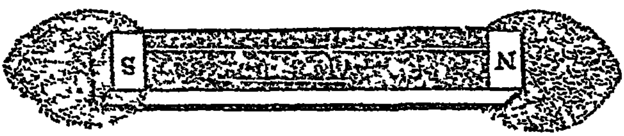


Fig 5

The two regions at the two ends where the attraction is the greatest are defined as the poles of the magnet A magnet has got two poles N and S at its two ends The poles are situated not exactly at the two ends but are just within the ends as shown in fig. 7

(2) **Directive** :—Suspend a bar magnet AB at its middle point O by a thread so that it can rotate freely. To start with, it goes on rotating but after some time you will see that it comes to rest in north-south direction Disturb it Again it will come to rest in the same direction It clearly shows that when suspended freely a magnet always comes to rest in north-south direction. The end which points towards the geographical north is defined as north pole The end pointing towards geographical south is called south pole as shown in fig 6

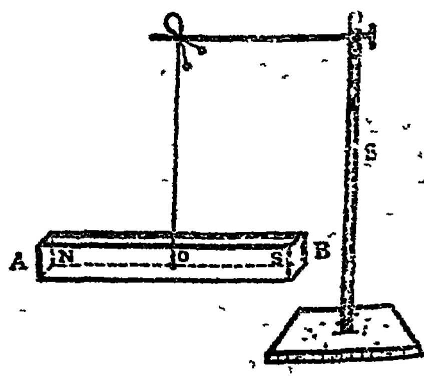


Fig 6

If you join the two poles N and S and produce the line, it is defined as magnetic axis as in fig. 7.

(3) **Like poles repel each other and unlike poles attract each other :—**

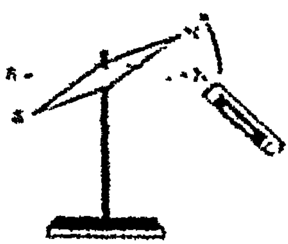


Fig 8

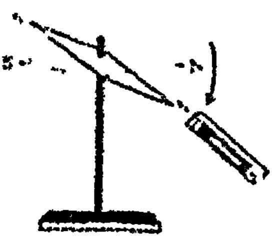


Fig. 9

Bring the north pole of a bar magnet NS near the north pole of a magnetic needle. You will see that they will repel each other as in fig. 9. If you bring the south pole of the magnet near N pole of the needle they will attract each other as in fig 9 It is clear from this experiment that *like poles (N and N) repel and unlike poles (NS) attract each other.*

(4) **Isolation :—**The two poles of a magnet cannot be isolated. Break a magnet in two or three parts as shown in fig. 10 Test each part Each part will behave like a complete magnet with two poles N and S at its two ends It clearly shows that the two poles cannot be isolated.

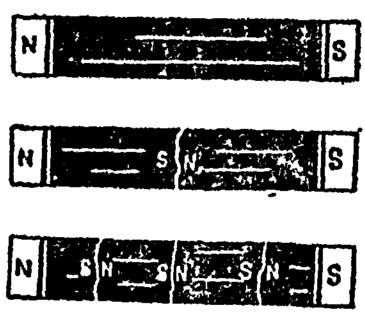


Fig 10

(5) **Both the poles of a magnet have equal strength :—**Both the poles of a magnet have equal strength They have equal attractive property Theoretically, it is impossible to imagine a magnet having unequal poles

Experiment :—Take a big piece of cork and float it on the surface of water Keep a magnet on it. You will find that the cork would rotate and set in such a way that the magnetic meridian may pass through the axis of the magnet Had the poles been of unequal strength it would have started moving in the direction of the pole of greater strength

(6) Induction :—Take a bar magnet *NS*. Bring a soft iron nail near its south pole. You will observe that it clings to it. This is possible only when the nail it self acquires magnetism under the influence of the magnet. The end of the nail nearer to the pole developes north polarity whereas the farther end developes south polarity. Then alone attraction between the two is possible. Bring second nail near the first. It will also acquire magnetism under the influence of the magnet and clings to the first nail. In this way you can make a long chain of soft iron nails. The length of the chain will depend upon the strength of the pole. If you remove the magnet, the chain will break. All the nails will fall to the ground except a few as shown in the diagram. It means that as soon as the magnet is removed the nails also lose their temporary magnetism. Thus, this development of temporary magnetism in iron nails by a magnet with or without actual contact is known as magnetic induction.

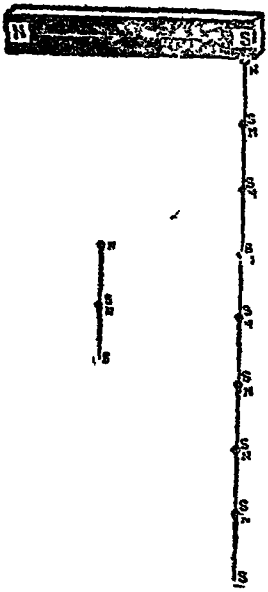


Fig 11

In the above experiment, if, the magnet is removed the chain breaks, only a few nails, say one or two, remain in the chain. It shows that soft iron can be easily demagnetised. On the other hand, take steel nails as in fig 12 and repeat the experiment. Now you cannot form a chain as long as in the case of soft iron nails. But in this case, even when you remove the magnet, most of the nails will remain clinging, showing that that have not lost magnetism. It clearly indicates that steel cannot be magnetised easily but once when it has been magnetised it retains its magnetism for a longer time.

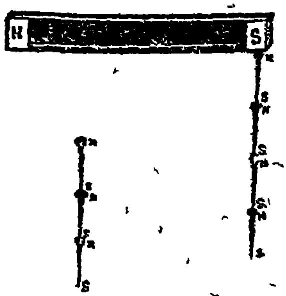


Fig. 12.

The phenomenon of magnetic induction can be verified in another way also. Take a small magnetised needle *NS* pivoted on an axle. Bring a soft iron piece *PQ* near the north pole of the needle. You will find that attraction will take place. It means that the end *Q* has developed south polarity under the influence of the needle. If you bring the same end near the south pole of the needle still attraction takes place, showing that now it becomes a south pole.

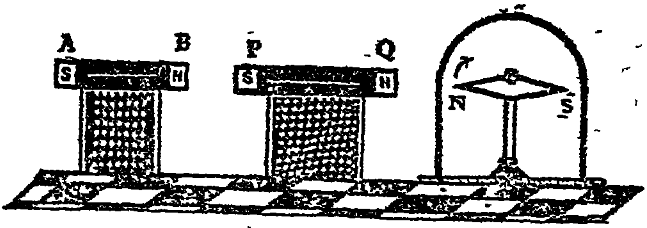


Fig 13

But if you put a strong bar magnet *AB* near *PQ* on the opposite side of the needle as shown in the diagram, you will see a marked change in the behaviour of *PQ* In this position the needle will have little influence on *PQ* as compared to the influence of the magnet Under this influence of *AB*, the nearer end *P* will become a south pole, while the farther end *Q* will become a north pole As *Q* has become a south pole there will be repulsion between *Q* and *N* pole of the needle. Though in this case the *PQ* has not been touched by the magnet yet the presence of the later alone has produced magnetism in *PQ*. This is also known as induction The pole which produces magnetism is called the inducing pole You can deduce the following results from this experiment —

(i) *The end which is nearer to the inducing pole developes polarity opposite to that of the inducing pole* The remote end becomes similarly magnetised.

(ii) *Stronger is the inducing magnet greater will be the magnetism induced*

(iii) *As you increase the distance between PQ and AB, i e*, between the inducing magnet and the body to be magnetised, weaker and weaker will be the induced magnetism

(iv) The *induced magnetism will also depend upon the material of the body.* Steel will not be easily magnetised in comparison to soft iron Similarly, steel will not be demagnetised soon

(v) **Induction preceds attraction** :—In the above experiment you have seen that, if, you do not put *AB* near *PQ*, both the ends of the magnetic needle attract the end *P* or *Q* As soon as the end *P* is brought near the poles of the needle, temporary magnetism is induced in it opposite to that of the inducing pole Hence, attraction takes place Thus the conclusion is induction precedes attraction

(7) **Effect of a very strong pole near a similar but weak pole** :—Suppose you bring *N* pole of a strong bar magnet *AB* towards the *N* pole of the weaker magnet *CD* As you take it towards *CD* induction will take place. The inducing action may be so strong that *N* pole of *CD* may change the nature of its polarity and become south pole In that case you will notice attraction taking place between the two.

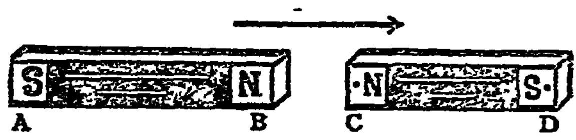


Fig 14

(8) **Repulsion is the surest test of magnetisation** —Suspend the specimen *AB* under test at its middle point by a thread. Now bring the north pole of a bar magnet *M* near the end *A* If there is attraction between the two there are two possibilities (i) Either the end possess south polarity (ii) or it was unmagnetised but inductively becomes south pole due to the inducing action of *N* pole of *PQ* So attraction alone cannot tell you whether *AB* is a magnet or not. On the other hand if, there is repulsion between the end *A* and the north pole of *M*, the end must be a north pole Hence, *AB*

is a magnet. So repulsion is the surest test of magnetisation In the previous case, if, there is attraction between N pole and the end A, N pole is taken near the end B. If still there is attraction, the piece AB is unmagnetised. If there is repulsion it is the south pole. Thus, from this simple experiment you can test whether a given specimen is a magnet or an unmagnetised magnetic substance

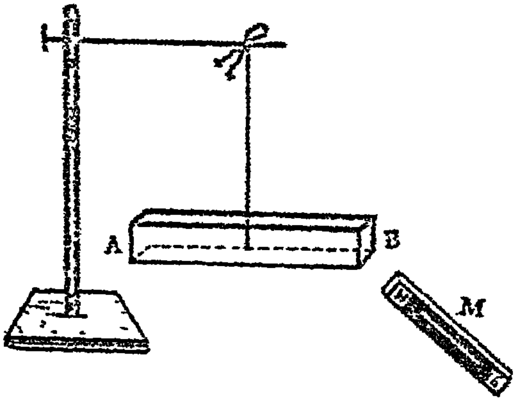


Fig. 15.

(9) Magnetic saturation :—You have seen in the above articles as to how a substance can be magnetised. But the question is whether it can be magnetised to any strength. If you take the case of rubbing, to start with as you rub more and more, the strength of magnetism will increase. But a stage will come when even more rubbing will not increase magnetisation. This is called saturation, *i.e.*, this is the limiting value to which that specimen can be magnetised. This is true for induction also.

(10) Demagnetisation :—If the magnets are not handled properly they will lose magnetism. This is called demagnetisation. This is due to heating it or striking it against irregular bodies etc. That is why it is an important precaution that magnets must be placed gently on the table. Apart from this, every magnet tends to lose magnetism on account of the inductive action of its own poles To avoid it, place a soft iron piece across the poles of a horse shoe magnet. Keep the bar magnets in pairs with opposite poles placed, side by side across a soft iron piece as in fig 16. The soft iron pieces are called keepers.

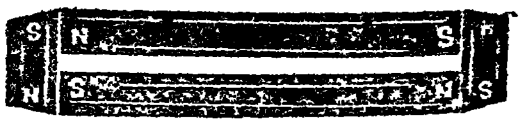


Fig. 16.

§ 5. Methods of magnetisation :—You have already learnt that natural magnets are irregular in shape. They cannot be conveniently used in practice There are certain metals and alloys in which the magnetic properties can be easily developed. This act of developing magnetic properties in a magnetic substance is called magnetisation. Magnets so produced are known as artificial magnets. They can be produced by the following methods.

(i) **By rubbing :**—(*a*) **Method of single touch** :—Take a thin piece of steel *PQ* and place it on a table. Take a bar magnet *NS*, and put its *N* pole in contact with the end *P* of the specimen as in fig. 17. Draw the magnet towards the end *Q* keeping it in the inclined position. The north pole should always remain in contact with the piece *PQ*. When you reach the end *Q*, lift the magnet as shown by the dotted line. Bring it back and put the

north pole again in contact with P. Repeat the same procedure a number of times always ending at Q This process will develop magnetism in the piece PQ. The end P where the process starts becomes a north pole The end Q where the stroking pole is lifted becomes a south pole

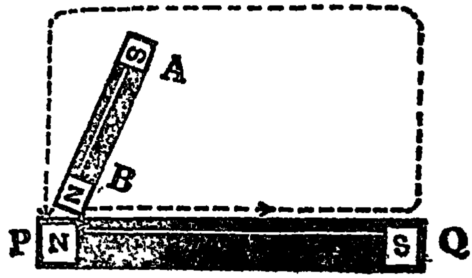


Fig 17

(b) **Method of divided or separate touch**.—Place the specimen PQ on the opposite poles of two bar magnets Take two magnets AB and CD with their opposite poles in contact Place them at the middle of the specimen as shown in fig. 18 Move the magnets in opposite directions indicated by dotted lines (The

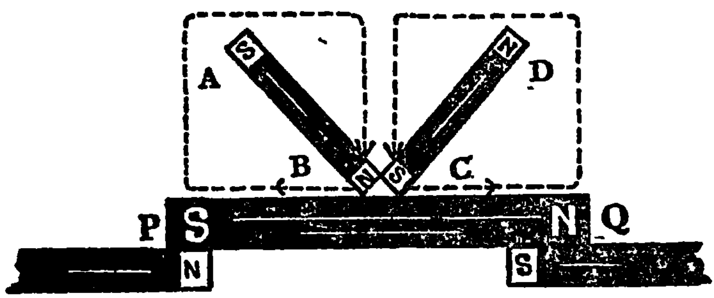


Fig. 18

poles should always remain in contact with PQ) When you reach the ends PQ, lift the magnets Put them back in the same position in the middle of PQ Repeat this procedure for a few times Turn PQ upside down and repeat the same thing The end P will become a south pole The end Q will become a north pole (the polarity developed will be opposite to that of the stroking pole) The presence of the two magnets at the bottom increases magnetisation

(c) **Method of double touch** —Place the specimen PQ on the opposite poles of two bar magnets Put a cork piece between the opposite poles of two bar magnets AB and CD Put the magnets on PQ such that their opposite poles put together touch the middle portion of PQ, as shown in fig 19 Draw both of the magnets together towards Q Without raising, again draw them towards P, and then again towards the middle In this case, do not lift them Always finish at the middle. Repeat the procedure for a few times. P and Q will respectively develop north and south polarity

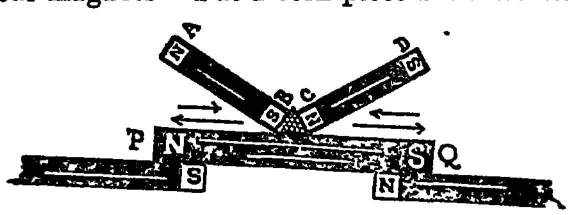


Fig 19.

(ii) **By electric current**:—Wind a number of turns of insulated copper wire round the specimen XY of soft iron Connect the two ends of the coil to the two terminals of a battery through a

variable resistance and a key. Pass a strong current through the coil for some time. Now test the rod PQ. It will behave like a magnet which shows that it has been magnetised. It is called an electro-magnet because the magnetism has been induced due to the passage of an electric current. If the rod is made of soft iron, it will lose the induced magnetism as soon as the current is switched off. Steel pieces will retain magnetism even after the passage of the current. In electromagnets generally soft iron cores are used.

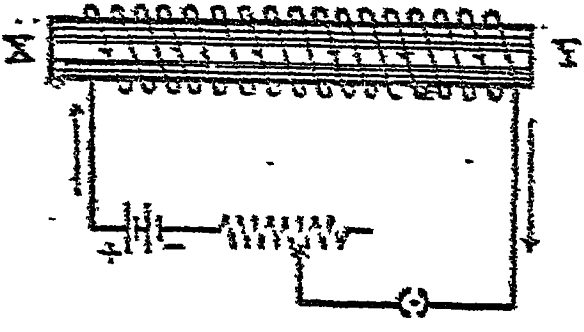


Fig. 20.

To determine the polarity developed look from the end A. If the current in the circular coil appears to flow in the clock wise direction the end A will acquire south polarity. You can remember this rule conveniently by putting arrows at the ends of S as in fig. (21), the direction will become clockwise. If looking from A the current in the coil appears to flow in the anticlockwise direction it will become a north pole as in fig. 22. This you can remember by putting arrows at the two ends of N. The other end will acquire polarity opposite to that of A, because looking from that side the current will appear to flow in the opposite direction.

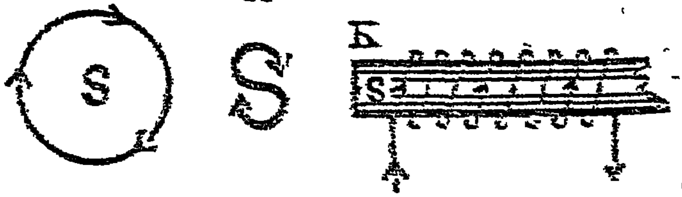


Fig. 21.

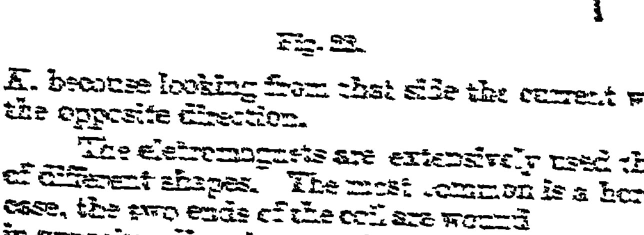


Fig. 22.

The electromagnets are extensively used these days. They are of different shapes. The most common is a horse shoe type. In this case, the two ends of the coil are wound in opposite directions as in fig. 23. In electric bells, telephones, factories and other technical appliances they find wide application.

Fig. 23.

(ii) Magnetisation by induction :—It has already been described in magnetic induction.

(iii) By the magnetism of the earth :—When a magnetic needle is

suspended freely it comes to rest in N-S direction It is because the earth behaves like a powerful magnet So, if, you take an iron piece and strike it on the earth for a number of times in N-S direction, it will acquire feeble magnetism. It is due to the inductive action of the earth's magnetism

§ 6. **Consequent poles** :—If you do not magnetise a specimen properly, you will find that similar poles will develop at the two ends and the opposite poles in the middle as in fig. 24 They can be tested by putting iron filings They are called consequent poles. They are short lived

Fig. 24

§ 7. **Magnetic Substances** :—All most all the substances are influenced by magnets But this influence is almost negligible in case of most of them There are only a few substances, *e g* iron, steel, nickel, cobalt, manganese and a few alloys which exhibit strong magnetic properties They are attracted even by weak magnets They are called magnetic substances All permanent magnets are prepared by these substances

§ 8. **Permanent and temporary magnets** :—Permanent magnets are prepared from hard steel They retain magnetism for a very long time even after the magnetising force is removed On the other hand temporary magnets are made of soft iron They exhibit magnetic properties only under the influence of some magnetising force *e g* electric current or some permanent magnet As soon as the magnet is removed or current is switched off, they immediately lose magnetism The best form of this type is an electro-magnet They are extensively used in electric bells, telephones, electric fans etc

§ 9 **Molecular theory of magnetism** :—You have already learnt that the two poles of a magnet cannot be isolated If you split a magnet in parts, each part will behave as a separate magnet It led Weber to develop a theory, which is called the molecular theory of magnetism It was later developed by Ewing According to this theory each molecule of a magnetic substance is a complete magnet with two usual N and S poles They are called weber elements In an unmagnetised state of a magnetic substance, these elements arrange themselves in closed haphazard chains as shown in fig 25 This is due to the molecular forces acting with in the substance Due to the formation of the closed chains the effect of one pole is neutralised by the effect of another opposite pole Consequently

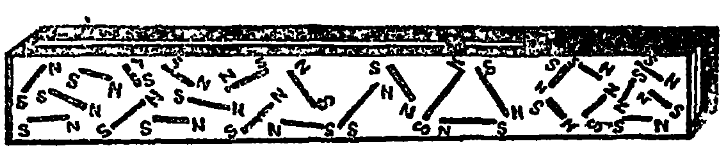


Fig 25

there exists no free poles Hence, there is no resultant magnetic intensity That is why in unmagnetised state a magnetic substance shows no magnetism.

When you magnetise it, the closed chains split up, and the alignment changes. They set in a particular order as shown in fig 26. At the two ends free poles are developed which do not neutralise. This is on account of the free poles, that a specimen exhibits magnetic properties. The more you rub, the more will be the

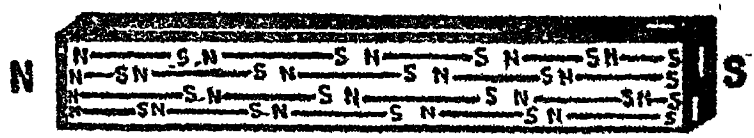


Fig 26.

numbers of chains which will split, and more will be the intensity of magnetisation

The following are a few facts which clearly show that the theory is correct —

(i) The two poles of a magnet cannot be separated. According to this theory each molecule behaves as a separate magnet. If you split a substance, ordinarily you can break it in to molecules, and not beyond that But a molecule is a magnet with two poles

(ii) If a substance is magnetised a state of a saturation comes when it cannot be magnetised further. It can be easily explained on the basis of this theory. When the substance is magnetised the closed chains break forming linear chains. But after certain limit no more chains can be any more broken.

(iii) If a magnet is roughly handled the regular chains will break up forming again the closed chains. Thus, the magnet will lose magnetism which is what actually happens

QUESTIONS

1. Describe a good method of magnetising a piece of iron rod In what respects does an electromagnet differ from an ordinary magnet? (See § 5)

2 How would you distinguish between a magnet, a magnetic substance and a non magnetic substance? Describe experiments to illustrate your answers (See § 4, 6, 8)

3. What is magnetic induction? Describe suitable experiments to illustrate this. (See § 4)

4 Distinguish between a permanent and a temporary magnet (See § 8)

5 Repulsion is the surest test of magnetic condition of a body, explain this. (See § 4, 6, 8)

6 Write a short note on Molecular theory of magnetism. (See § 9)

7. Explain why two poles of a magnet cannot be separated from each other? (See § 9)

8 Explain why a bar of iron cannot be magnetised beyond a certain limit? (See § 9)

CHAPTER II

INVERSE SQUARE LAW AND A FEW DEFINITIONS

§1 **Inverse Square law** :—This law was first of all discovered by coulomb. It gives the magnitude and direction of the force acting between the two magnetic poles. It was later experimentally verified by Grimshel. It can be put in the following two forms :—

(i) *The force of attraction or repulsion between two poles is inversely porportional to the square of the distance between them* If F and d represent the force and distance respectively, according to this law,

$$F \propto \frac{1}{d^2} \qquad \text{.. (1)}$$

This is called the law of inverse square because F is inversely proportional to d^2. As the distance increases, force will go on decreasing If you double the distance the force will reduce to one fourth If you reduce the distance by one third the force will become nine times greater.

(ii) It was further found that the *force was directly proportional to the product of their pole* strengths If m_1 and m_2 are the pole strengths of the two poles, we have

$$F \propto m_1 m_2 \qquad (2)$$

[when d is kept constant]

If you take stronger poles F will increase For example if the pole strength of one of the poles is doubled, the force will also become doubled If one pole is doubled while the other is made four times, the force between them will increase eight times

If the two laws are combined, from (1) ond (2) we get

$$F \propto \frac{m_1 m_2}{d^2}$$

or

$$F = K \frac{m_1 m_2}{d^2} \qquad (3)$$

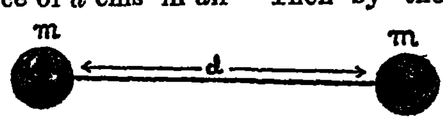
Where K is a constant called coefficient of permeability.

It depends upon the nature of the medium For air or any non magnetic medium K is equal to unity.

So the law becomes for such medii

$$F = \frac{m_1 m_2}{d^2} \qquad . \ (4)$$

§2 **Unit pole** :—Suppose there are two similar poles each of pole strengt m situated at a distance of d cms in air Then by the law of inverse square the force of repulsion F between them will be from above



$$F = \frac{m\,m}{d^2} = \frac{m^2}{d^2} \qquad (5)$$

Fig 27

If we put in equation (5) $F = 1$ dyne, and $d = 1$ cm

$$m^2 = 1$$

or $m = \pm 1$ *i.e*, the pole will be of unit strength.

Thus, *If the force of repulsion between two similar poles placed in air at a distance of 1 cm is 1 dyne, each pole is a unit pole.* [This is in C.G.S system of units).

§3 **Magnetic Field :**—The space around the magnet in which its influence is exerted is called its magnetic field. As you move away from the magnet its influence will deminish. The space over which the strength of the field is the same is said to have uniform field.

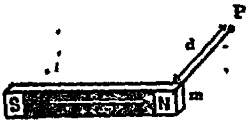


Fig 28.

§4 **Intensity of magnetic field** —If a unit north pole is placed at a point P in the magnetic field of a magnet NS, it will experience a certain force. Thus the *intensity of magnetic field is defined as the force in dynes experienced by a unit north pole placed at that point. It has been assumed here that the unit pole is so weak that it has no magnetic field of its own* If the force is one dyne, the intensity will be unity It is measured in oersteds. *The intensity will be 1 oersted if the force experienced by a unit north pole is 1 dyne*

If m is the pole strength of the magnet and d the distance between its north pole and the point P, the force F experienced by a unit north pole placed at P willbe by inverse square law

$$F = \frac{m}{d^2}$$

$$\therefore \text{ Intensity of magnetic field} = \frac{m}{d^2} \qquad (6)$$

Due to a pole of unit strength at a unit distance from it the field would be unity A unit pole in a unit field experiences unit force If a pole of strength m units is placed in a magnetic field of strength H oersteds, the force experienced by it will be mH dynes

§5 **Lines of force :**—M is a bar magnet P is a point situated in its magnetic field Place a unit north pole at P It will experience a force of repulsion due to N pole, and force of attraction due to Spole of M as shown in fig 29 Under the influence of these two forces the unit north pole will tend to move along the direction PQ, the direction along which the resultant of the two forces acts. Similarly if you take another point instead of P, the direction of the resultant will also change Consequently the direction of the movement of the unit north pole will also change So a unit

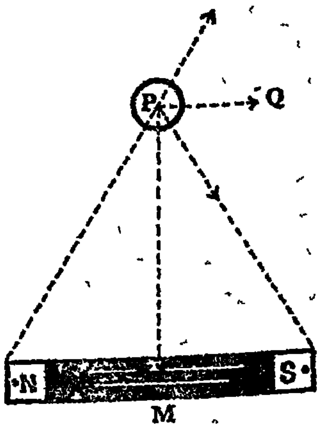


Fig 29

north pole—if free to move—will move along a curve The curve starts from the north pole and ends at the south pole. This curve is called a line of force.

Thus, a *line of a force can be defined as the curve along which an isolatd north pole will travel in a magnetic field, if free to move The tangent at any point along this imaginary curve gives the direction of the resultant force and hence of field at that point* (see fig 30). Conventionally the direction from north pole to south pole is taken as positive

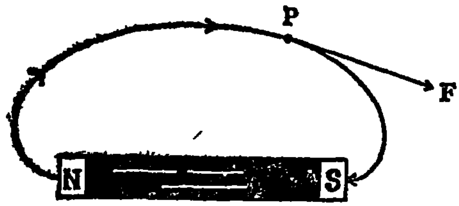


Fig 30

§6 **Drawing lines of force :**—(*i*) **By iron filings :**—Take a plate of glass or sheet of card board Place it on the magnet Put iron filings on this sheet Due to induction these filings will become weak magnets Now tap the glass plate by your finger You will see that these filings arrange them selves in particular curves as shown in fig 31. The curves will begin from *N* pole and end at *S* pole and *vice-versa* These curves are nothing but lines of force This method is applicable only in the case of strong magnets The lines so obtained are also not very well defined

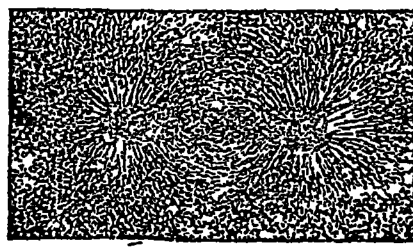


Fig 31

(*ii*) **By Compass needle :**—(For details see "A text book of practical physics" by authors) For tracing the lines of force *an isolated north pole is required, but in practice* an isolated north pole cannot be obtained A small compass needle when placed in a magnetic field will set it self in the direction of the magnetic field at a particular point So if you find the magnetic axis of a small compass needle at point, it will give you the direction of the line of force at that point Thus lines of force can be drawn by a compass needle

Put the magnet on a piece of a white paper Put a compass needle near the north pole of a magnet in the position *A* as in Fig. 32 S pole of the needle will be attracted by the *N* pole of the magnet mark the position of the *S* pole of the needle Put the compass needle in such a position that its *S* pole should now occupy the position previously occupied by its *N* pole Now mark its *N* pole and put it further Repeat this process till you reach south pole of the magnet Now join all these marked points, you will get a line of force In a similar way you can

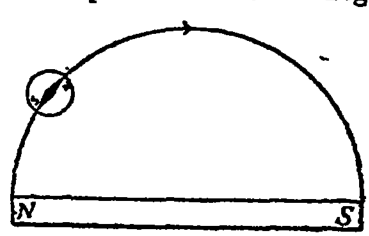


Fig 32

draw other lines of force around it. [The smaller is the needle better will be the accuracy obtained]

§7. **A few important maps of lines of force :**—[*A*] *Lines of force due to the horizontal component* [*H*] *of the earth's magnetic field or* [*The details about H you will find in the next chapter*] *or drawing of the magnetic meridian*

By now you know it very well that earth acts as a big magnet. The horizontal component H of its magnetic field acts in the horizontal direction from south to north To draw the lines of force due to take white paper and fix it on the table. Take a compass needle, and follow the procedure described in the above paragraph You shall obtain straight lines parallel to each other running from south to north Any of these lines represent the magnetic meridian

[B] **Lines of force due to a bar magnet placed in the—magnetic meridian with its north pole pointing north :**—First of

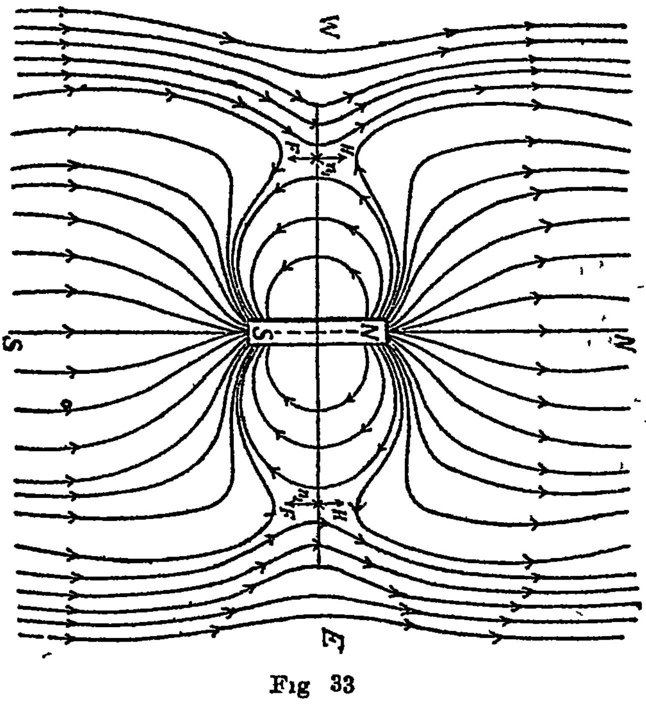


Fig 33

all determine the direction of the magnetic meridian by a compass needle, and draw it on a paper as explained above Put a magnet AB along this line so that its north pole—faces north Two fields are acting in the space around the magnet One due to the magnet and other due to the earth's horizontal component H. Take a compass needle and draw lines of force as explained previously. In the vicinity of the magnet the field due to the magnet will be predominating, and you will get curves as in fig 33. As you move away from the magnet this field will be influenced by H. Therefore the lines will become distorted Ultimately H becomes stronger in comparison to the magnetic field of the magnet. and the lines

become straight running from south to north The lines of force drawn on one side of the magnet are shown in fig 33 :- A similar map can be obtained on the right hand side also It is clear from the figure that along the magnetic meridian the field due to the magnet and the earth's horizontal field are in the same direction But along the equitorial line these two fields are in the opposite direction H is always constant where as the field due to the magnet decreases as the distance from the magnet increases. Some where near X the two fields are equal and opposite. As they are equal and opposite the resultant field at that point is zero Such a point is called a Neutral point *Thus a neutral point can be defined as that point in a magnetic field where the resultant intensity is zero or where the magnetic field due to the magnet is equal and opposite to the magnetic field due to the earth's horizontal component* As there is no field at X, if you place a compass needle at X it will set in any direction The point X will be some where at the centre of the curvilinear quadrilateral formed by the lines of force as shown in the figure Similary there will be another neutral point on the left hand side of the magnet Thus to sum up, when n pole is facing north two neutral points can be obtained on the equitorial line equidistant from the centre of the magnet

[C] **When the magnet is placed in the magnetic meridian with its S pole facing north :—**Put the magnet along the magnetic

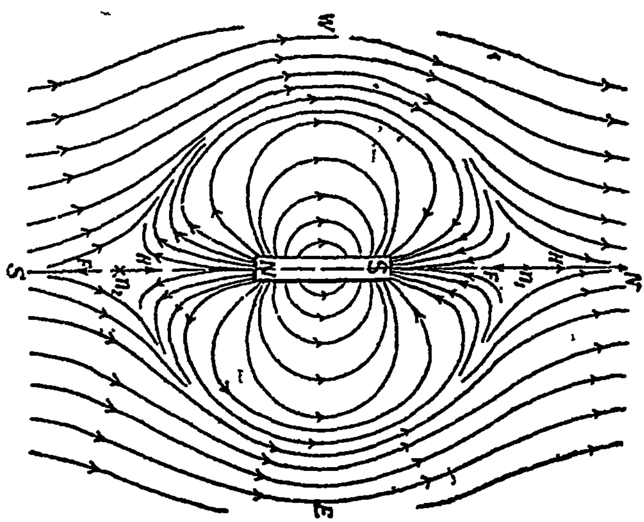


Fig 34

meridian with its south pole facing north Draw lines of force in a similar way You will find now the direction of the lines reversed On the equitorial line the two fields are in the same direction. But on the magnetic axis they are acting in the opposite direction Hence the neutral points will lie on both the sides of the magnetic axis produced as shown in fig 34.

[D] When magnet is placed at right angles to the magnetic meridian :—You can draw the lines of force in a similar way. You would get two neutral points XX as shown in fig 36. They will be situated mid way between the equitorial line and the magnetic axis of the magnet. They will be diagonally opposite to each other. Fig. 35 (a)

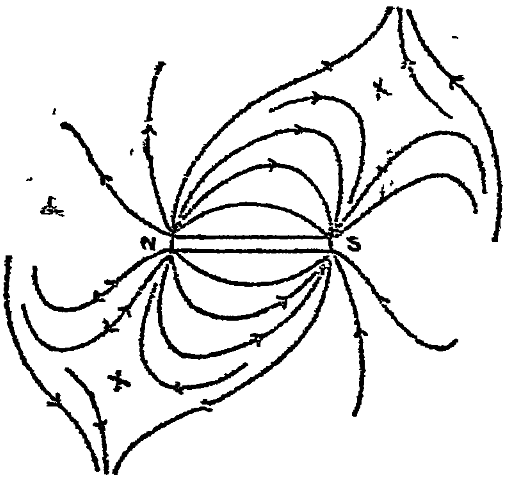


Fig 35(a).

[E] When a long bar magnet is kept vertical with one of its ends on a table :—It can be assumed as equivalent to a single isolated pole Hence lines of force will start radially from it in all directions In its south (if N pole on table) or due north (if S pole is on table) we shall get the curvilinear quadrilateral and the neutral point. In this case there is only one neutral paint [see fig. 35(b)].

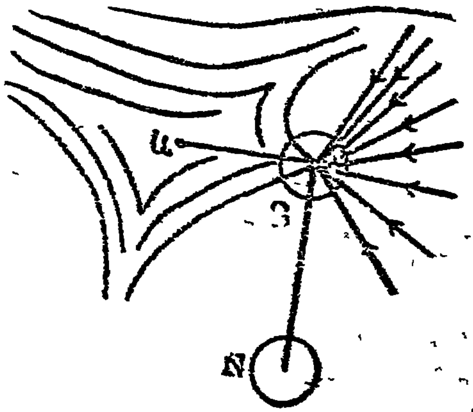


Fig. 35(b).

It is always to be remembered that two lines of force will never intersect Because if they do so at the intersecting point there would be two fields which is not possible we have only one resultant field acting at a point

§8. **Numerical Problems :—1.** *Two exactly similar poles are placed at a distance of 8 cms apart, and the force between them is 9 dynes. Calculate the force in grams weight when they are 4 cm apart.*

Let the strength of each pole be m units. By Coulomb's law the force F between them is given by

$$F=\frac{mm}{d^2}=\frac{m^2}{d^2}$$

$$\therefore \quad 9=\frac{m^2}{8^2} \text{ (for } d=8 \text{ cm.)}$$

$$\therefore \quad m=\pm 24 \text{ units}$$

Suppose the force between them is F dynes when they are 4 cm. apart,

$$\therefore \quad F=\frac{24^2}{4^2}=36 \text{ dynes.}$$

As $$981 \text{ dynes} = 1 \text{ gm. wt.}$$

$$36 \text{ dynes} = \frac{36}{981} \text{gm. wt}$$

$$F_1 = \frac{4}{100} \text{ gm wt.}$$

Problem 2. *Two poles one of which is 5 times as strong as the other exert on each other a force equal to the weight of 800 m. gm. when placed 10 cm apart. Find the strength of each pole*

Let the pole strength of one of the poles be m units, then that of another will be $5m$ units

Force exerted between them $= wt$ of $800m. gm.$

$= \cdot 8$ gm. wt

$= \cdot 8 \times g$ dynes (where g is the acceleration due to gravity).

or by Coulomb's law we have

$$\cdot 8g = \frac{m . 5m}{10^2}$$

or $$m^2 = \frac{100 \times 8}{10 \times 5} g = 16g$$

$$m = 4\sqrt{g}$$

∴ The strength of one pole $= 4\sqrt{g}$ units

" " anoter " $= 20\sqrt{g}$ "

QUESTIONS

1 State the law of force between two magnetic poles and hence derive definition of a unit pole (See §1 and §2)

2 Calculate the force acting on a magnetic pole of strength 75 units placed at a point 15 cm from either pole of the magnet 10 cm long and having a pole strength of 45 units (*Ans* 10 dynes)

3 Define the strength of magnetic field and a uniform magnetic field (See §3 and §4)

4 What do you understand by a line of magnetic force ? Why do the lines of force never cross ? Why do lines of force never pass through a neutral point. (See § 5 and § 6)

5 Define neutral points

Trace the lines of force surrounding a bar magnet when the magnet is placed along the magnetic meridian with the N-pole north Indicate the positions of neutral points in your diagram (See § 6)

6 Find the force between two like magnetic poles each of strength 20 units and placed 5 cm apart What is the intensity of the magnetic field due to these poles at a point 5 cm from each ?

(*Ans* 16 dynes , any value from 0 to 1 6 oersted)

CHAPTER III

TERRESTRIAL MAGNETISM

§1. **Earth as a magnet :**—If you suspend a magnet freely at its centre of gravity, it will always rest in the north-south direction. This is due to the magetism present in the earth. The behaviour of the earth is as if a powerful magnet is lying at its centre running from north to south direction. The south pole of earth's magnet lies some where near the geographical north while the north pole of the magnet lies some where near geographical south. This is the reason why the north pole of magnet points towards north [Attraction takes place between the north pole of the magnet and the south pole of the earth's magnet] If the magnetic axis of a freely suspended magnet is produced on both the sides it won't pass through the two geographical poles. It indicates that the magnetic poles of the earth are a bit removed from its geographical poles. Remember, there is no magnet inside the earth actually.

Let *NG* and *SG* be respectively the geographical north and south poles of the earth. If *S m* is the south pole of earth's magnet, it lies at a place with latitude 71°*N* and longitude 96°*W*. It is about 600 miles from the geographical north pole. Similarly if *Nm* is the north pole of the earth's magnet it is situated at a place with latitude 73°*S* and longitude 156°*E*. It is also about 900 miles away from the geographical south.

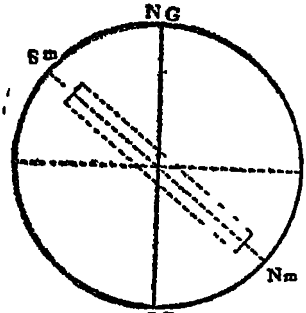


Fig 36

Now the question is as to what is the cause of earth's magnetism? So Many theories have been advanced by a number of persons. One of the theory is that it is due to the electric currents flowing through the earth. The electric currents are produced on account of its rotation. You shall read in Electricity that where there is electricity, there is a magnetic field. For the sake of simplicity you may regard a magnet with its usual poles, placed at the centre of the earth.

§2. **Elements of terrestrial magnetism :**—To get a complete picture of the magnetic field at a certain place you must know three things very accurately. They are (*i*) declination (*ii*) Dip (*iii*) *H*, horizontal component of the earth's magnetic field. They are defined as the elements of terrestrial magnetism.

§3. **Declination :**—Suspend a magnet freely at its centre of gravity. Find out the direction in which it comes to rest at a place *P*. Let *NG* and *SG* be respectively the north and south poles of the earth. The magnetic axis of the magnet will give you the direction of the magnetic field at that place. If you produce the axis on both the sides, it will cut the surface of the earth at S_m and N_m (Not at

NG and *SG*) They are respectively the magnetic south and north poles of the earths magnet.

If you draw a plane passing through *NG*, *SG* and *P*, you will get the geographical meridian The plane passing through S_m, *P* and N_m will give you the magnetic meridian You can clearly see in fig 37 that these two plane do not concide but make an angle δ between them This angle is the declination at that place *Thus, declination can be defined as an angle between the true north and south direction and the direction along which a freely suspended magnet would rest The declination varies from place to place.* **The place where the *S* pole of the earth's magnet is situated is popularly known as magnetic north and vice-versa.**

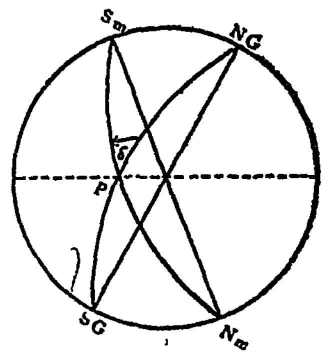


Fig. 37

§4. **Dip** :—Suspend a magnetic needle freely at its centre of gravity *O* When the needle comes to rest, you will notice

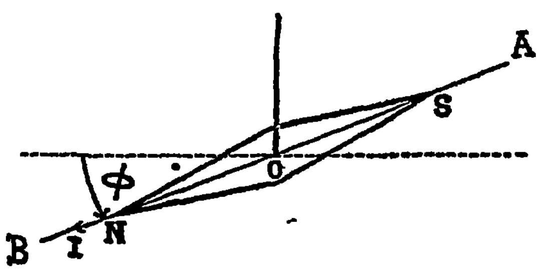


Fig. 38.

that its magnetic axis *AB* will not remain horizontal as shown in Fig 39 [Though it will lie in *NS* direction] Obviously *AB* is the direction in which the total magnetic intensity of the earth at that place acts. In northern hemisphere, *N* pole of the needle will dip down In southern hemisphere *S* pole of the needle will dip down This dipping is due to the attraction between the poles of the needle and the pole of the earth's magnet This fact can be explained by taking a magnet *NS* If you hold a compass needle near *S* pole of the magnet, *n* pole of the needle dips At the north pole *S* pole of the needle will dip due to attraction. In the middle, the attraction due to both the poles will be equal, and hence, the needle will remain horizontal

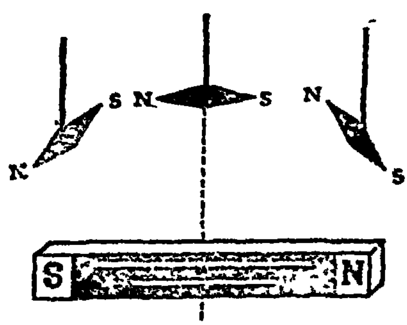


Fig 39

Thus you can easily see in fig 38 that the magnetic axis *AB* makes an angle ϕ with the horizontal direction This is called the angle of dip or inclination *Hence, the angle of dip can be defined as the angle between the magnetic axis of a freely suspended magnet suspended from its centre of gravity*

and at rest and a horizontal line passing through its centre. It varies from place to place. Near about the equator, the needle remains horizontal. It means that the angle of dip at these places is zero. [As already explained this is because at these places the attraction due to the earth's magnetic north pole is counter balanced by the attraction due to earth's magnetic south pole] The imaginary line joining these places is called magnetic equator. The angle of dip will go on increasing as you go away from the magnetic equator. At poles it will attain the maximum value of 90°, *i.e.*, the needle will become vertical.

If you say that the dip at Delhi is 40°*N*, it means at Delhi needle will make an angle of 40° with the horizontal and *N* pole will dip.

§5. Horizontal and vertical components of the earth's magnetic field :—Let *ACBF* represent the plane of the magnetic meridian, and *AMN* that of the geographical meridian. Let the total intensity of the earth's magnetic field *I* at a certain place be represented in magnitude and direction by the line *AD*. This is naturally the direction in which the magnetic axis of a freely suspended magnet will lie. I can be resolved in two ⊥ directions One along *AC* in the horizontal direction, and the other along *AF*, the vertical direction. Let ϕ be the angle between *AB* and *AC*. By definition it is the angle of dip at that place.

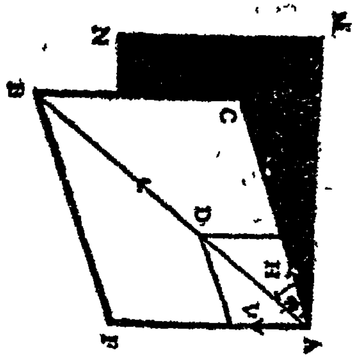


Fig. 40.

If *H* and *V* are the resolved parts in the horizontal and vertical directions respectively, from fig. 40 we shall have

$$H=I\cos\phi \qquad \ldots (i)$$
$$V=I\sin\phi \qquad \ldots (ii)$$

Thus, *H* can be defined as the component of earth's magnetic field acting along the horizontal direction [Note that it shall always act in *N*—*S* direction]

Squaring and adding (*i*) and (*ii*) we get

$$V^2+H^2=I^2(\sin^2\phi+\cos^2\phi)$$
$$=I^2 \text{ (because } \sin^2\phi+\cos^2\phi=1)$$

or

$$I=\sqrt{V^2+H^2} \qquad \ldots (iii)$$

By dividing (*ii*) by (*i*) we get

$$\frac{V}{H}=\tan\phi. \qquad \ldots (iv)$$

Hence if you know the values of *H* and ϕ at any place the value of I can be easily calculated

§ 6. Determination of declination. To determine declination you shall have to find out both magnetic and geographical meridians.

To find out the geographical meridian—*Fix a* thin straight rod *AB* vertically on the ground. The ground should be perfectly smooth and horizontal It should also be open to the sun. Draw a circle of 1 or 2 feet radius around the rod The sun would cast a shadow of the rod. The shadow will become shorter and shorter as the sun rises in the sky At some time in the morning the shadow will touch the circumference of the circle. Mark that point upon the circumference Let it be *D* Again in the after noon the shadow will touch the circumference Again mark the position Let it be *C*. Naturally the shadows in both of these positions are of equal length Join the points *C* and *D* to the point *A*, the base of the rod *i e.*, the centre of the circle Bisect the angle *CAD* by the line *AE* The biseceor *AE* will give you the true north. and south direction The vertical plane through this line will be the geographical meridian.

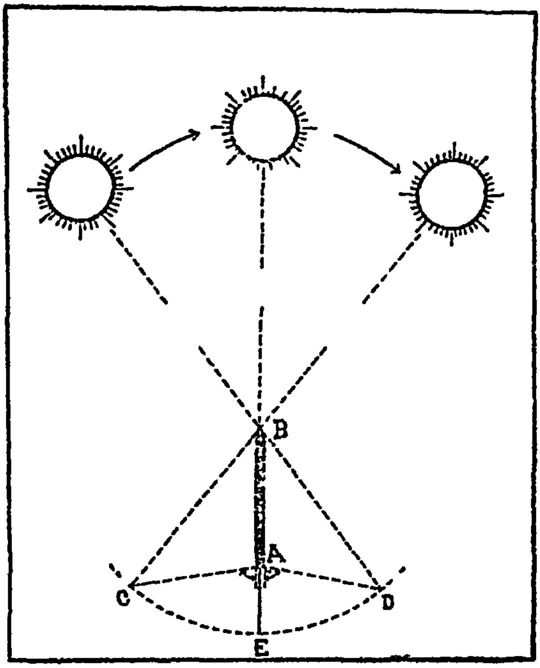


Fig 41.

To find out the magnetic meridian —

Suspend a bar magnet *AB* upon a table by means of an unspun silk loop. Attach two thin vertical wires 1 and 2 at the middle point of each end of the magnet When the magnet comes to rest fix two pins *C* and *D* on the table against each end The pins should be so fixed that these two pins and the two vertical wires 1 and 2 shold appear in the same straight line This is possible only when there is no parallax between the four pins Mark the positions of *C* and *D* and join them by a line as in fig 42 Turn the magnet up side down in the silk loop Repeat the same process fixing pins at *E* and *F* join *E* and *F* by a straight line.

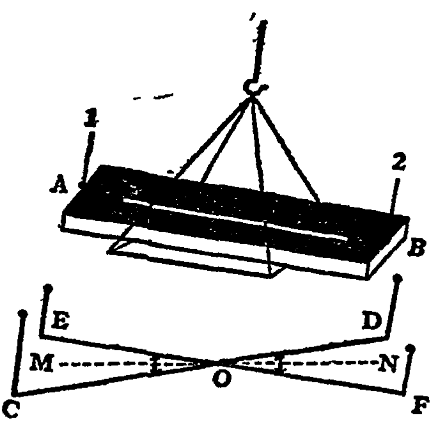


Fig 42

These two lines instersect at *O* Bisect the $< EOC$ or the $> DOF$. The bisector will give you the direction *of the magnetic meridian.*

A vertical plane through this line will be the magnetic meridian. The angle of declination, then will be the angle between two lines *AE* and *MN*.

§ 7. **Determination of the angle of dip :—**It can be determined in the laboratory with the help of a dip circle.

Dip circle :—As shown in fig 43 (*b*). H. S. is a circular scale marked on a horizontal disc supported on three levelling screws

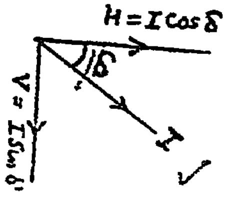


Fig 43 (*a*).

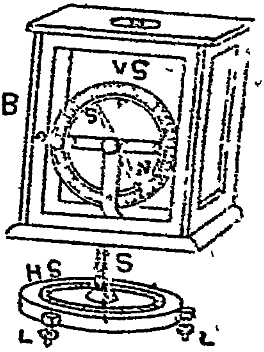


Fig. 43 (*b*)

(*LL*). *S* is a stand which passes through the centre of the circular scale, and is capable of being rotated about a vertical axis. A pointer attached to *S* gives the position of the stand on *H.S.* scale. *B* is a rectangular box with glass panes supported on the stand. It carries a vertical graduated circle *VS*, also capable of rotating in the vertical plane. The *O*—0 line of the circle is perfectly horizontal. A small compass needle *N.S.* supported on a horizontal axis of agate passing through its centre of gravity is placed on the stand at the centre of the *V.S.* scale.

Since the needle is not free to rotate in horizontal plane the plane of the dip needle is to be adjusted *i e.*, the vertical scale is to be brought in the magnetic meridian

Adjustment :—(*i*) Level the disc of the circle with the help of levelling screws It will be indicated either by spirit level or by plumb line

(*ii*) Rotate the dip circle till the needle becomes vertical [*i e.* (90—90)] In this position the plane of the circle is perpendicular to the magnetic meridian because the influence of earth's horizontal component on the needle is zero. Read the position of the pointer on the horizontal scale.

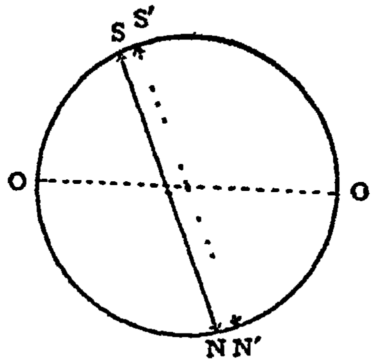
(*iii*) Rotate the circle about the vertical axis by 90° on the horizontal scale This will bring the plane of the circle, and hence the magnetic needle in the magnetic meridian

(*iv*) Take the readings of both the ends of the needle on the scale *V S.* Find the mean of these two readings which will give you the angle of dip.

Errors :— The dip circle may have the following errors .—

(i) The axis of the magnetic needle may not pass through the centre of the vertical scale as shown in fig 44).

To eliminate this error readings of both the ends of needle are taken Mean of the two is free from this error



Fig, 44

(ii) the *O—O* line of the *V S* scale may not be horizontal.

To remove this error the dip circle is rotated through 180° Again two readings of both the ends are taken Mean of the four removes error (*i*) and (*ii*) See fig 45

(*iii*) The geometric axis of the needle may not coincide with it magnetic axis To remove this, reverse the face of the needle on its bearing and repeat the procedure as described in errors (*i*) and (*ii*) In this way eight readings are taken Mean of these eight removes these three errors

(*iv*) The Centre of gravity of the needle may not coincide with the axis about which the needle is rotating To remedy this error,

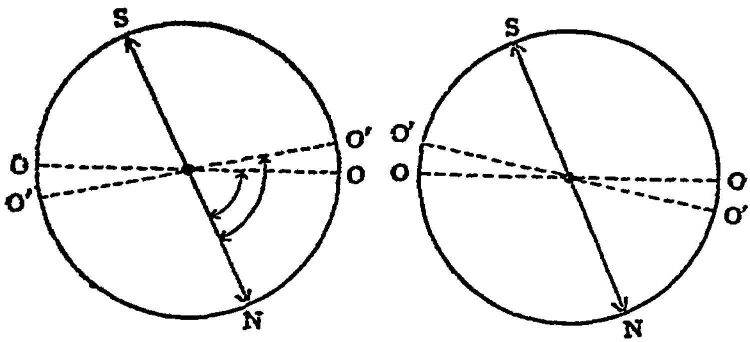


Fig 45

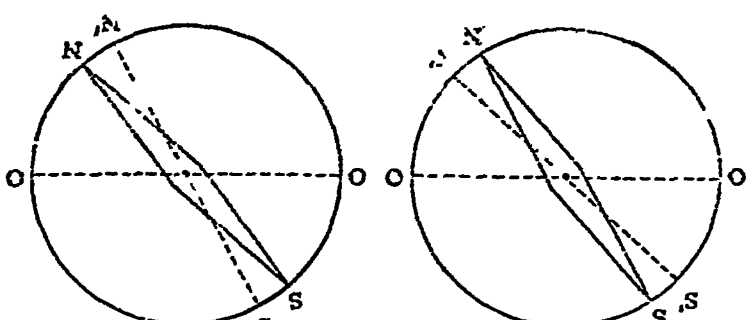


Fig 46

remove the needle Demagnetise it and remagnetise it so that the polarity is reversed *i e.*, the north pole becomes south pole and *vice*

versa. fig. 47. Again mount the needle and repeat the procedure as described in (*i*), (*ii*) and (*iii*). In this way sixteen readings are

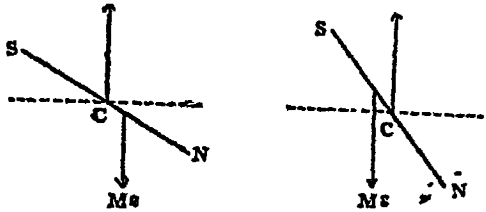


Fig. 47.

obtained. The mean of these sixteen readings will be free from all errors and give true angle of dip

§ 8. **Determination of dip even when dip circle is not placed in the magnetic meridian :**—You have seen in the previous article that to determine the angle of dip the dip circle must be brought in the magnetic meridian. But you can find out the true angle of dip at any place even by determining apparent angles of dip in any two planes mutually at right angles to each other.

First find out the apparent angle of dip in plane OA. Let it be ϕ_1 Now determine the apparent angle of dip in a mutually perpendicular plane Let it be ϕ_2 If the true angle of dip in the magnetic meridian plane OB is ϕ, it is given by the relation $\cot^2 \phi = \cot^2 \phi_1 + \cot^2 \phi_2$

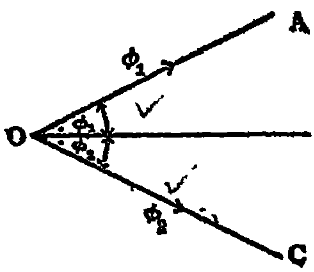


Fig 48.

§ 9. **Variation of magnetic elements :**—The values of the magnetic elements change from place to place and time to time. Following are the important variations

[A] **Secular changes :**—The values of the elements gradually vary extending over a very long time. After a long time they again attain their former values. They are called secular changes They are generally large

[B] **Annual Changes :**—They are periodic changes. During the year the value of the element varies between a maximum and a minimum As for example at a particular place declination becomes maximum in February and minimum in March

[C] **Durinal changes :**—The valule of an element changes even within 24 hours It becomes maximum at a certian particular hour and minimum at some other hour. These changes are also periodic

[D] **Irregular changes :**—They are non-periodic, and abrupt. They originate due to sun spots, volcanic eruptions etc They are called magnetic storms.

§ 10. **Change of magnetic elements with change of place :**—They change from place to place, but there are certain places where they are equal. Maps have been accordingly drawn. Following are the important lines :—

[A] **Isogonic lines :**—These are the lines which join places possessing the same angle of declination The particular line joining places having zero declination is called Agonic line

[B] **Isoclinic lines :**—These are the lines which join places having the same angle of dip Similarly the line joining places having zero dip is called Aclinic line or magnetic equator.

[C] **Isodynamic lines :**—These are the lines joining places having same value of H.

Numerical Problems :—1. If the horizontal component of the earth's field is 0 36 $C\ G\ S$ Units, and the dip is 42° at a certain place. What is the total intensity of the earth's field at that place ?

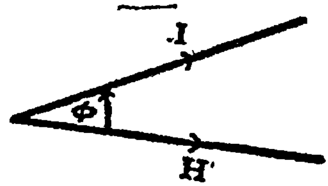


Fig 49

If I and H are respectively the total intensity and horizontal component of the earth's field at a certain place, we have $H=I\cos\phi$ (i) [where ϕ is the angle of dip].

Substituting the values in equation (i) we get

$$\cdot 36 = I\cos 42$$

or $$I=\frac{36}{\cos 42}=\frac{\cdot 36}{7431}=\cdot 49$$

Q. 2. *A dip circle is placed so that the magnetic needle is vertical. The circle is then rotated through θ° about a vertical axis and the angle of dip measured in this position is found to be φ°. Find the value of the angle of dip at that place*

The needle is vertical, shows that the dip circle lies in a direction ⊥ to magnetic meridian *i e*, it lies in east west direction OE When you rotate it through $\theta°$, its plane makes an angle $(90-\theta)$ with the magnetic meridian.

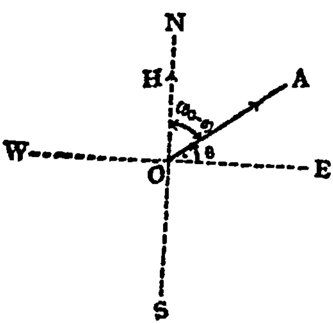


Fig. 50

The resolved part of H in this direction therefore, will be $H\cos(90-\theta)=H\sin\theta$. The vertical component V remains unaltered, and therefore in the position OA we have

$$\tan\phi=\frac{V}{H\sin\theta}$$

$$\text{or } \frac{V}{H}=\tan\phi\sin\theta \qquad (1)$$

If ϕ^1 is the true angle of dip

$$\tan\phi^1=\frac{V}{H} \qquad (2)$$

Substituting the value of $\frac{V}{H}$ from (1) in (2) we get

$$\tan\phi^1=\tan\phi\sin\theta$$
$$\text{or } \phi^1=\tan^{-1}(\tan\phi\sin\theta)$$

QUESTIONS

1. Give a short account of the earth's magnetic field. Discuss its variation with place and time on the surface of the earth.

(see §1 and see 8)

2 What are the magnetic elements of a place, and how are they related to each other? How would you measure the angle of dip at a place correctly? (see §4, see §5 and see §7)

3 Describe a dip circle How will you use it to determine the magnetic inclination of a place. (see §7)

4 What do you understand by the magnetic elements at a place? How can you find the total intensity of the earth's magnetic field at a place, if the value of the magnetic elements there are known to you?

(see §4 and see 5§)

5. What do you understand by (*a*) magnetic equator, (*b*) Isodynamic lines, (*c*) Isogonic, and (*d*) isoclinic lines (see §10)

6 At *A* the total magnetic intensity is 0 5 gauss and the angle of dip is 68°, while at *B* the total intensity is 0 55 gauss and the angle of dip is 72°. Compare the horizontal intensities at the two places (cos 72°=·3090 and cos 68°= 3748)

(*Ans.* 0·1874 gauss ; 0 1699 gauss)

7 A dip circles is placed so that the needle sets vertical. The circle is then rotated through 30° about a vertical axis and the dip as measured in position is 45°. Find its true value.

(*Ans.* 26°·6.)

CHAPTER IV

MAGNETIC MEASUREMENTS

§ 1. **Intensity of magnetic field** —It has already been defined in Chapter 2. You have also seen that the intensity of the magnetic field due to a pole of strength m at a point distance d cm from it is equal to m/d^2 oersteds. Now we are going to find out the couple acting on a magnet when it is deflected in a uniform magnetic field.

§ 2. **Couple experienced by a magnet in a deflected position :—** Let m and $2l$ be respectively the pole strength and length of the magnet PQ. Suspend it freely in a uniform magnetic field of intensity H acting from south to north as shown in the fig 51. It shall rest in the direction AB, the direction of the intensity H. Now deflect it through an angle θ (It is the angle between the mean and the deflected positions) by applying some force. In the deflected position N pole of the magnet will experience a force of mH dynes in the upward direction. Similarly S pole will experience the same force of mH dynes in the downward direction As these forces are equal, opposite and parallel, they constitute a couple. This couple acts in the direction as shown by arrows, and tries to bring the magnet back in its original position AB. Hence it is called the restoring couple.

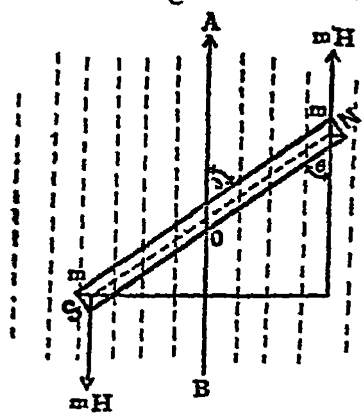


Fig 51

Moment of the restoring couple C about the axis of the magnet

= (Force × perpendicular distance between the forces)

or $\quad C = mH \; NS \sin\theta$

or $\quad C = mH.\, 2l \sin\theta \; [NS = 2l]$

$\quad = MH \sin\theta$, [if $M = 2\,ml$] ... (1)

The couple will be largest when $\theta = 90°$ and the least when $\theta = 0°$. As the couple is zero when $\theta = 0$, the magnet freely suspended always rests in the direction along which magnetic intensity acts.

§ 3. **Magnetic moment :—**M in equation (*i*) above is called magnetic moment of the magnet. It is equal to the product of the pole strength and length of the magnet [$M = 2\,ml$]

If we put $\theta = 90°$, and H = unity in equation (*i*) we get $C = M$.

Hence *magnetic moment of a magnet is defined as the moment of the couple experienced by the magnet when the magnet is placed at right angles to a uniform field of unit intensity.*

§ 4. **Field due to a bar magnet at any point in end on and broadside on positions :—** When the point lies on the prolonged axis of the magnet, the point is regarded to be situated in end on position. When the point lies on the right bisector through the middle point of the magnet it is said to lie in broad side-on position.

[A] Field in end on position :— Let NS be a bar magnet of length $2l$. Let m be the pole strength of each of its poles. P is a point on its axis produced at a distance d from the centre of the magnet. We are to find out the intensity of the magnetic field at the point P It is obvious from the figure that the distance between P and N pole of the magnet is $(d-l)$ while between P and S pole is $(d+l)$.

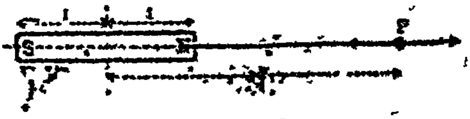


Fig. 52.

To find out the intensity of the magnetic field, consider a unit north pole placed at P. The force of repulsion on this pole due to N pole of the magnet will be.

$$= \frac{m}{(d-l)^2} \text{ (acting from } N \text{ to } P).$$

Where as the force of attraction on this pole due to S pole of the magnet will be $\frac{m}{(d+l)^2}$ (acting from P to S)..

The resultant force on P which will be the intensity of magnetic field (F_1) at P due to the whole magnet will be

$$F_1 = \frac{m}{(d-l)^2} - \frac{m}{(d+l)^2}$$

[acting from N to P, conventionally the force of repulsion is taken to be positive].

$$= m\left\{\frac{1}{(d-l)^2} - \frac{1}{(d+l)^2}\right\}$$

$$= \frac{4\,m\,l\,d}{(d^2-l^2)^2}$$

$$= \frac{2\,M\,d}{(d^2-l)^2} \qquad (2)$$

for $2\,ml = M$, the magnetic moment of the magnet.

If l^2 is smaller in comparison to d^2 equation (2) becomes

$$F_1 = \frac{2\,M}{d^3} \text{ oersteds approximately} \qquad (3)$$

[B] **Field in Broad side on position** :—Let NS be a bar magnet of length $2l$ Let the strength of each of its poles be m. Let P be the point on its right bisector at a distance d from its middle point O. We are to find out the intensity of magnetic field due to this magnet at P. Join PN and PS consider a unit north pole placed at P.

The force of repulsion on the unit pole due to N pole of the magnet will be equal to $\frac{m}{NP^2}$ acting along PB. While the force of attraction on the pole due to S pole will be $\frac{m}{SP^2}$ acting along PS As these two forces are acting inclined at angle BPS, the resultant can be found out by the law of parallelogram of forces.

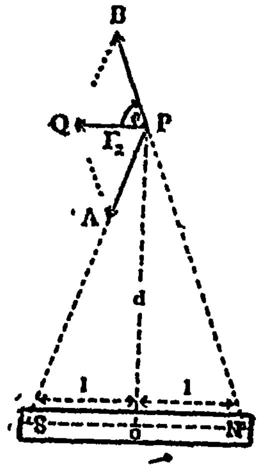


Fig. 53.

Let PB and PA respectively represent in magnitude and direction the two forces $\frac{m}{NP^2}$ and $\frac{m}{SP^2}$. Complete the parallelogram $PBQA$ Evidently the diagonal PQ will represent in magnitude and direction the resultant of the two forces Let it be F_2. Then F_2 will be the resultant magnetic intensity at the point P.

Suppose the resultant makes an angle θ with PB

Now from figure we have,

$\angle BPQ = \angle QPS = 2\theta$ [The diagonal bisects the angle in a parallelogram].

But. $\angle BPS = \angle PNS + \angle PSN = 2\theta$

and $\angle PNS = \angle PSN = \theta$

$\therefore \quad \angle BPQ = \angle PNS$

Thus F_2 will act parallel to the magnetic axis

Now $\triangle^s$ BPQ and PSN are similar

$$\therefore \quad \frac{PQ}{PB} = \frac{SN}{PN}$$

Now $PQ = F_2$, $PB = \frac{m}{NP^2}$, $PN^2 = l^2 + d^2$

and $SN = 2l$ substituting these values we get

$$\frac{F_2}{\frac{m}{(l^2+d^2)}} = \frac{2l}{(d^2+l^2)^{\frac{1}{2}}}$$

$$\text{or} \quad F_2 = \frac{2\,ml}{(d^2+l^2)^{\frac{3}{2}}} = \frac{M}{(d^2+l^2)^{\frac{3}{2}}} \text{ oersteds} \left[M = 2\,ml \right] \qquad \ldots(4)$$

If $d >> l$

$$F_2 = \frac{M}{d^3} \quad .. (5)$$

From (3) and (5) it is clear that at the same distance from O, the field in end on position is double the field in broadside—on position

§ 5 To determine the pole strength of the magnet by locating neutral points :—You already know the method of finding out the neutral points If S pole of the magnet faces north, two neutral points XX will be found on the magnetic axis produced as shown in fig. 34 Evidently these points lie in end—on position. Measure the distance between the neutral point and centre of the magnet. Let it be d. If F is the intensity of magnetic field at the neutral point

$$F = H = \frac{2\,M\,d}{(d^2-l^2)^2} \quad \left\{\begin{array}{l}\text{where } m \text{ is the magnetic moment and}\\ 2l \text{ the length of the magnet}\end{array}\right\}$$

From this equation if H, d and l are known M can be found out. From the value of M, the pole strength m can be calculated by the relation $M = 2\,ml$.

If N pole of the magnet faces north, the neutral points will lie on the equitorial line. Measure the distance between the neutral point and centre of the magnet let it be d Then we have

$$F = H = \frac{M}{(d^2 + l^2)^{\frac{3}{2}}} \quad \text{(with usual notations).}$$

From this relation M can be found out. From the value of M the pole strength m can be calculated.

Numerical problems :—*1. A magnet of magnetic moment 1000 C G S. units lies in the field of intensity 0·18 gauss. What couple will be required to keep it at an angle of 30° to the direction of the field.*

Let C be the required Couple then it is given by the relation

$$C = MH \sin\theta$$

In this problem $M = 1000$ C.G S. units

$H = 0{\cdot}18$ gauss

$\theta = 30°$

$C = 1000 \times 0\,18 \times \sin 30°$

$= 90$ C.G S. units

2 A bar magnet of 4 cm length is placed with its north pole pointing north The neutral points are found to be 20 cms. from the centre of the magnet calculate the strength of earth's horizontal field, if the pole strength of the magnet is 140 C. G. units

Let x be the position of the neutral point as shown in the diagram.

Then at x

$F = H$ [where H is the required strength of earth's horizontal field].

$$= \frac{M}{(d^2+l^2)3/2} \quad \text{(Field in broadsde on position).}$$

In this problems $M = 2\,ml = 2 \times 140 \times 2 = 560$ C.G.S. units

$$l = 2 \text{ cms.}$$

$$d = 20 \text{ cms.}$$

$$H = \frac{M}{d^3} \text{ as } d >> l.$$

$$= \frac{560}{02^3} = \frac{560}{20 \times 20 \times 20} = \cdot 07 \text{ oersteds.}$$

20^3

EXERCISE

1. Explain the term magnetic moment of a magnet. Find the general expression for the moment of the couple acting on a magnet placed in a uniform magnetic field. (see §2 and see §3

2. A freely suspend magnet of moment 980 C G.S units is deflected through 60° by a couple. Calculate the magnitude of the couple
(*Ans.* $490 \times \sqrt{3} \times H$ *dynes—cm*)

3 Obtain an expression for the intensity of [the magnetic field at a point on the pronlongation of the axis of the magnet. (see §4)

4 Find the strength of the [field due to a bar magnet at a point on the line bisecting the magnet at right angles (see §4)

5. A short bar magnet is placed in the magnetic meridian with its north pole pointing south The neutral point is 24 cm. north of the south pole of the magnet and upon its axis produced. Find the intensity of the field at a point an the axis 20 cms from the south pole and north of it. (H=0·18)
(*Ans* 0 131 oersteds)

6 Two magnets each of 8 cms. length and moment 10 G.G S. units lie in one straight line with their north poles 6 cms apart Find the *force of* repulsion between them. (*Ans.* 1 964 dynes)

7 Calculate the field due to a bar magnet 10 cm long and having a pole strength of 100 units at a point 20 cm. from each pole.
(*Ans.* 1·25 oersteds)

Section V

ELECTRICITY

CHAPTER I

ELECTRO STATIC PHENOMENON

§ 1. **Introduction** :—You must have often noted that when you draw rubber comb through dry hair crackling noise is produced. Apart from this noise, the rubber comb developes the property of attracting light bodies. This property was first of all discovered by a Greek philosopher Thales of Miletus, near about 650 B.C *He showed that amber (a kind of yellow resin) when rubbed with fur acquires the property of attracting light bodies, e g*, bits of paper, piece of straw etc Near about 1600 A D Dr. W Gilbert, the celebrated physician to queen Elizabeth made further discoveries in this line. He demonstrated that in addition to amber, there are other substances also which when rubbed with suitable substances develop this attractive property. For example substances like glass, ebonite, resin etc rubbed with silk, or flannel exhibit this type of property

The property so developed in the bodies is *called electricity or simply electric charge* The word 'electricity has been selected from electron, the Greek word for amber. The bodies which possess this property are *said to be electrified or charged* When the charge does not move in the body in which it has been produced, it is called static electricity (*i e* electricity at rest) In electrostatics you shall study the properties of electrified bodies on which the charge is static

§2. **Types of electrification** :—Suspend a glass rod *AB* by means of a silk thread (fig. 1). Rub it with silk so that it becomes charged Bring another glass rod *CD* also rubbed with silk near *AB*, You will find repulsion taking place between the two rods If instead of *CD*, you bring an ebonite rod rubbed with flannel near *AB*, attraction will take place between the two rods You can deduce two important conclusions from this experiment

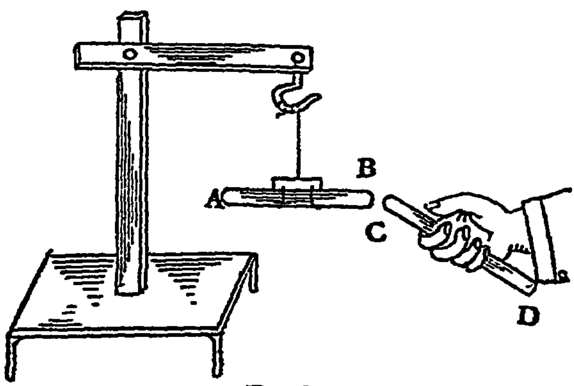


Fig 1

(1) **There are two types of charges produced on bodies when they are rubbed.** (*i*) *On glass when it is rubbed with silk* (*ii*) *on ebonite when it is rubbed with flannel* The kind of electricity produced on glass on rubbing with silk was named vitrous electricity derived from 'viterum' the Greak word for glass. While the electricity generated on ebonite was called resinous electricity. *These days the former is called positive and the latter is called negative*

Thus, *there are two types of electrifications or charges* :—(*i*) **Positive** (*ii*) **Negative.** This is true not only for glass or ebonite but it holds good for all substances. Whenever two substances are rubbed together, they acquire one or the other charge depending upon the nature of the bodies. Not only this, when you rub glass with silk, if glass acquires positive charge, the silk will simultaneously acquire negative charge. If after charging you put the glass rod and silk together, they will lose charge, proving that they were equally and oppositely charged.

(2) You have also seen in the above experiment that when the charges on the bodies are the same as in the case of glass rods, repulsion takes place. Where as if they are oppositely charged they attract (as in the case of glass and ebonite). *Thus, the law is that like charges repel and unlike charges attract each other.*

§3. **Repulsion is the surest test of electrification** :—As explained in Magnetism, Chapter I, two bodies will attract each other, when one of them either carries no charge or is oppositely charged. But if two bodies repel each other, they can do it only when they are similarly charged. *Thus repulsion is the surest test of electrification.*

§4. **Conductors and insulators** :—Hold a brass rod in your hand and rub it with fur. Test it whether it is charged or not You will find that tests do not show any charge on fur. From such experiments people thought that there are certain substances, prominently metals which could not be charged by friction, *i.e.*, by rubbing. But this line of thought was proved to be wrong by subsequent experiments If you charge the same rod of brass after mounting it on a glass handle it will indicate negative charge. So the question is where then the charge leaked in the previous case ? When brass is rubbed with fur it developes charge, but the charge flows to the earth through the hand by which it is held. Hence, the rod shows no charge. When the rod is mounted on a glass handle the charge cannot flow through the hand, and the charge remains on the rod. Thus, there are two types of substances with respect to the passage of electricity through them.

(*i*) *Substances which allow electricity to flow through them freely, e.g., metals, charcoal, acids, human body, earth etc. are called conductors.*

(*ii*) *Substances which do not allow electricity to pass through them are called insulators, e.g., ebonite, glass, sulphur, oil, shellac, quartz, paraffin wax etc.* Due to glass handle the charge could not leak in the above experiment Electric switches are made of plastic on account of latter being a non-conductor of electricity.

In addition to these there are a few substances which are neither conductors nor insulators. They are called partial Conductors *e.g.*, wood, paper, marble, cotton, etc.

§5. **Nature of the charge produced** :—To determine the nature of charge produced, suspend at a fairly large distance two rods, one of glass *AB* rubbed with silk, and the other *CD* rubbed with flannel, by the help of silk threads. Bring the charged bodies near them. If they are repelled by *AB* they are positively charged. Whereas if they are repelled by *CD* they are negatively charged.

As the nature of the charge depends upon the two bodies rubbed, a list has been prepared in which substance are so arranged in order that if you rub two substances from the list, *the one situated earlier in the list will acquire positive charge, rendering the other negatively charged* The following is the list —

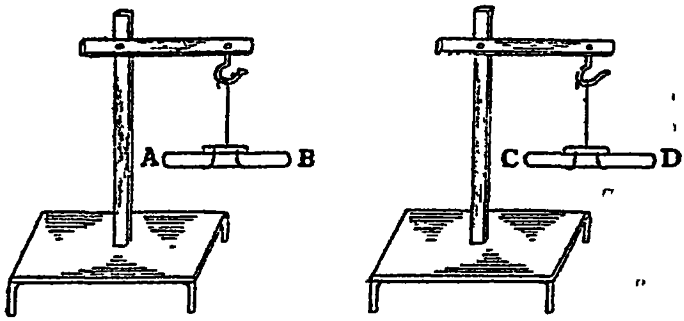


Fig. 2

Fur	Glass	Metals	Ebonite
Flannel	Paper	Resin	Gutta parcha
Shellac	Silk	Amber	
Sealing wax	Wood	Sulphur	

§6. **Sharing of charge by conductors :**—*A* is a brass rod

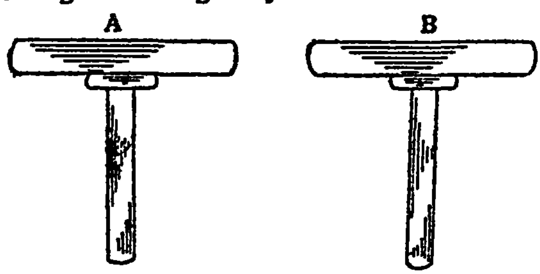


Fig 3

mounted on a glass handle Charge it negatively by rubbing it with fur. Take another metal rod *B* also mounted on a glass handle and bring it in contact with *A* The charge will flow from *A* to *B*, making *B* also negatively charged Thus when brought in contact both will share the same charge This is known as charging by conduction.

§7. **Gold leaf electroscope :**—This is an apparatus widely used in electrostatics for detecting different charges. It consists of a bell jar of glass resting on a wooden base Its mouth is tightly closed by a rubber or an ebonite stopper *GH*. Through this stopper passes a rod At its upper end the rod carries a metal disc *D* and knob *K* At its lower end two thin golden leaves *LL* are attached to it At the base of the instrument two tin foils *PQ* are attached rising from bottom to the middle of the jar (parallel to the side of the jar). The base is earthed by a wire The electroscope can be used for the following purposes :—

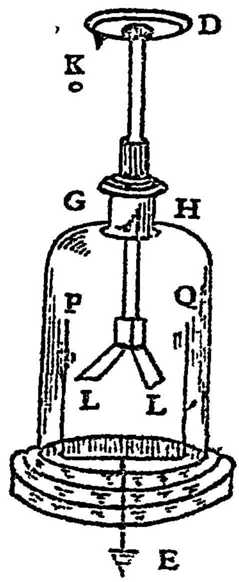


Fig 4.

(1) *To test whether a body is charged or not?*—In the beginning under their own weight the leaves will remain collapsed Now bring the body under test near the disc of the electroscope and touch it with K. If the leaves do not diverge the body carries no charge. If the body carries charge leaves will diverge. As both of the leaves acquire the same charge, they tend to repel and divergence takes place.

(2) *To find out the nature of the charge*:—Take a brass-rod mounted on a glass handle. Charge it negatively by rubbing with fur. Bring the rod near the disc D and touch it with the knob K The charge will flow to the leaves through the rod AB, making the leaves negatively charged It will result in the divergence of the leaves. Remove the rod, the leaves still remain diverged. Hence the electroscope has become negatively charged Now bring the body under consideration in contact with the disc D. *If the divergence increases the body is negatively charged. On the other hand if the divergence decreases or the leaves collapse the body is either positively charged or uncharged.* To test this touch the knob K with the finger, so that the electroscope is discharged (the charge flows to the earth), and the leaves collapse. Now bring the body in contact with the knob. *If the leaves diverge the body is positively charged If there is no divergence the body carries no charge.*

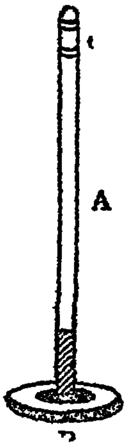


Fig. 5.

§8. Proof plane:—To test the charge on the bodies they should not as a whole be brought near the disc of the electroscope But a proof plane should be employed for this purpose It is nothing but a metal disc B mounted on an insulated handle A First of all take the proof plane and touch its disc with a body whose charge is to be detected. It will also develop the same charge as is on the body Now test for the charge on the proof plane by bringing it near the knob of the electroscope as explained in §7. It simply acts as a carrier of the charge It is easy to move a proof plane as compared to the body itself.

§9. Electrostatic induction:—Mount an uncharged conductor PQ on an insulated stand Charge a glass rod AB positively by rubbing it with silk Bring the charged conductor AB near to the uncharged and insulated conductor PQ (Fig 6). Under the influence of the charge on AB, charges will develop at the two ends P and Q. To test this bring the disc of the proof plane in contact with the end P. Now bring the disc of the proof plane in contact with the knob of an electroscope. The leaves will diverge showing that the plane and hence the end P is charged Similarly the other end Q will also show the presence of a charge Now if you remove the rod AB, PQ will be discharged. This can also be tested with the help of a proof plane.

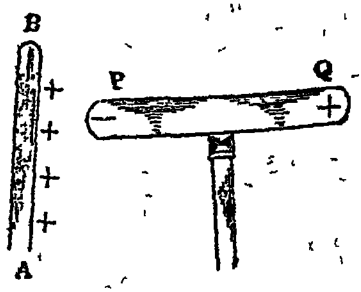


Fig 6.

The above phenomenon in which under the influence of a charged body, charges are developed at the two ends of a conductor is called **electrostatic induction.** The charge on AB is called *inducing charge* The charges produced on PQ on account of induction are called *induced charges.* Induction usually means influencing from a distance

§10. **Nature of the charges produced :**—Repeat the above experiment Touch the disc of the proof plane with the end P, and put it in contact with the knob of a negatively charged electroscope The divergence of the leaves will increase It means that the end P acquires negative charge. Discharge the proof plane by touching its disc with the hand, and put it now in contact with the other end Q Now put the disc of the proof plane in contact with the knob of a positively charged electroscope Again the divergence will increase showing that the end Q is positively charged On removing the rod AB, it will be found that the charges on PQ disappear It is possible only when the positive charge developed at the end P is equal to the negative charge developed at the end Q Thus, the following conclusions can be derived from this simple experiment —

(1) *On account of electrostatic induction charges are produced at the two ends of a conductor simultaneously*

(2) *The nearer end* (*i. e*, P) *becomes oppositely charged whereas the remoter end* (*i e*, Q) *becomes similarly charged.*

(3) *The two induced positive and negative charges are equal in magnitude.*

§11. **Charging a gold leaf electroscope by induction :**—To charge it positively bring on ebonite rod A charged negatively near

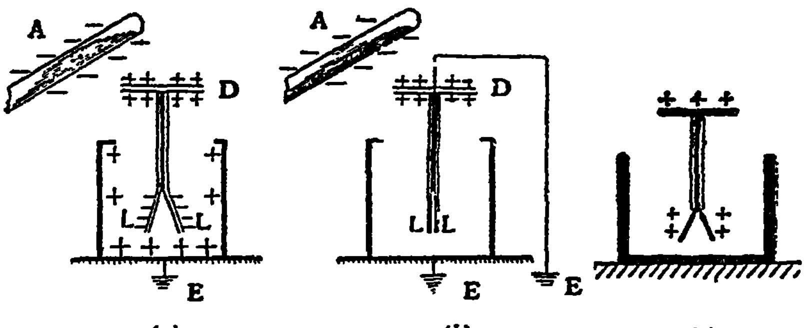


Fig 7.

its disc as shown in fig 7 (*a*) The rod will induce positive charge on the nearer disc D and the negative charge on the farther leaves LL The leaves will diverge on account of the similar charges developed on them Leaves in turn will induce positive charge on inside of the tin foils and negative charge on the outside of the tin foils As the foils are earthed the negative charge will flow to the earth, and the positive charge on insides of the tin foil will help in increasing the divergence of the leaves

Touch the disc D momentarily by your finger without removing the rod AB The negative charge on the leaves will leak to the earth as it is free Consequently they will collapse as in fig 7 (*b*) As inducing charge has vanished the tin foils will also lose their

charge. But on account of the presence of the rod AB the bound positive charge on the disc remains.

Now remove the rod. The positive charge will spread on the rod and leaves. The leaves will again show divergence. The tin foils now acquire negative charge due to induction. Thus the electroscope has been positively charged. Similarly it can be charged negatively.

§12. Seat of the charge on a conductor :—A is a sphere of brass mounted on an insulated handle. Charge it negatively by

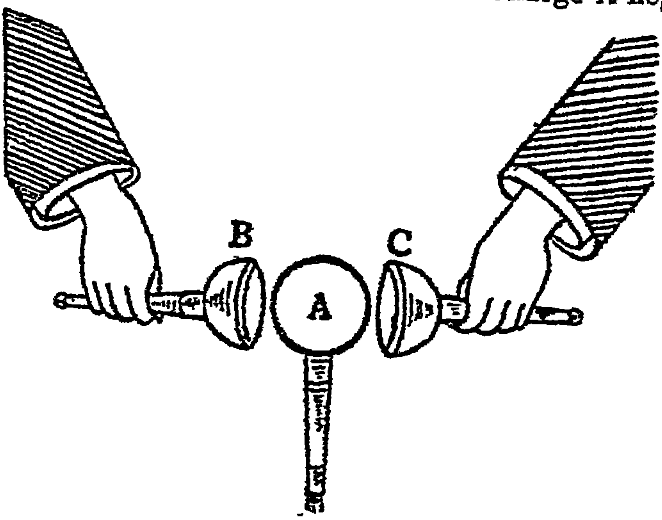


Fig. 8.

rubbing it with fur Test this charge by the help of an electroscope. Now take two hemispherical caps of brass B and C mounted on glass handles. These brass caps are so made that when they are placed on the sphere they tightly fit it. Put them on the sphere so that they completely enclose the sphere Now separate them, and test the three for charges by a gold leaf electroscope. You will find that the sphere A will not show any charge, whereas the two caps will show all charge. This experiment was first of all performed by Biot and *is known as Biot's, experiment*. It clearly shows that the charge goes to the outer surface and does not remain inside the body. The same results are obtained with a hollow conductor also.

Thus in case of solids as well as hollow conductors the charges reside only on their outer surfaces

§13. Faraday's butterfly net experiment :—Take a conical net made of muslin and mount it on an insulated stand. It has two silk threads attached to it, by which it can be turned up side down. Charge the net. and test the outer and inner surface by the help of a proof plane and an electroscope You will find that only outer surface shows charge. Now turn the net up side down Again the outer surface alone will indicate charge. *This ex-*

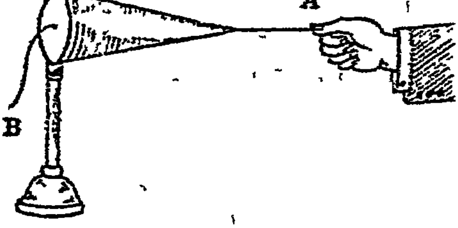


Fig. 9.

periment also illustrates the fact that the charge resides on the outer surface.

§14. **Faraday's ice pail experiment** :—In this experiment Faraday took a tin vessel B of fairly large size, open at the upper end, mounted on an insulated stand. The outer surface of the vessel was connected to the disc of a gold leaf electroscope by a wire A body A was charged positively. It was suspended by means of a silk thread and lowered into the can as shown in fig 10. The can completely coveres the body. As it was lowered in the can, negative charge was induced on the inner surface of the vessel and positive charge on the outer surface, the disc and the leaves On account of this positive charge the leaves diverged. When the body A was removed the leaves collapsed It is possible only when the two induced charges produced on the two sides of the vessel are equal and opposite *Thus, it was proved that the two induced charges produced are equal and opposite.*

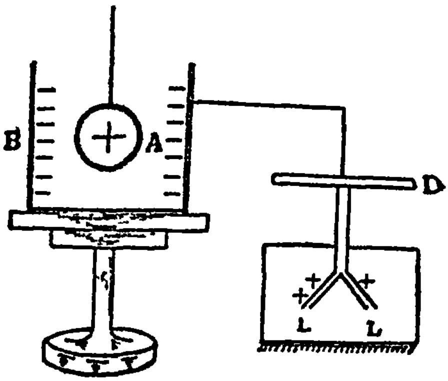


Fig. 10.

Now the body A was again lowered in the vessel. Again the leaves diverged due to positive charge Now the inner side of the vessel was touched by the body momentarily. This did not affect the divergence of the leaves. Neither the divergence increased nor decreased. On the other hand when the body A was put to test it was found to be completely discharged This was possible only when the charge on the body was equal and opposite to the charge on the inner surface of the vessel, so that when they touched each other, their charges were neutralised and the charge on the leaves remained uneffected

Thus, the inducing charge and the two induced charges are each equal to one another This is rigourously true only when the inducing conductor is completely-surrounded by the induced conductor

§15. **Charge density** :—When a conductor is charged the charge is distributed over whole of its surface, but the distribution is not uniform all over the surface. It depends upon the following factors :—

(1) **The shape of the conductor.** If the curvature of that body

(a) (b) (c)

Fig 11

at that point is *greater, greater will be the accumulation of charge at*

that point. As *shown in fig. 11* the more is the end pointed the more is the accumulation of charge there. In fig. 11 (*c*) as the end is extremely pointed, the accumulation of charge at that point is the greatest.

(2) **Neighbourhood of other Conductors :—**If there are other conductors placed in the vicinity they also influence the distribution of charge over a surface.

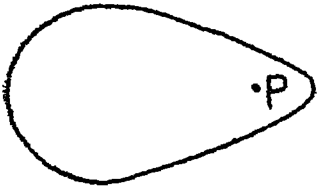


Fig 12.

Charge density :—*It is defined at any point on the surface of a conductor as the charge per unit area surrounding that point.* If Q is the charge and A is the area surrounding the point P.

Charge density$=Q/A$. If it is a sphere of radius r, then charge density on its surface due to the charge $Q=Q/4\pi r^2$.

§16. **Action of points :—**When *a* charged conductor possesses sharp pointed ends, *charge accumulates at these sharp points*, The density of the charge is so great that the charges develop the tendency of repelling each other. Therefore the charge leaks from these ends and charge the atoms of the medium around the conductor by conduction. The atoms of the medium are imparted the same charge as that of the conductor. Consequently they experience a force of repulsion, and are moved away from the conductor. Fresh atoms fill up their place and are similarly repelled. In this way the medium is continuously charged and the *charge leaks from the pointed ends.* This principle has been utilised in constructing the lightning conductor.

§17. **Lightning Conductor :—**It was first of all suggested by Benjamin Franklin in the year 1749. It is used to protect buildings from destruction by the atmospheric lightning. Its construction is extremely simple. It is in the form of a long rod or a strip of metal (copper) running from the top of a building to its bottom. The upper end of the strip is provided with sharp pointed ends The lower end is fixed to a plate buried in the wet earth.

When the charged clouds pass over the conductor they induce opposite charge on the conductor. The charge gets accumulated at the sharp pointed ends As explained above this charge leaks through the ends and charges the atoms of the medium surrounding the pointed ends. The atoms receive the same charge as that on the conductor. *Therefore repulsion takes place and the charged atoms of the medium travel towards the cloud discharging the latter.* Hence the danger of lightning is averted. Apart from this if the potential difference between the clouds and the conductor is very large and discharge occurs, the lightning conductor *provides a straight and easy path for the discharge.* Thus, the charge from the cloud quietly flows to the earth along this conductor without damaging the buildings All most all the big buildings are provided with lightning conductors.

QUESTIONS

1. What do you mean by the statement that the body is electrically charged? (See § 1)

2. How would you prove that positive and negative electrifications are produced in equal quantities? (See § 2)

3. Why repulsion is the surest test of electrification? (See § 3)

4. Describe the construction and use of a gold leaf electroscope. (See § 7)

5. Explain what is meant by electrostatic induction. (See § 9)

6. Describe a gold leaf electroscope and explain how it can be charged by induction and conduction. (See § 11)

7. Describe an experiment to show that the induced charge equals the inducing charge. (See § 14)

8. Show that the charge on an insulated conductor lies entirely on its surface. (See § 12)

9. What is meant by 'the surface density at a point'? How does it depend upon the shape of the conductor? (See § 15)

10. Explain the discharging action of points and hence describe a lightning conductor. (See § 16 & 17)

11. What charge is required to electrify a sphere of 25 cm radius until the surface density of electrification is $5/2\pi$. (*Ans.* 2500 units of charge)

CHAPTER II

INVERSE SQUARE LAW

§1. **Inverse square law** :—It was discovered by Coulomb. It can be put in two forms :

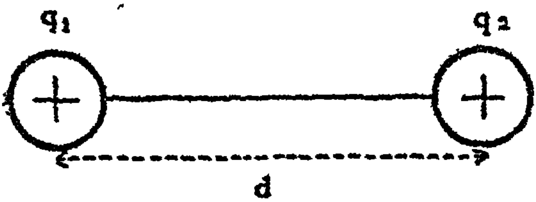


Fig. 13.

(i) *The force of attraction or repulsion between two charges is inversely proportional to the square of the distance between them.* Mathematically if F and d represents the force and distance between the two charges q_1 and q_2

$$F \propto 1/d^2 \quad \ldots (1)$$

As the distance increases F will decrease If d is doubled F will be reduced to one fourth

(ii) *The force is directly proportional to the product of the two charges* Mathematically

Then, $$F \propto q_1 q_2 \quad \ldots (2)$$

If the two laws are combined, from (1) and (2), we get

$$F \propto \frac{q_1 q_2}{d^2}$$

or $$F = \frac{1}{k} \frac{q_1 q_2}{d^2} \quad \ldots (3)$$

where k is a constant depending upon the nature of the medium between the two charges. It is called **di-electric constant or specific inductive capacity of the medium.** For air $k=1$. Thus relation (3) becomes

$$F = \frac{q_1 q_2}{d^2} \quad \ldots (4)$$

This law is known as **Coulomb's law.**

§2. **Electrostatic unit charge** :—Let q and q be two similar charges placed at a distanc of d cms in air. Then by Coulombls law the force of repulsion F between them will be

$$F = \frac{qq}{d^2} = q^2/d^2$$

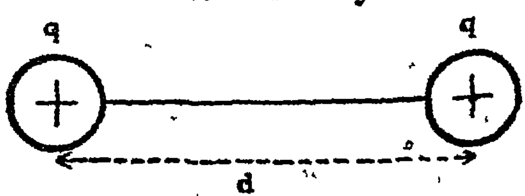


Fig. 14.

If we put $d=1$ cm
and $F=1$ dyne
$q^2=1,$
or $q=\pm 1$, *i e*, the charge will be of unit strength.

Thus electrostatic unit of charge can be defined as that charge, which when placed at a unit distance from a similar charge repels the latter with a force of one dyne.

§3. **Electric field** :—The space around a charge in which its influence is exerted is called its *electric field.*

§4. Intensity of electric field :—Suppose q is a charge and P is any point in its electric field If you place a unit positive charge at P it will experience a force of repulsion This force experienced in dynes by a unit positive charge placed at that point is defined as the intensity of electric field at that point It is assumed that the unit charge has no field of its own The direction of this field is given by the direction of the force experienced by the unit +ve charge. If F is the intensity, then by Coulomb's law, we have

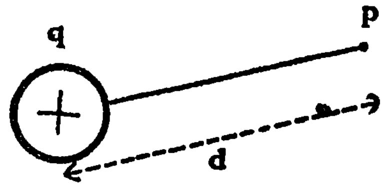


Fig 15

$$F = q/d^2 \quad . \quad (5)$$

§5. Electrostatic potential :—You know it very well that water flows from a higher level to a lower level As for example take two vessels A and B containing water at different levels Join them by means of a tube Water will flow from A to B till the level is the same in both the vessels Similarly take two bodies at different temperatures and join them. Heat will flow from the body at a higher temperature to the body at lower temperature

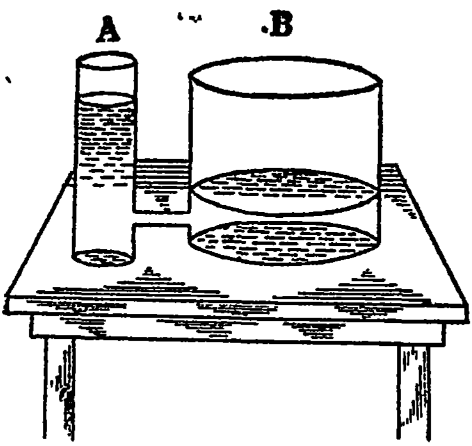


Fig 16 (*a*)

The same analogies can be extended to electrostatics also Suppose P and Q are two insulated metallic charged spheres. If you join them by a wire charge will flow from one to the other. If it flows from P to Q, P is said to be at a higher potential than Q *Thus potential is analogous to level in hydrostatics and temperature in heat.* Charges flow from a body at higher potential to a body at lower potential till both the bodies acquire the same potential. In thermometry temperatures are measured with reference to the ice point as the standard temperature The level of the sea is taken to be standard in hydrostatics. Similarly in electrostatics the potential of the earth is regarded as constant It is regarded as the standard of potential and *taken to be equal to zero,* Now connect the charged conductor to the earth, if it sends charge to the earth it is regarded

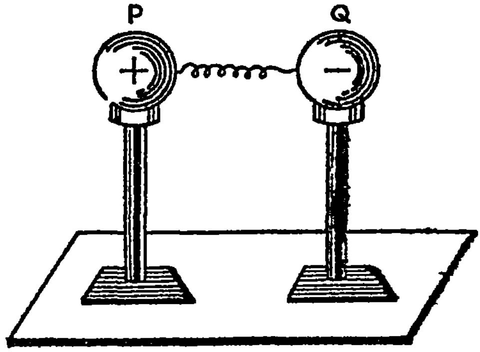


Fig 16 (*b*)

to be positively charged. If it begins to receive electricity from the earth it is regarded to be negatively charged.

Thus, if a charged body is connected to the earth by a conductor, *the potential of a body can be defined as its electrical condition which determines, whether the charge will flow from the body to the earth or vice versa.*

§6. **Measure of potential :—***The potential at a point is measured by the amount of work done in bringing a unit positive charge from infinity to that point.*

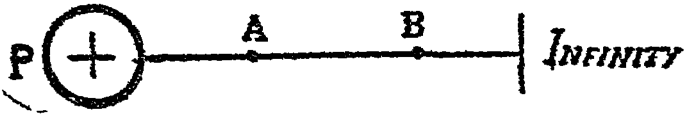


Fig. 17.

Let a positive charge be placed at the point P. It will have its electric field around it. If you bring a unit positive charge from infinity to some point B, you shall have to do work against the force of repulsion between the charge at P and the unit positive charge. Let it be equal to w_1 ergs. Then w_1 will be the potential at the point B due to the charge at P. If you want to bring the unit positive charge to some nearer point A, you shall have to do more work. Let it be w_2 ergs. Then the potential at the point A will be w_2 e s units. Hence the potential difference between the points A and B will be equal to (w_2-w_1) e s. units. *Thus, the potential difference between two points will be equal to the work done in bringing a unit positive charge from one point to the other.*

Therefore, as you move away from the charge its potential will go on decreasing, becoming *zero at infinity.*

If at P instead of a positive charge you place a negative charge, the unit positive charge at infinity will move towards this charge due to attraction. Hence no work is done Rather the charge does some work, and so the *potentials at A and B will be negative.*

§7. **Unit potential :—**In the above experiment if *the work done in bringing a unit positive charge from infinity to any point is unity, the potential at that point will be one electrostatic unit.*

§8. **Potential at a point :—**Let a charge of $+Q$ units be placed at the point O. Let A be the point at a distance of d cms.

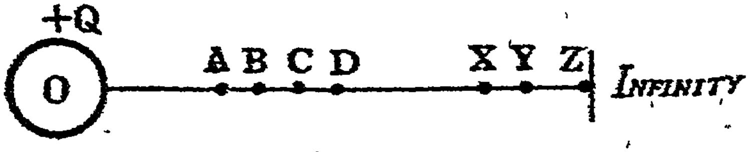


Fig. 18.

from it placed in its electric field. We want to find out the potential at the point A due to this charge. According to the definition the potential will be equal to the work done in bringing a unit positive charge from infinity to this point.

To calculate it, join O to A by a straight line and produce it almost to infinity. Imagine the point Z to be lying at infinity.

Divide the distance AZ into a number of equal parts by the points $B, C, D,$. X, Y Obviously the work done in bringing a unit positive charge from Z to A will be equal to the algebric sum of the quantities of work done in moving the charge through short distances as Z to y ; y to x C to B and B to A.

The intensity of the electric field due to this charge at the point A is Q/OA^2, and at B is Q/OB^2 (*See* §3). These are the forces acting on a unit positive charge at these points Evidently the average force F between A and B will be more than Q/OA^2 and less than Q/OB^2 The average can be determined by taking the geometric mean of the two [when AB is small].

$$\therefore \quad F=\sqrt{\frac{Q}{OA^2}\;\frac{Q}{OB^2}}=\frac{Q}{OA\;\;OB} \qquad \text{... (6)}$$

Since the work done is equal to force multiplied by the distance through which the force is moved, the work done (W) in moving a unit positive charge from the point B to the point A will be equal to $F\times AB$

$$\therefore \quad W=F\times AB$$

$$=\frac{Q}{OA\;OB}\,.\,AB$$

$$=\frac{Q(OB-OA)}{OA.OB}$$

$$=\frac{Q}{OA}-\frac{Q}{OB} \qquad .\ (7)$$

Similarly the work done from C to B, *i e*, for the distance BC

$$=\frac{Q}{OB}-\frac{Q}{OC} \qquad ..\ (8)$$

For the distance ZY it will be

$$=\frac{Q}{OY}-\frac{Q}{OZ} \qquad ..\ (9)$$

and for the distance YX it will be

$$=\frac{Q}{OX}-\frac{Q}{OY} \qquad ...(10)$$

Thus the work done between the points Z and $Y=Q/OY-Q/OZ$

„ , „ „ Y and $X=Q/OX-Q/OY$

.. . .

„ „ „ „ C and $B=Q/OB-Q/OC$

„ „ „ „ B aad $A=Q/OA-Q/OB$

By adding all these small amounts of work done, we get the total amount of work $=\dfrac{Q}{OA}-\dfrac{Q}{OZ}$, rest of the terms getting cancelled This is the amount of work done in bringing a unit positive charge from Z to A, in the electric field due to the charge Q As Z has been supposed to be situated at infinity, by definition this

work is equal to the potential at the point A due to charge Q. If V is the potential

$$V = \frac{Q}{OA} - \frac{Q}{OZ}$$

As $\qquad OZ = \infty$

$\therefore \qquad \frac{Q}{OZ} = 0$

or $\qquad V = \frac{Q}{OA}$

$$= \frac{Q}{d} \qquad (ii)$$

Thus, the potential at a point due to a charge is equal to the charge divided by the distance between the charge and the point.

It is clear from the equation above that as the distance increases the potential tends to fall, becoming zero at infinity

§ 9. **Potential inside a hollow sphere** :—No charge resides inside a hollow conductor, and therefore no work is done in moving a charge inside it. Therefore, potential everywhere inside it is constant The value of the potential inside the conductor is equal to the potential of its surface For the determination of the potentials at external points the charge residing outside may be regarded as situated at the centre.

§ 10. **Solved prblems** :—1 *Two equally charged spheres repel each other when their centres are half a metre apart with a force equal to the weight of 6 milli. gm. What is the charge on each in electrostatic units ?*

We know that $\quad F = \frac{q_1 q_2}{d^2}$, Here $F = \cdot 006 \times 980$ dynes

$\therefore \qquad F = \frac{q^2}{d^2}$

$\therefore \qquad \cdot 006 \times 980 = \frac{q^2}{50^2} \; [q_1 = q_2]$

$\therefore \qquad q = 121\,3$ C G.S. units

2. *A hollow spherical conductor whose radius is 13 cm is charged with 10 units of electricity find the potential (a) at the surface of the sphere (b) inside it and (c) at a point 25 cm. from the centre*

For all calculation purposes the charge can be regarded to be situated at the centre of the sphere. The potential inside the sphere is equal to the potential on its surfce. p d. is given by $V = \frac{Q}{d}$; $Q = 10$ units, $d = 10$ cm

$\therefore$ Potential on the surface $= \frac{10}{10} = 1$ e s unit

$\therefore$ Potential inside the sphere and on the surface of the sphere $= 1$ e s. unit.

Potential at a point 25 cm from the centre.

$$= \frac{10}{25} = 0 \cdot 4 \text{ e s unit.}$$

QUESTIONS

1. Discuss Coulomb's Law of the forces between electric charges and define a unit electrostatic charge (see § 1 and 2)

2. What do you understand by (a) electric field and (b) intensity of electric field at a point? (see § 3 and 4)

3. Show clearly what do you understand by the term 'electrostatic potential giving analogies? (see § 5)

4. Find the value of the potential at a point due to a single electric charge $+q$ at a distance d from it (see § 8)

NUMERICAL PROBLEMS

1. Two charged bodies carrying $+10$ and $+40$ e s u respectively are placed at 6 cm. apart Find the point midway between them where the intensity of the field is zero (*Ans* 2 cm from $+10$ e s.u)

2. A charge of 100 e.s u. is placed at a point What work will be required

   (a) to bring a unit positive charge from infinity to a distance 40 cm from it

   (b) to carry a unit positive charge once completely round it in a circle of 20 cm radius? Give reasons for your answers.

   (*Ans.* 2 5 ergs , no work)

3. Charges of 6, 12 and 24 units of electricity are placed at the three corners of a square Find what charge must be placed at the fourth corner in order that the potential at the centre of the square may be zero

   (*Ans.* -42 units)

CHAPTER III

ELECTRIC MACHINES

§ 1. **Introduction** :—*Mechanical devices* capable of producing continuous supply of electric charges are called *electric machines*. They are either *frictional machines* or *induction machines*. Induction machines are based on the principle of electrostatic induction. The important machines based on the latter principle are as follows :—

(1) **Wimshurst machine.**

(2) **Voss machine.**

(3) **Van de Graff generator.**

(4) **Electrophorus.**

Electrophorus is the simplest of all these machines, and has been described in this chapter.

§ 2. **Electrophorus** :—It is based upon the principle of electrostatic induction. It essentially consists of three parts. (*i*) A circular metal plate *D* mounted on an insulated handle. It is called the *collector because it collects the charges produced*. (*ii*) A circular disc *C* of ebonite or resin of a slightly bigger diameter than that of the former plate. [(see Fig 19 (*a*) and 19 (*b*)]. Its upper surface is generally rough so that when *D* is placed over it, the former touches the latter only at a few points. *It is called the cake.* (*iii*) A metal plate *S* generally of tin foil upon which rests the cake *It is called the sole.* The lower face of the sole is connected to the earth

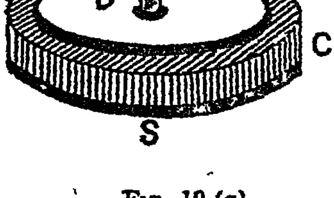


Fig. 19 (*a*).

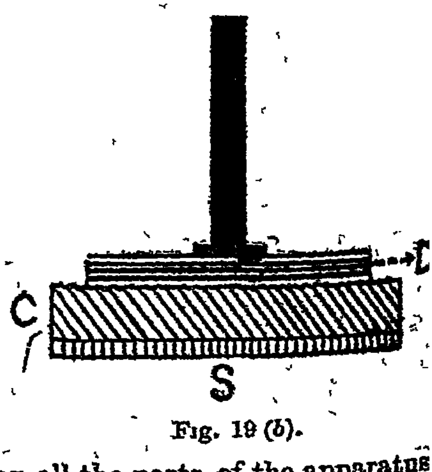
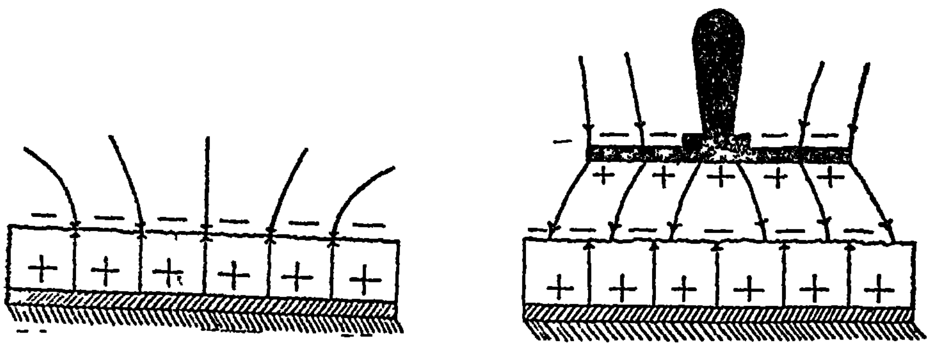


Fig. 19 (*b*).

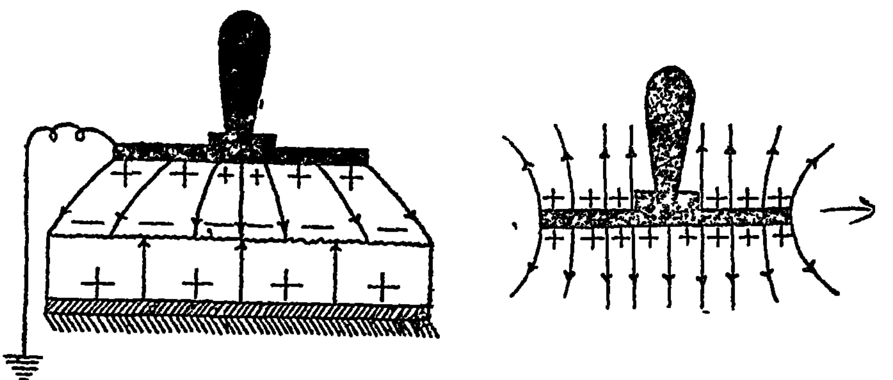
Working :—To start with, clean all the parts of the apparatus thoroughly. Rub the cake *C* with flannel, fur or catskin. It will develop negative charge on the cake. Put the collector-plate *D* on the cake. Earth the collector plate by momentarily touching by hand. Now remove the plate *D* and test. It will be found to have acquired a positive charge. Thus, in this way the plate has

been charged. This charge can be utilised in charging any other body. When the plate *D* gets discharged it is again put on the disc *C* and is again charged in the same way. Hence, for a considerable time a continuous supply of charge can be obtained without freshly charging the cake.

Principle :—When the cake is given the negative charge it induces charges on the surface of the sole The inner side acquires the positive charge The negative charge developed on the outer surface of the sole flows to the earth The remaing positive charge on the sole attracts the negative charge on the cake and does not allow the latter to leak. Hence the function of the cake *C* is to prevent leakage of charge from the cake *C* [See fig. 20 (*a*)].



(*a*) Fig. 20. (*b*)



(*c*) Fig. 20. (*d*)

When the collector plate is placed on the cake, it also developes charges on its two surfaces due to induction. As the plate touches the cake at only a few points there is a layer of air in between them. Therefore there is very little conduction between the two plates. The charging is due to induction only. The inner surface of the plate *D* acquires positive charge, while its outer surface becomes negatively charged [See fig 20 (*b*)] When the plate is momentarily earthed [See fig. 20 (*c*) the negative charge leaks to the earth. But under the influence of the cake the positive charge remains on the plate When the plate is removed away from the cake the

charge spreads over the whole surface of the collector plate D, which has thus become charged [See fig. 20 (d)]. This can be made use of in charging other bodies. As already described, the process can be repeated a number of times. To obtain the negatively charged plate the cake should be given a positive charge.

QUESTIONS

Describe giving diagrams the action and working of an electrophorous Is it an electric machine why? [See § 2]

CHAPTER IV

PRIMARY CELLS

§1. **Requirement for an electric current :**—Take two insulated spherical conductors P and Q Give positive charge to P and

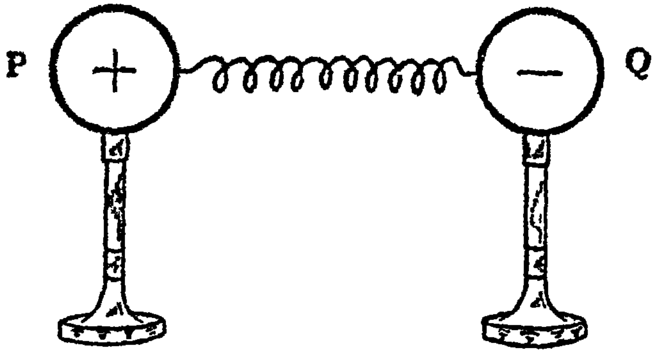


Fig 21.

negative charge to Q If you connect the spheres by a wire as shown in fig 21 charge will flow from the sphere P which is at a higher potential to the sphere Q which is at a lower potential The flow of the charge would be instantaneous *When both of the spheres attain the same potential the flow will stop* This flow of charge from a body at a higher potential to a body at lower potential *constitutes an electric current* Though actually electrons travel from Q to P conventionally electric current is supposed to flow from P to Q, *i e*, the direction along which the positive charge flows. From this experiment it is quite clear that the current will flow only when there exists a potential difference between two bodies When P and Q attain the same potential, the current stops So if you want to maintain a constant flow of current between PQ, P should always be maintained at a higher potential than Q, or in other words there should always exist a potential difference between P and Q This can be achieved by the help of simple voltaic cells, and secondary cells in which chemical action takes place or dynamos in which electro-magnetic induction takes place First of all we shall consider primary cells which are simple in construction and easy in use

§2 **Voltaic Cells :**—*It was* Volta *who discovered in the year* 1800 A.D *that if you join two similar conductors there will develop* a potential difference between the two, at the point of contact The two conductors will become oppositely charged The two conductors can be metals, liquids or a metal and a liquid.

A simple cell consists of a glass vessel A containing dilute solution of sulphuric acid. Two rods, B of copper and C of Zn are partially dipped in the solution There exists a number of positively charged ions of Hydrogen and negatively charged ions of

SO_4 in the dilute solution of sulphuric acid When the rods of copp and zinc are dipped in the solution, *the positive ions of hydrog move toward the copper plate. The negative ions of SO_4 travel towards the zinc plate.* This is on account of *chemical affinity* existing between the ions and the plates Ions are not neutral atoms but parts of atoms carrying either a positive or a negative charge. The agency which does the work of moving these ions within the cell **is called electromotive force.** This comes into existence on account of *chemical reactions* Thus, within the cell the positive charge moves from the zinc to copper plate giving rise to an electric current As explained above this current which flows from zinc to copper within the cell is due to the electromotive force It remains constant so far as the solution and the plates are the same.

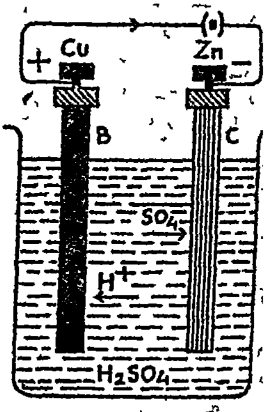


Fig 22.

Due to the movement of the positive charge towards the copper plate, its potential increases above that of the zinc plate. Hence a potential difference is set up between the two plates. This potential difference so developed causes the current to flow from copper to zinc Hence, the potential difference acts in a *direction opposite to that of electromotive force* So the potential difference goes on increasing till it becomes equal to the e.m f. When the two are equal the current stops within the cell *Hence, in the open circuit the potential difference across the two plates is equal to the electromotive force* When the circuit is closed current begins to flow in the outer circuit from copper to zinc *This results in the lowering of the potential difference Again the e m f starts acting maintaining the p. d across the plates.* **Remember** *that it is the electromotive force which is responsible for the development of potential difference.* The e m f. of a such a cell is equal to 1 04 volts

§ 3. **Defects of a simple cell :**—Simple cells described above suffer from the following defects .—

(1) **Polarisation.** You have seen that hydrogen ions charged with positive electricity travel towards the copper plate. They raise the potential of the plate and neutral hydrogen gas escapes into the atmosphere But all the molecules of the gas do not escape, and a few of them form a neutral layer of hydrogen gas arround the copper plate This layer prevents the fresh ions to go towards the positive plate. *Thus, the potential of the copper plate falls and the current ultimately stops. This defect is known* **as polarisation.** This can be explained in two ways ·

(*a*) The layer of neutral gas formed around the positive plate offers a great resistance to the current with in the cell. As the thickness of the layer increases the resistance also increases. Ultimately it becomes so high that the current totally stops flowing within the cell

(*b*) As the fresh incoming hydrogen ions carrying positive charge cannot reach the copper plate, they hand over their charges to the neutral layer. Thus, an electric field is set up between the

layer of hydrogen and zinc plate. This is called *back electromotive force* It tends to send current in the opposite direction If it becomes quite high it completely impedes the motion of hydrogen ions towards the copper plate. It results in the stoppage of the current within the cell.

Polarisation can be removed by preventing the formation of hydrogen layer around the plate This can be achieved by oxidising hydrogen, as soon as it is formed Different oxidising agents have been employed by different persons in different cells *The chemicals which are used to remove hydrogen in these cells are called* **depolarisers**

(2) **Local action :**—Pure zinc does not react with H_2SO_4 unless a contact is establised between zinc and copper Certain impurities, *e g*, carbon, arsenic, iron, lead etc. are always present in the ordinary zinc used in making zinc plates These impurities act with acid forming miniature cells consising of impurity, acid and zinc. *These miniature cells so formed causes local currents to flow on the zinc rod* This unnecessarily consumes the zinc rod because the local currents so formed do not contribute to the main current. *This is a sheer wastage of zinc and is known as* **local action.**

This defect can be remedied by *coating the zinc rods with a mercury layer* This process is known as *amalgamation of zinc* Zinc dissolves in mercury and comes on the surface layer, while others remain inside the mercury coating Thus the contact between the two is broken Thus the local action stopes

§4. **Cells** ·—Following are a few important primary cells. Each one of them has got a different polariser and a different electrolyte

§5 **Leclanche Cell** —

Construction —It consists of a glass vessel containing a solution of ammonium chloride A porus pot is placed in the middle of the vessel A carbon rod is placed at the centre of the pot Powdered MnO_2 mixed with pieces of carbon is packed around the rod in the porous pot A zinc rod is immersed in the solution Carbon and zinc respectively form the positive and negative plate of the cell The electrolyte is NH_4Cl

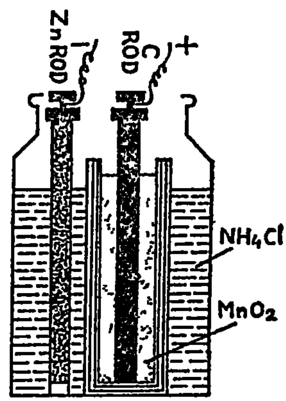


Fig 23

Working —Zinc acts on ammonium chloride forming $ZnCl_2$ and positively charged ions of hydrogen

$$2Zn+2NH_4Cl=2ZnCl_2+2NH_3+2H^+$$

The ions penetrate the porous pot and carry the charge to the carbon rod The potential of the carbon rod increases MnO_2 acts as a depolariser It acts on hydrogen forming water

$$2MnO_2+H_2=Mn_2O_3+H_2O$$

Being a solid it is a *weak oxidising agent.* Therefore, the cell gets polarised after a little use But if some rest is given to the cell the deposited hydrogen will be converted into water and the cell will again work. Thus, this type of cell is suitable only for

those places where *intermittent current is needed* e.g, in telegraphs, telephones electric bells etc It is quite cheap and sturdy. Its emf is 1 45 volts. The local action is eliminated by amalgamating the zinc rod

§6 Daniell Cell :—

Construction :—It consists of a copper vessel filled up with concentrated solution of copper sulphate The vessel acts as the positive plate In the middle of the vessel is placed a porous pot containing dilute sulphuric acid and an amalgamated zinc rod. The zinc rod forms the negative plate

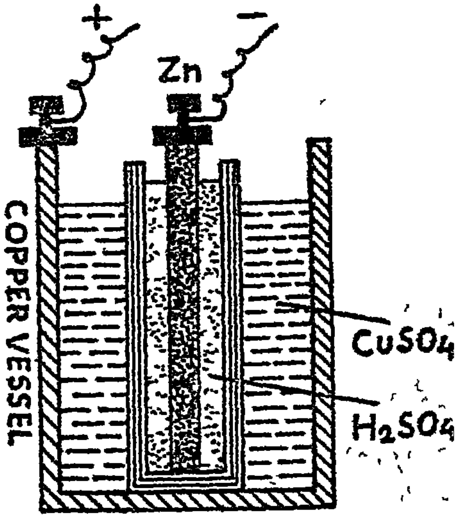


Fig. 24.

Working :— Zinc ions which are positive go into the solution reacting with H_2SO_4 forming $ZnSO_4$ Positive ions of hydrogen are liberated in this reaction, and the potential of the zinc plate is lowered

$$Zn^{++}+H_2SO_4=ZnSO_4+2H^+$$

The hydrogen ions so produced travel towards the copper vessel reacting with $CuSO_4$. **Copper Sulphate Solution acts as a depolariser.**

$$CuSO_4+2H^+=H_2SO_4+Cu^{++}$$

Neutral solution of sulphuric acid is formed and positively charged ions of copper are liberated. The ions travel towards the copper vessel. They give charge to the vessel, and are deposited there Thus, the potential of the copper vessel increases. The e. m. f. of this cell is 1 1 volt The liquid depolariser used is better than the solid depolarisers Therefore, the cell is almost free from polarisation. Local action is eliminated by amalgamating the zinc rod This type of cell can be used where steady current is to be drawn.

§7. Bunsen's Cell :—

Construction :—It consists of a porous pot filled with concentrated solution of nitric acid. A carbon rod is placed at the centre of the pot. The rod act as the positive plate of the cell. The porous pot is placed in a larger procelain vessel containing dilute solution of sulpharic acid. An amalgamated zinc cylinder is placed between the vessel and the porous pot. It remains immersed in H_2SO_4, and acts as the negative plate

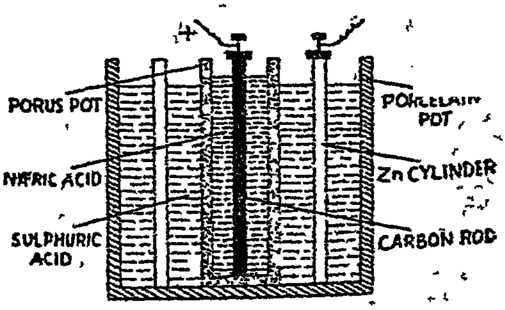


Fig. 25

Working. The reactions take place in H_2SO_4 while HNO_3 acts as a *depolariser*.

$$Zn^{++} + H_2SO_4 = ZnSO_4 + 2H^{++}$$

The positive ions of zinc react with H_2SO_4 forming $ZnSO_4$ and positive ions of H_2. These ions travel through the porous pot acting with HNO_3

$$HNO_3 + 2H^{++} = 2H_2O + 2NO^{++}$$

Molecules of NO_2 carry the positive charge to the carbon rod, and the potential of the rod increases. NO_2 dissolves in concentrated HNO_3. This cell is not much in use because the fumes of NO_2 are very injurious and disagreeable. Its e m f is 1·95 volts. Polarisation is also not completely removed in this cell.

§8. **Grove's cell :**—It is exactly similar to that of Bunsen's except that the carbon rod is replaced by a platinum foil. It is not in use now.

§9. **Bichromate cell :—Construction.** It consists of a glass bottle containing dilute solution of sulphuric acid. A few crystals of potassium dichromate are placed in the acid. The crystals of potassium dichromate act as a *depolariser*. Two inter connected carbon plates *c c* are placed in the bottle as shown in fig 26. The plates act as the positive plate. A zinc rod forming the negative plate is placed between the carbon plates.

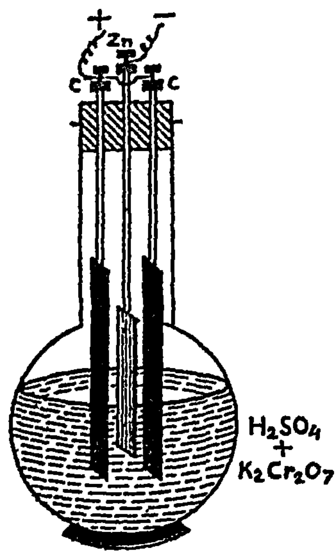


Fig 26.

Working. Zinc ions go into solution reacting with H_2SO_4.

$$Zn^{++} + H_2SO_4 = ZnSO_4 + 2H^{++}$$

The positive ions of hydrogen so liberated hand over their charge to the carbon plates. They are converted into water by the depolariser $K_2Cr_2O_7$. Actually it is the chromic acid formed which acts as a depolariser. Its e m f is 2 volts, and has an extremely low internal resistance. As the depolarisation is not complete, the current falls off soon. It is employed only when strong currents are required for a very short duration.

§10. **Standard cells :**—As the current is drawn from the cells described above, generally their e m f s decrease. Therefore, they cannot be used where constant e m f is required. Hence, for calibration and comparison purposes standard cells are required. Their e m f s remain constant and do not change with temperature. They are used only for calibration purposes. They are mainly of two types :—

(1) **Cadimium cell :—Construction :**—It consists of two limbs made up of glass joined by a horizontal tube as shown in fig 27. It forms a *H* shaped vessel. Pure and dry mercury is placed at the bottom of one of the limbs. It acts as the positive pole. Above

the level of mercury paste of mercurous sulphate is placed which acts as the depolariser. At the bottom of the other limb an amalgam of mercury and cadmium is placed which acts as the negative pole. In the vessel saturated solution of cadmium sulphate is filled in. The level of the solution in the vessel is kept a little above the horizontal tube. To ensure the saturation of cadmium sulphate solution, crystals of cadmium sulphate are placed as shown in fig 27. Two platinum wires are fused at the bottom of both the limbs. Its e m.f. is 1·0183 volts at 20°C. Strong currents are never drawn from this cell It is mainly used for comparison purposes only.

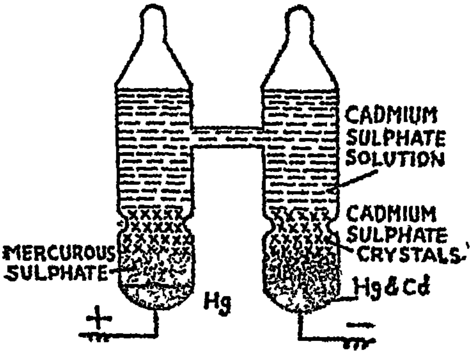


Fig. 27.

(2) **Latimar clarke cell** —It is similar to cadmium cell except that cadmium is replaced by zinc through out It is often shaped like dry cells.

§11. **Dry cells** : —They are nothing but modified forms of Leclanche cells They are extensively used in torch lights, radios, etc. Every body is quite familiar with these types of cells

Construction :—It consists of a carbon rod to which is attached a brass cap It forms the positive pole. The rod is placed in a muslin bag containing powdered charcoal, MnO_2 and a little gum. Around the bag is placed a paste of NH_4Cl, saw dust and a little zinc chloride in a zinc container The zinc container forms the negative plate of the cell A non conducting diaphram *D* is placed at the bottom and the top of the container to insulate it from the carbon rod. To allow the ammonia gas to escape outlets are provided in the muslin bag. The e.m f. of this cell is 1·4 volts.

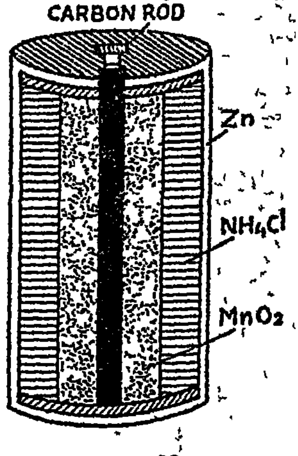


Fig 28

QUESTIONS

1. Describe a simple voltaic cell. (See § 2)
2. What are the defects of a simple cell and how can they be removed (See § 3)
3. Describe and explain a Leclanche cell What is the function of MnO_2 in this cell ? (See § 5)
4. Giving a neat diagram describe a Daniell cell, why is it more efficient than a Leclanche cell ? (See § 6)
5. Describe a good standard cell giving a neat diagram. (See § 10)

CHAPTER V

MAGNETIC EFFECTS OF CURRENT

§1. Introduction :—When an electric current passes through a wire magnetic field is produced around it It shows that a current carrying conductor behaves like a magnet. This was first of all demonstrated by Arago.

Fix a card board *ABCD* in a horizontal plane. Sprinkle iron filings upon its surface Take a copper wire *PQ* and pass it in a hole made in the centre of the card board The wire should remain perfectly vertical Now pass current in the wire from *P* to *Q*. Tap the card board. The iron filings will arrange themselves in a particular way. Just near the wire they will arrange in *concentric circles* as shown in fig. 29. If you take a compass needle its north pole will travel along these curves which are nothing but *lines of force.* This simple experiment illustrates the fact that when you pass current through a wire *magnetic field is produced.* In this chapter you shall study the magnetic effects of current. The direction of the lines of force is *anticlockwise if the current flows upwards* If the direction of the current is reversed, *i e.*, it is made to flow downwards the direction of the lines of force will become *clockwise.*

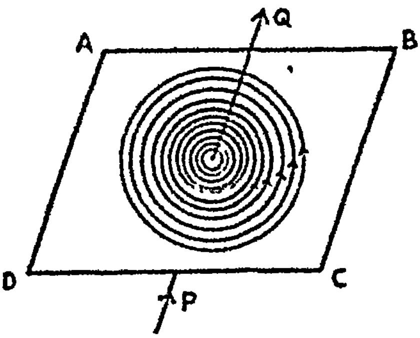


Fig. 29.

Oersted further studied this phenomenon and performed an experiment in 1819 which demonstrates the effect of an electric current on a magnetic needle.

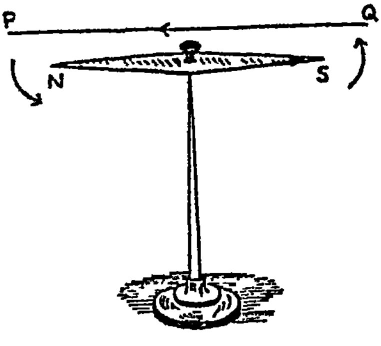


Fig. 30.

Take a pivoted needle *NS*. Place a straight wire *PQ* above the needle in such a way that it is parallel to the axis of the needle. Now pass current as shown, *i e*, from *Q* to *P* The needle will be deflected. It will tend to set itself at right angles to the wire Its north pole will be deflected as shown by the arrow If you reverse the current the needle will be deflected in the opposite direction

§2. Amperes's rule :—You have already seen in the above article that when the current is passed in a wire, magnetic needle

placed near it is deflected. Ampere studied these effects and enunciated a rule which is called **Ampere's rule.** It gives the direc-

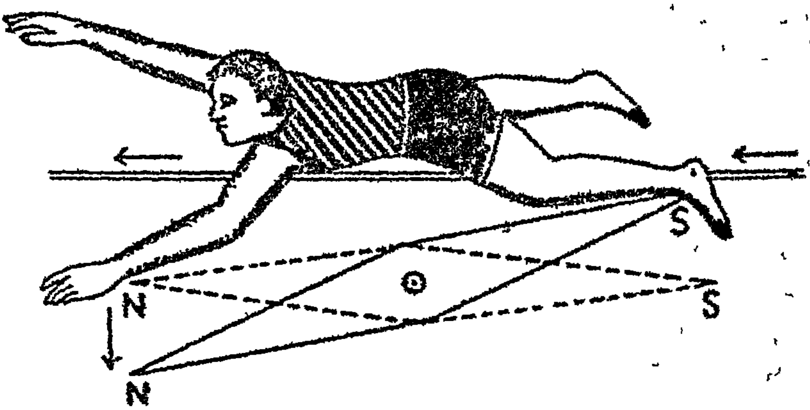


Fig. 31.

tion of deflection of a magnetic needle due to the passage of current. It can be summed up as follows :—

Imagine a man swimming in the direction of the current along the wire with his face always turned towards the needle (see fig 31) *Then the north pole of the needle will be deflected towards his left hand. Obviously the south pole will be deflected towards his right hand.*

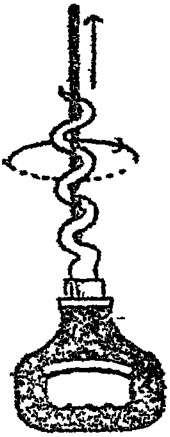


Fig. 32.

§3. MaxWell's cork screw rule :—If you turn an ordinary right-handed cork screw so that its point moves in the direction in which the current is flowing. Then, the direction along which the thumb is rotating will give you the direction of the magnetic lines of force as shown in fig. 32 From this the direction of deflection of the north pole can be easily found out.

§4. Field due to a current flowing in a straight wire :—Laplace's theorem :—Whenever a current passes through a wire a magnetic field is produced around it. The intensity of the magnetic field produced at any point is given by **Laplace's theorem.**

Laplace's theorem :—It states that the magnetic field F produced at the point P distant d from a length l of a short conductor carrying current i is

(i) directly proportional to the strength of the current i, *i.e.,*

$$F \propto i$$

(ii) directly proportional to the length of the conductor, i e ;

$$F \propto l$$

(iii) directly proportional to the sine of the angle between the conductor and the line joining the middle point of the element l *to the point* P, *i.e.*

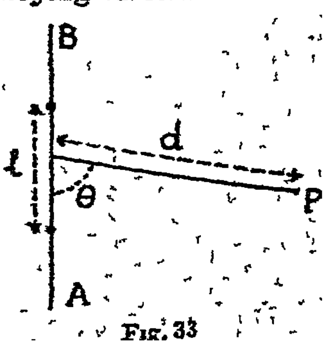


Fig. 33

$$F \propto \sin\theta$$

(iv) inversely proportional to the square of the distance between the element and the point P, i.e.

$$F \propto 1/d^2$$

or embodying all the four in one equation we get

$$F \propto \frac{i\,l \sin\theta}{d^2}$$

or $\quad F = K\dfrac{i\,l \sin\theta}{d^2}$, where K is a constant depending upon the units in which other quantities are measured. The direction of the field is at right angles to the plane containing the element and the point, and the sense is given by the cork screw rule.

§ 5. Field at the centre of a circular coil carrying current :— Let a current of strength i flow through a circular coil of radius r. If only the small element AB of length l_1 on its circumference is taken into consideration, by Laplace's law the intensity of the magnetic field F_1 at the centre of the coil due to this element of length l_1 is given by

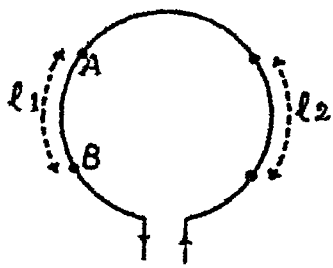


Fig. 34

$$F_1 = K\frac{i\,l_1}{r^2} \sin\theta$$

$$= K\frac{i\,l_1}{r^2} \sin 90$$ [because the line joining the centre to any point on the circumferance is always normal to the element, and so θ is always 90°].

or $$F_1 = K\frac{i\,l_1}{r^2}.$$

Similarly take another element l_2. The field F_2 at the centre due to that element is given by

$$F_2 = K\frac{i\,l_2}{r^2}$$

The two fields will be acting in *the same direction* Also field due to every element on the circumference will be in the same direction. Therefore, the total field F will be equal to the sum of these fields

$$\therefore \quad F = F_1 + F_2 + F_3 + \ldots$$

$$= \frac{K\,i\,l_1}{r^2} + \frac{K\,i\,l_2}{r^2} + \frac{K\,i\,l_3}{r^2} + \ldots$$

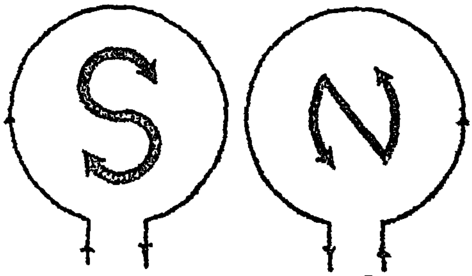
$$= \frac{K\,i}{r^2}\,(l_1 + l_2 + l_3 + \ldots)$$

$$= K\; i\,\frac{2\pi r}{r^2}$$ [because $l_1 + l_2 + l_3 + .$ is the length

of the circumference of the whole coil which is equal to $2\pi r$].

or $$F=K\,\frac{2\pi i}{r}. \qquad ...(1)$$

The direction of the field will be perpendicular to the plane containing the conductor and the point The sense will be given by right handed cork screw rule. The direction of the field can also be found out by noting the direction of the current. If you look at the face of the coil, and find that the *current is flowing in the clockwise direction, the positive direction of the line of force inside the coil will be away from you, i.e.* the field will be acting away from you. If the direction of the current is *anticlockwise the positive direction of the lines will be towards you, i e.,* the field will act towards you. Obviously in the former case the face towards the observer will behave as the *south pole* while in the latter it will behave as a *north pole* shown in fig 35.



(a) Fig. 35. (b)

§ 6. **Electromagnetic unit of current :**—From equation (1) of the last article, the field F at the centre of a circular coil is given by

$$F=\frac{2\pi i}{r}.$$

If we select the unit of current in such a way that when $r=1$ cm , $i=1$; $F=2\pi$, K is reduced to unity, our expression for field becomes

$$F=K\,\frac{2\pi i}{r}$$

This unit is known as *electromagnetic unit of current*.

Hence, an absolute electromagnetic unit of current can be defined as the current which when flowing through a circle of unit radius will exert a force of 2π dynes on a unit north pole placed at the centre of the coil. It is written as e m.u

The e. m u. is very large for practical purposes. The practical unit is an *ampere* which is one-tenth of an electromagnetic unit of current.

1 ampere$=\frac{1}{10}$ e. m. u of current If the current is measured in amperes the field at the centre of the coil will be

$$F=\frac{2\pi i}{10\,r}\ \text{oersted}$$

or $$F=\frac{2\pi n i}{10\,r} \qquad ..(2)$$

where n is the number of turns in the coil.

§ 7. **Fleming's left hand rule :**—So far we have considered only the action of current on magnets. On the other hand if a conductor carrying current is placed in a magnetic field, *the conductor experiences a force and tries to rotate* As action is equal to reaction and the magnet is fixed, it is the conductor which moves The direction of motion of the conductor is given by Fleming's left hand rule The rule can be stated as follows —

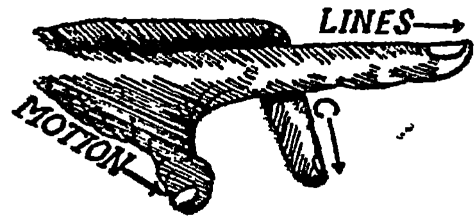


Fig. 36

Hold the thumb, first finger and the middle finger of the left hand in three directions mutually at rightangles to each other If the first finger points in the direction of the magnetic field due to the magnet, and the middle finger in the direction of the current then the thumb gives the direction in which the conductor moves (See fig 36).

§ 8. **Solved problem :**—*A study current is flowing through a circular coil of 10 cms. radius and having 10 turns When kept in a vertical plane magnetic east west, there is a neutral point at the centre. If H=0 35 gauss. Calculate the strength of the current.*

Field F at the Centre of the coil is given by

$$F=\frac{2\pi n i}{10r}$$

At the neutral point $F=H$

or
$$\frac{2\pi n i}{10r}=H$$

In this problem $n=10$, $r=10$ cm and $H=0\ 35$ gauss

$$\therefore \quad 0{\cdot}35=\frac{2\pi\ 10\ i}{10\ 10}$$

or
$$i=0{\cdot}557 \text{ amp}$$

QUESTIONS

1 Describe experiments to show that a circuit carrying current behaves like a magnet (See § 1)

2 Assuming Laplace's law for the magnetic field due to an electric current, obtain an expression for the field at the centre of a circular coil carrying current [See § 4 and § 5]

3 Define electromagnetic unit of current, what is its practical unit, and what is the relation between the two types of units [See § 6]

4 What is Fleming's left hand rule ? [See § 7]

NUMERICAL PROBLEM

Calculate the force experienced by a magnetic pole of strength 6 units placed at the centre of a coil of 400 turns and 20 cm radius through which a current of 10 amp. is flowing. [See § 5]

(*Ans* 754 26 dyns)

CHAPTER VI

GALVANOMETERS

§1. Introduction :—Galvanometers are the instruments by which electric current can be detected or measured. *They are of two types* :—

(1) **Moving magnet type :**—In such types of instruments the coil which carries the current is fixed. The magnet moves at the centre of the coil, *e g*, tangent galvanometer, astatic galvanometer, sine galvanometer etc

(2) **Moving coil type :**—In such types of instruments the magnet is fixed. The coil moves in the magnetic field of the magnet They are of two types :

(*i*) Suspened coil type.

(*ii*) Pivoted coil type.

§2. Tangent galvanometer :—It consists of a circular coil of insulated wires of a few turns wound upon a wooden frame (see fig. 37). The frame is itself capable of rotating about a vertical axis fixed at the centre of a circular horizontal disc fixed at the base. The base rests upon three levelling screws At the centre of the coil there is placed a cylindrical box covered with glass. The box carries a horizontal circular scale fixed at its base. The scale is divided into four quadrants reading from 0° to 90°. A small magnetic needle is freely pivoted at the centre of the circular scale. A long and light alluminium pointer is attached to the needle at right angles to its length The pointer moves over the circular scale. The base of the box also carries a mirrior in which the image of the pointer can be seen. The box can also rotate about a vertical axis

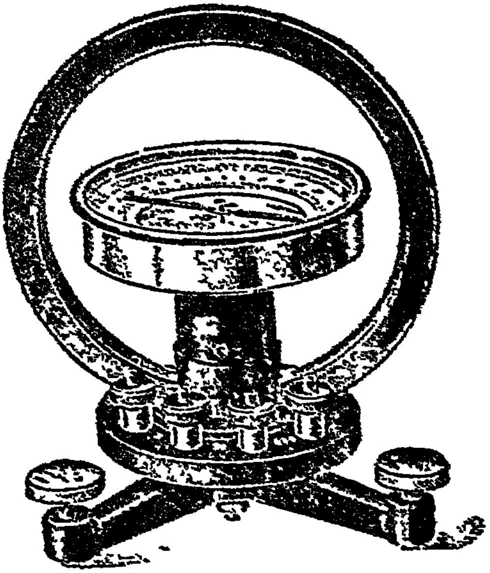


Fig 37.

The two ends of the coil are connected to two binding screws fixed on the base of the instrument The number of the turns are usually written between the two binding screws. Generally the instrument carries three or four coils of different turns each connected to a different binding screw on the base of the instrument The coils are so arranged that any one or all of them can be put in the circuit in series

Adjustment :—(*i*) First level the instrument by a spirit level. This will ensure free movement of the needle at the centre of the scale.

(*ii*) Rotate the coil till it comes in the magnetic meridian. In this position its plane will be parallel to the length of the magnet, *i e*, both of them will be in the same vertical plane.

(*iii*) Adjust the box carrying the needle such that the pointer reads 0—0 on the circular scale

(*iv*) Connect the two required terminals on the base to the desired circuit, and pass current through the coil. Due to the passage of the current magnetic field is produced and the needle will be deflected Note down the deflections at both the ends of the pointer and find the mean of the two readings

Theory :—When the current is passed in the coil, magnetic field F is produced at its centre The field acts at right angles to the plane of the coil, *i e.*, at right angles to the length of the freely pivoted magnetic needle. *The field is almost uniform in a small region near the centre* As the needle used is quite small it more or less moves in a uniform magnetic field

Now two fields at right angles to each other are acting upon the needle. (*i*) *Earth's horizontal component H acting along the plane of the coil in the magnetic meridian* (*ii*) *The field F acting at right angles to H as shown in fig.* 38.

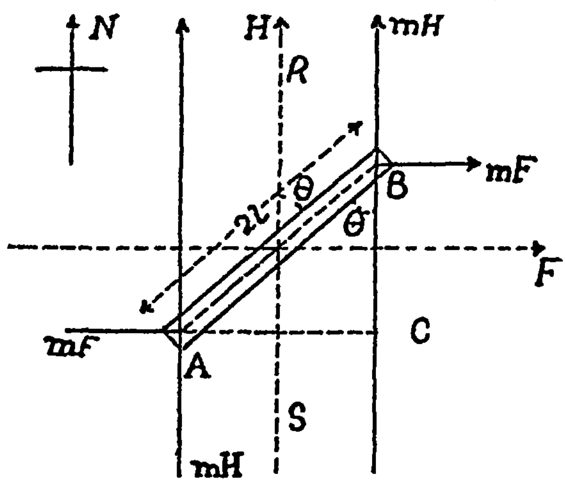


Fig. 38.

F will try to set the needle at right angles to the magnetic meridian, while H tries to bring it back to its original position Hence, the needle is acted upon by two couples (*i*) The deflecting couple provided by the field F (produced on account of the flow of current) (*ii*) The restoring couple provided by H Under the influence of the two couples the needle will come to rest in some intermediate position making an angle θ with the magnetic meridian In equilibrium the moment due to one couple must be equal to that of the other.

Let the pole strength of each of the poles of the needle AB be m Let its mean position be represented by SR. Let it make an angle θ with the magnetic meridian in the equilibrium position (see fig 38) The force on each pole due to H will be mH dynes acting parallel to the magnetic meridian The force due to F on each pole will be mF dynes acting perpendicular to the magnetic meridian In equilibrium,

Restoring couple $=$Deflecting couple

or $$mH \times AC = mF \times BC$$

But $$AC = AB \sin\theta, \text{ and } BC = AB \cos\theta$$

$$\therefore \quad mH\, AB \sin\theta = mF\, AB \cos\theta$$

or $$mH \sin\theta = mF \cos\theta$$

or $$H \frac{\sin\theta}{\cos\theta} = F$$

$$\therefore \quad F = H \tan\theta \qquad (1)$$

It is called **tangent law.** As this galvanometer is based upon this law it is called **tangent galvanometer.**

Let i be the current flowing through the coil of a tangent galvanometer. Let the number of turns of the wire in the coil be n, and its radius r. Then by Laplace's law the field at the centre of the coil will be

$$F = \frac{2\pi n i}{r} \qquad (2)$$

From relations (1) and (2) we get

$$F = \frac{2\pi n i}{r} = H \tan\theta$$

or $$i = \frac{rH}{2\pi n} \tan\theta$$

Here i is measured in e.m. units. If it is *measured in amp.*

$$i = \frac{10rH}{2\pi n} \tan\theta$$

$$= H/G \tan\theta \quad \left[\text{where } G = \frac{10r}{2\pi n},\right.$$

which is a constant called the galvanometer constant because n is constant and r is constant]

But for the same place H is also constant, and therefore

$$H/G = k \text{ [where } k \text{ is another constant]}$$

or $$i = k \tan\theta \qquad (3)$$

or $$i \propto \tan\theta \qquad (4)$$

Thus, the current in the galvanometer is directly proportional to the tangent of deflection k is called the **reduction factor of the tangent galvanometer.** It is so named because by multiplying it with the tangent of the deflection, current can be obtained. If $\theta = 45°$, $i = k$ Thus, the reduction factor is numerically equal to the current in amp flowing through the coil when the deflection is 45°. As k depends upon H and G, it changes with change of place and G (on account of n and r).

The following points should be carefully noted regarding a tangent galvanometer .—

(1) *The coil must be exactly in the magnetic meridian,* otherwise the tangent law will not apply. It can be verified by reversing the current in the coil. If the coil is correctly set the deflection

will remain the same in both the cases If the coil is placed at right angles to the magnetic meridian the needle will not be deflected, because now the two fields are acting parallel to each other. Hence, no couple acts.

(2) As the field is more or less *uniform only over a small rigion at* the centre of the coil, the magnetic needle *should be small,* so that it may move in a uniform field The pointer is made longer in order to read the deflection with greater accuracy It has been assumed here that the magnetic needle is very weak, and therefore exerts no magnetic field of its own. Hence, there are only two fields F and H acting at right angles to each other at the centre of the coil where the magnetic needle is placed.

(3) The instrument will be accurate when the deflections are near about 45° Because in the vicinity of 45° a slight error in reading the deflections will not cause any appriciable error in determining the value of the current.

§ 3. Sensitiveness of a tangent galvanometer :—A galvanometer which produces large deflection for a small current is said to be sensitive *The larger will be the deflection for a given current the more will be its sensitivity.*

Hence for θ to be large for a given value of i, k, *i e*, H/G should be small. Thus for a galvanometer to be more sensitive G should be large and H should be small To make G large n should be more and r should be small. But n cannot be increased indefinitely for the following reasons :—

(1) If the number of turns is increased, the turns will require a certain width of the coil for their winding as shown in fig 39 on account of this their centres will not lie at the same point Furthermore their radii will also be different, and so the field will not be uniform and tangent law will not apply

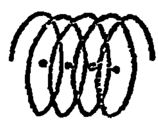


Fig 39

(2) As n increases the resistance of the coil will increase diminishing the value of the current Hence, n cannot be increased after a certain limit

Similarly r also cannot be reduced below a certain value because if r is reduced as shown in fig 40, the rigion having uniform field at the centre will be reduced. It means that the magnetic needle taken should be very very small which is not possible, because it also has got some length.

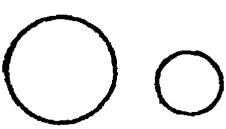


Fig 40

Therefore, to make the galvanometer more sensitive H should be decreased It can be done by putting a control magnet above the galvanometer needle in the magnetic meridian The magnet should be so placed that it may oppose H, *i e*, *its south pole should point in* north direction. It will reduce the value of the resultant field acting in the direction of magnetic meridian This can be adjusted by adjusting its distance from the needle.

The deflections can be increased by taking a long pointer, but it will increase the weight of the pointer This will result in more friction, and lower the value of deflections

We have assumed in the theory of the galvanometer that the magnetic needle being small does not have any field of its own. But however small it may be, it has got some magnetic moment of its own and therefore, it reduces the deflecting couple decreasing the deflection, and hence the sensitivity. To avoid this astatic pair of needles is taken. This increases the sensitivity.

§ 4. Defects of a tangent galvanometer :—

(1) Every time before taking readings it should be placed in the magnetic meridian which is a cumbersome affair.

(2) As the controlling field H is weak the deflection in the galvanometer is easily affected by the presence of small magnets or magnetic substances in its neighbourhood.

(3) The deflection produced is not directly proportional to θ but is proportional to tan θ. Therefore it can not be made direct reading.

(4) The needle takes time to come to rest.

§ 5. Moving coil galvanometers (*suspended coil type*) :— **Description :**—It consists of a permanent horse shoe magnet NS. The pole faces of the magnet are concave having a cylindrical air gap in between them. A coil of insulated copper wire of many turns is suspended between the two pole pieces of the magnet NS. The coil is either rectangular or circular in shape It is suspended by means of a phosphor-bronze strip A fixed to the tortion head H forming one of the terminals of the instrument The current enters the coil through this strip. The other end of the coil is connected to a coiled spring B also of phosphor-bronze. The spring is connected to the other terminal of the instrument. Current leaves the galvanometer through the spring. The spring and the strip also provide the controlling couple The strip carries a mirror M. By lamp and scale arrangement the deflection of the mirror can be found out

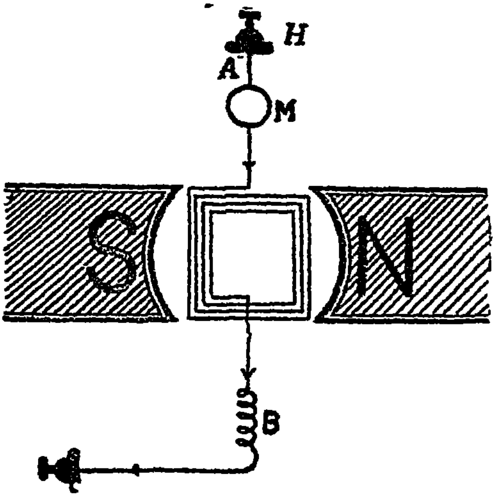


Fig. 41.

Usually an iron core of spherical or cylindrical shape as shown in fig. 42 is fixed at the centre of the coil This concentrates the lines of force in the coil *increasing the controlling field* The pole pieces are made concave or circular to make the field *radial* as shown in fig 42 The coil is so suspended in the air gap that the magnetic lines of force due to the permanent magnet are parallel to its plane As

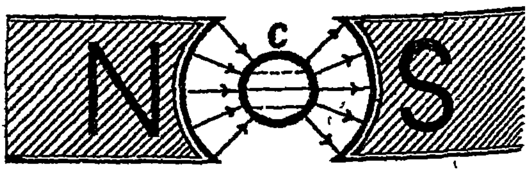


Fig 42.

the field becomes radial the lines of force always remain parallel to the plane of the coil, so long the coil rotates in the vertical plane. *The coil may lie in any position, the lines will always cut its vertical sides at right angles.*

Adjustments :—The instrument is carefully levelled so that the coil is free to rotate in the magnetic field

Working :—When the current is passed through the coil it developes polarity at its two faces and behaves as a magnet. Due to the interaction between its field and the field due to the permanent magnet a couple is developed This couple which is called the *deflecting couple tends* to set the coil at right angles to the direction of the magnetic field (field due to the magnet) On the other hand the tortion present in the phosphor bronze strip opposes the motion of the coil and generates *another couple* which opposes the former. This is called the *controlling couple* *Under the influence of these two couples the coil sets in an intermediate position where the moments due to both of the couples are the same* The deflection of the coil is measured by lamp and scale arrangement

Theory:—Let H be the field due to the permanent magnet, i the strength of the current passing through the coil, l the length of the vertical side of the coil, b the breadth of the coil and n the number of turns of wire in the coil When current passes through the coil each of its vertical sides experiences a force equal to $H\,i\,l$ dynes As there are n turns the total force experienced is equal to $H\,i\,l\,n$

Thus, the moment of the deflecting couple$=n\,i\,H\,l\,b$.

If θ is the deflection and T the moment per unit twist generated in the strip, the moment of the restoring couple will be$=T\theta$. In equilibrium the two are equal

$$n\,i\,H\,l\,b=T\theta$$

or $$i=\frac{T}{n\,H\,l\,b}\theta \quad [\text{but } l\times b=A \text{ the area of the coil}]$$

or $$i=\frac{T}{n\,H\,A}\theta$$

or $$i=k\,\theta, \quad [\text{where } k=\frac{T}{n\,H\,A}, \text{ as } T, n, H \text{ and } A \text{ are constants } k \text{ will be a constant}]$$

This is called the constant of the galvanometer

Hence $$i \propto \theta \qquad .. \quad (5)$$

Thus, the current passing through the coil is directly proportional to the angle of deflection If k is known, i can be calculated

§ 6. Sensitiveness of a moving coil galvanometer —From relation $i=\frac{T}{n\,H\,A}\,\theta$, for the same value of i, θ will be large when k is small For k to be small T should be small and n, H and A

should be large. If A and n are very much increased the resistance of the instrument will increase adding very little to the sensitivity of the instrument Hence, to increase sensitiveness T is decreased and H is increased As phosphor-bronze has got more tensile strength and smaller value of T, it is extremely suited for using it in suspending the coil. H is increased by taking a more powerful magnet

As this form of galvanometer was first of all developed by D'Arsonval, it is also called **D'Arsonval galvanometer.** It is very sensitive It can measure currents up to 10^{-9} amp

§ 7. **Damping of a galvanometer** :—Even when current ceases to pass, the coil does not come to rest quickly In order to make it dead beat, *i e*, in which the coil comes to rest quickly, its oscillations are damped *It is done by winding the coil upon a light conducting frame* The *eddy currents* produced in the frame oppose the motion of the coil, and brings it quickly to rest

§ 8. **Comparison between tangent galvanometer and moving coil galvanometer** :—

Tangent Galvanometer	**Moving Coil Galvanometer**
(1) The controlling field provided by earth's megnetic field is very weak. Hence its deflection is affected by iron pieces or other megnets placed in the vicinity	(1) The controlling field provided by the permanent megnet is very strong, and hence its deflections are not at all affected by other external fields.
(2) Before use always it has to be set in the magnetic meridian which takes time.	(2) It can be placed in any position
(3) The constant H/G of the galvanometer changes from place to place. Therefore, at each place it has to be determined separately	(3) Its constant remains the same at all places So once it has been determined it can be used any where.
(4) Once the megnetic needle is disturbed, it takes long time before it comes to rest	(4) By making it dead beat the coil can be brought to rest at once
(5) Current is proportional to tangent of deflection.	(5) Current is proportional to deflection

The moving coil galvanometer also suffers from some disadvantages In case of a tangent galvanometer current can be determined from its dimensions which is not possible in this case Furthermore, if very strong currents are passed in moving coil galvanometers they are damaged.

§9. Moving Coil Galvanometer Pivoted Type :—In this case a coil of copper wire is wound upon an aluminium frame. Instead of suspending the coil, the coil is pivoted by means of a specific fixed jewel bearings. The coil is placed between the two concave poles of a permanent magnet NS as shown in fig. 43 (b). The coil is so pivoted that its plane always remains parallel to the magnetic field. At the centre of the coil an iron core is fixed to concentrate the field within the coil. One end of the coil is connected to a hair spring placed above the coil. The other end of the hair spring is connected to one of the terminals of the instrument. Similarly the other end of the coil is connected to the other hair spring placed below the coil. The other end of this spring is connected to second terminal of the instrument. The two springs are coiled in the opposite direction. *They provide the controlling couple to balance the deflecting couple* [See fig. 43 c]. *A light pointer is attached to the spindle of the coil, at right angles to its plane.* The pointer moves over a circular scale calibrated in parts of equal length.

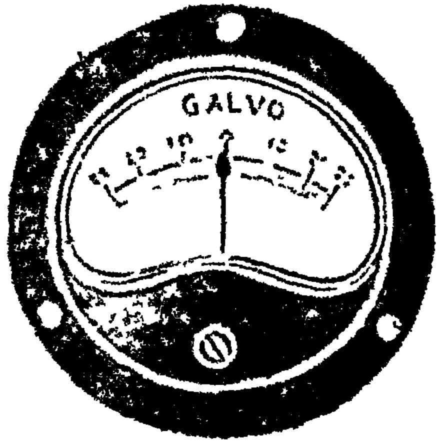


Fig. 43 (a)

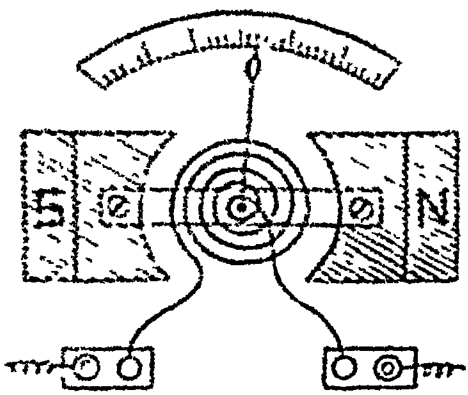


Fig. 43 (b).

When current is passed through the coil, the coil behaves like a magnet. *There is an inter-action* between its field and the field due to the permanent magnet, which provides the deflecting couple. Under the influence of the deflecting couple and the controlling couple the coil and hence, the pointer comes to rest in some intermediate position which can be read on the scale.

Though the pivoted type of galvanometers are a bit less sensitive than the former type, they are easily *portable and dead beat*. The pointer quickly comes to rest. They are generally employed in all electrical experiments.

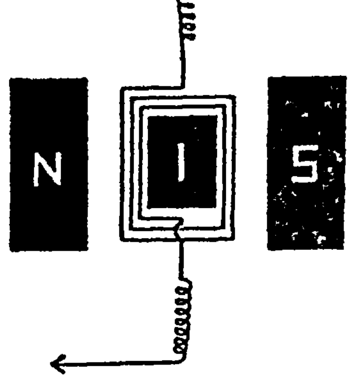


Fig. 43 (c).

§10. Ammeters and Voltmeters :—They are the instruments which measure current and potential difference respectively. The

former are low resistance galvanometers while the latter are high resistance galvanometers. They have been discussed in details in the next chapter.

§11. Solved problem :—*The coil of a tangent galvanometer consists of 5 turns and has a mean radius of 20 cms. A current of 2·1 amp in the coil produces a deflection of 45°. Find the horizontal component of the field*

From the theory of the tangent galvanometer we have

$$i=\frac{10rH}{2\pi n}\tan\theta$$

In this problem $i=2·1$ amp., $r=20$ cm, $n=5$; $H=?$

$$\therefore \qquad 2\ 1=\frac{10\times 20\times H}{2\times 3\ 14\times 5}$$

or $$H=·329 \text{ oersteds}$$

QUESTIONS

1 Describe the essential parts of a tangent galvanometer and explain its action Why is it called a tangent galvanometer ? (See § 2)

2. Explain the construction, action and use of a tangent galvanometer. Why is the instrument most accurate when the deflection is 45° ? Why should the magnet pivoted at the centre of the coil be small ? (See § 2)

3. What do you understand by reduction factor of a tangent galvanometer ? How can it be calculated (See § 2)

4. Discuss sensitiveness of a tangent galvanometer and show what are its defects. (See § 3 and 4)

5 Describe giving a neat diagram, suspended coil type of galvanometer and explain its theory (See § 5)

6 Discuss the sensitiveness of a moving coil galvanometer (See §6)

7 Compare and contrast moving coil galvanometer with that of tangent galvanometer. (See § 8)

8. Describe a moving coil pivoted type of galvanometer and show how it has been made dead beat (See § 9)

NUMERICAL PROBLEMS

1 A current of 2 amp. flowing through a coil of tangent galvanometer produces a deflection of 42°. Calculate the reduction factor of the galvanometer. (*Ans* 2 22 amp.)

2 A tangent galvanometer with 20 turns and 10 cm radius is placed in the magnetic meridian Calculate the current which will deflect the galvanometer through 45° [H=·35 gauss] (*Ans.* 0 278 amp)

3 The coil of a tangent galvanometer is 10 cm in radius. How many turns of wire should be wound on it, so that a current of 0 01 amp may produce a deflection of 45° ? [H=0 18 C G S. units] (*Ans* 287)

CHAPTER VII

OHM'S LAW

§1. Ohm's law :—Connect a coil of wire R in series with an ammeter A, a cell and a key (see fig 44). Connect a voltmeter V across the two ends of the conductor to find out the potential difference Pass current through the coil Note down p d. across R in the voltmeter Note the current through it by the help of the ammeter. Now increase the number of cells in the circuit. The p d. across R will increase, resulting in the increase of current in R. In this way change the number of cells for a few times, always noting down the values of corresponding p.d and current. Find the ratio between p d. and current for each pair *It will come out to be a constant quantity,* if the temperature of the coil is kept constant. This type of experiment was first of all performed by Ohm and hence, the result arrived at due to this experiment is known as **Ohm's law.**

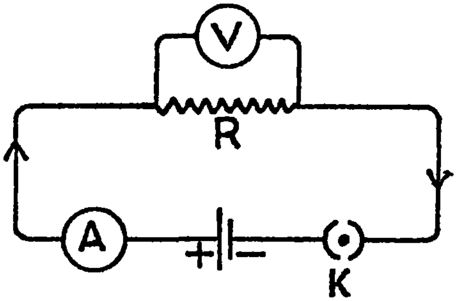


Fig 44

The law appears to be extremely simple, but its importance in physics is enormous. *It states that at constant temperature, the potential difference across the ends of the conductor is directly proportional to the current flowing in it.* If E is the p.d. and i the current, then mathematically

$$E \propto i \quad \text{or} \quad E/i = \text{a constant}$$

This constant is called the **resistance** of the conductor and is generally denoted by the symbol R Thus Ohm's law can be expressed in the following ways :

$$E/i=R \quad (1)$$

or $$i=E/R \quad \ldots (2)$$

or $$E=iR \quad . (3)$$

It is clear from relation (2) that if p d. is kept constant current in the circuit is inversely proportional to resistance, *i e*, if R increases i will fall and *vice versa* Thus, it resists the flow of current in the circuit and hence is called *resistance*

§2. Factors upon which the resistance depends :—It can be easily demonstrated by a simple experiment that the resistance R of a given conductor is ·

(*i*) *directly proportional to the length of the conductor provided its material and cross-section do not change*

(*ii*) *inversely proportional to the cross-section provided its material and length remain the same.*

Hence if l and S is respectively its length and cross-section,

$$R \propto l$$
$$\propto 1/S$$

or
$$R \propto l/S$$
or
$$R = \sigma \frac{l}{S} \quad \sigma \text{ (Sigma)}$$

where σ is a constant This constant depends upon the material of the conductor and is called the *specific resistance or the resistivity of the material of which the conductor is made of.*

If we put $l=1$ cm, and $S=1$ sq. cm. in the above equation R becomes equal to σ

Hence the specific resistance of a substance can be defined as the resistance offered by a wire of that substance of unit length and unit cross-section. Or it can be expressed as ~~the~~ resistance offered between two opposite faces of a unit cube made of that substance

From the above relation

$$\sigma = \frac{R\,S}{l}$$

putting units for R, S and l, we get

$$\sigma = \frac{\text{ohm} \times \text{cm.}^2}{\text{cm}} = \text{ohm} \times \text{cm.}$$

Thus it can be expressed as ohm-cm. As it is the resistance offered between the two opposite sides of a unit cube, its unit can also be expressed as ohms per cm.^{3}

Specific resistances of a few substances

Copper	.	$1{\cdot}7 \times 10^{-6}$	ohm-cm
Silver	..	$1{\cdot}46 \times 10^{-6}$	,,
Iron	.	$8\ 6 \times 10^{-6}$	,,
Manganin		$40{\cdot}0 \times 10^{-6}$	,,
Eureka	..	$49\ 0 \times 10^{-6}$	,,

§3. Difference between resistance and specific resistance :— Resistance of a conductor can increase or decrease depending upon its length and cross-section, but specific resistance is a constant quantity. So long as the material is the same it does not change. It is just like specific gravity or specific heat.

§4 Variation of resistance with temperature ·—The resistance of a wire generally increases with the rise of temperature. If the resistance changes from R_o at 0°C to R_t at t°C, for small value of t, R_t can be given by the relation

$$R_t = R_o(1+\alpha t),$$

where α is the *temperature coefficient for resistance* Smaller will be the value of α, lesser will be the changes produced in the resistance of a wire due to change of temperature. For pure metals the value of $\alpha = {\cdot}00366$ which is equal to the coefficient of cubical expansion of a gas.

§5. Difference between the behaviour of pure metals and alloys —By mixing certain metals, certain alloys can be obtained. So far as resistance is concerned they are superior to pure metals for the following reasons

(i) They have got very *high specific resistance*, and therefore

for a given value of a resistance, only a small length of the conductor is required

(ii) They possess very *low temperature coefficient.* Hence they are used in constructing standard resistances which do not change with temperature.

The alloys commonly used are :

(1) Manganin.—It is an alloy of copper-manganese and Zinc.

(2) Eureka or Constantan.—It is an alloy of copper-nickel.

(3) Nichrome.

(4) German silver.

§6. **Different units —(1) Strength of the current.**

Electric charge.—The quantity of electricity carried by 1 e. m. Unit of current in one second is defined as one e. m. unit of electricity.

Its practical unit is **Coulomb,** *i e* , the quantity of electricity conveyed by one ampere in one second

$$\text{Coulomb} = \text{amperes} \times \text{seconds}$$

$$1 \text{ Coulomb} = \tfrac{1}{10} \text{ e.m unit}$$

$$= \tfrac{1}{10} \times 3 \times 10^{10} = 3 \times 10^{9} \text{ e s units.}$$

Strength of the current is defined as the quantity of electricity flowing through a conductor in unit time. It is thus, the rate of flow of electricity. If Q is the quantity of electricity flowing for t seconds, then the current i is given by

$$i = Q/t$$

The practical unit of current is ampere

$$1 \text{ ampere} = \tfrac{1}{10}\text{th of an e.m. unit of current}$$

$$= \tfrac{1}{10} \times 3 \times 10^{10} = 3 \times 10^{9} \text{ e s. units.}$$

(2) Potential difference.—The two ends of a conductor PQ (see fig 45) are said to possess a potential difference of one C G S. (electromagnetic unit), when unit work (1 erg) is done in transferring one e m unit of charge from Q to P.

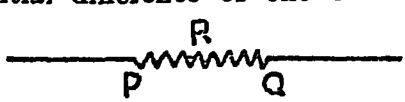


Fig. 45.

The practical unit of p d is a volt. *Two points are at a potential difference of one volt when one joule of work is done in moving one coulomb of charge from one point to the other*

$$1 \text{ volt} = 10^{8} \text{ e m u of p d} = \tfrac{1}{300} \text{ e s units.}$$

(3) Resistance.—The electromagnetic unit of resistance can be defined as the resistance offered by a conductor when one e m u of p d. across its ends causes a unit e m u. of current to flow through it. This can be easily deduced from ohm's law.

The practical unit of resistance is **ohm** *A conductor is said to have a resistance of one ohm when a p d of one volt across its ends causes a current of one amp to flow through it.*

Thus,

$$1 \text{ ohm} = \frac{1 \text{ volt}}{1 \text{ amp.}} = \frac{10^{8} \text{ e.m u.}}{\frac{1}{10} \text{ e.m u.}} = 10^{9} \text{ e.m.u.}$$

The units can be expressed in a tabular form also :—

S. N.	*Quantity.*	*Practical units*	*Equivalent e.m. units.*	*Equivalent e.s units.*
1	Current	1 amp	1/10 e m u	3×10^9 e s u
2	Potential Difference	1 volt.	10^8 e.m.u.	1/300 e s u
3	Resistance	1 ohm.	10^9 e m u	

§7. Resistances in Series :—When number of resistances are connected end to end as shown in fig 46, such that the same current flows through each of them in succession, they are said to be arranged in series. *AB, CD, EF* are the resistances

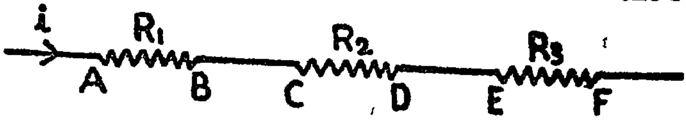


Fig 46.

arranged in series. Let their resistances be R_1, R_2 and R_3 respectively. Let the current passing through them be i. Let the p. d. across the three conductors be respectively V_1, V_2 and V_3. Then applying ohm's law to each conductor we get,

$$V_1=R_1C\ ,\ V_2=R_2C \text{ and } V_3=R_3C$$

If V is the total p d. between the two extreme ends A and F, and R the total resistance in the circuit we have

$$V=V_1+V_2+V_3$$

or

$$RC=R_1C+R_2C+R_3C$$

Hence

$$R=R_1+R_2+R_3$$

Thus, the total resistance of a number of conductors arranged in series is equal to the sum of the individual resistances

§8. Resistance in parallel :—When one end of every resistance is connected to a common point C, and the other end to another common point D as shown in the fig. 47 they are said to be arranged in parallel. In this arrangement the current i divides into as many branches as there are conductors at C and again combine at D.

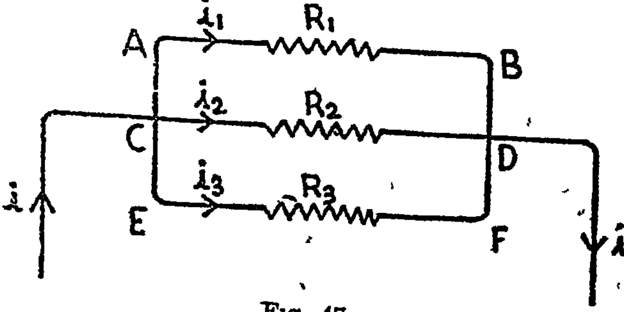


Fig 47.

Evidently all the conductors will have the same potential difference across their ends. Let it be V. Let R_1, R_2 and R_3

be respectively the resistances and i_1, i_2 and i_3 be respectively the currents through these resistances Let i be the total current passing through these resistances and R the total resistance of the system (i e, equivalent resistance), by applying ohm's law, we get

$$i=i_1+i_2+i_3$$

$$\therefore \quad \frac{V}{R}=\frac{V}{R_1}+\frac{V}{R_2}+\frac{V}{R_3} \quad \left[\because \; i=\frac{V}{R},\; i_1=\frac{V}{R_1},\; i_2=\frac{V}{R_2},\; i_3=\frac{V}{R_3}\right]$$

or

$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$$

Thus when the resistances are joined in parallel the reciprocal of the equivalent resistance is equal to the sum of the reciprocals of individual resistances

As *conductance* is defined as the *reciprocal of the resistance*, if a number of conductance is the sum of individual conductances.

It is clear that if the resistances are arranged in parallel their equivalent resistance is smaller than the resistance of any one of them.

Thus, to introduce more resistance in a circuit arrange the resistance in series while to have less resistance arrange them in parallel

§ 9. **Shunts** :—When a strong current is passed through the galvanometer there is always the danger of its being burnt It is on account of the excessive heat produced due to the flow of the current To protect the coil from this danger, usually a small resistance is connected in parallel to the coil of the galvanometer. Most of the current flowing in the circuit is shunted off through this low resistance, and a very small current passes through the coil. It ensures the safety of the instrument *This low resistance connected across the terminals of the galvanometer is called a shunt*

Let G be the resistance of the galvanometer. Let the resistance of the shunt connected in parallel to it be S. If i is the current flowing in the main circuit, it will divide into two parts at the point A to combine again at the point B Let the current through the galvanometer and the shunt be respectively ig and is. If V is the potential difference across the galvanometer and the shunt by applying ohm's law we get —

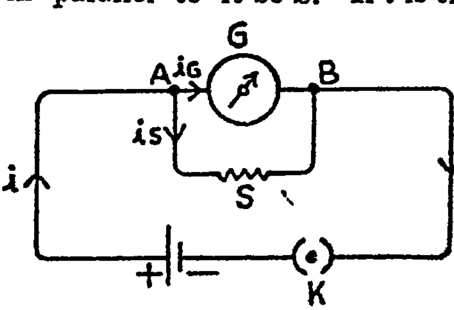


Fig 48

$$iS=\frac{V}{S} \qquad \text{(i)}$$

$$iG=\frac{V}{G} \qquad \text{(ii)}$$

By dividing (i) by (ii) we get

$$\frac{is}{iG}=\frac{G}{S} \qquad \text{(iii)}$$

Adding one to both sides, we get

$$\frac{iG+iS}{iG}=\frac{G+S}{S} \quad \text{...(iv)}$$

But

$$i=iG+iS$$

$\therefore$

$$\frac{i}{iG}=\frac{G+S}{S}$$

or

$$iG=i\frac{S}{G+S} \quad \text{...(v)}$$

By using the proper value of S, iG can be made very very small. $\frac{G+S}{S}$ is called the *multiplying power* of a shunt. To get the main current in the circuit the current in the galvanometer should be multiplied by this factor.

If only $\frac{1}{n\text{th}}$ of the current is to be sent through the galvanometer, we have

$$\frac{iG}{i}=\frac{1}{n}=\frac{S}{G+S}$$

or

$$S=\frac{G}{(n-1)} \quad \text{...(vi)}$$

Hence if the shunt resistance is $\frac{1}{(n-1)}$th of the galvanometer resistance, only $\frac{1}{n}$th of the total current passes through the galvanometer, and it will not be damaged.

For example if we want to pass only $\frac{1}{10}$th of the main current through the galvanometer.

$S=\frac{G}{n-1}=\frac{G}{10-1}=\frac{G}{9}$, the resistance of the shunt should be $\frac{1}{9}$th of the resistance of the galvanometer :—

Similarly to get $\frac{1}{100}$th of the main current in the galvanometer the resistance of the shunt should be $\frac{1}{100}$th of that of the galvanometer.

Thus, by selecting the required value of S, current in the galvanometer can be reduced to any extent.

When a shunt is connected paralled to any resistance, the equivalent resistance of the circuit decreases To compensate it a suitable resistance should be placed in series.

§10. Ammeters. They are moving coil pivoted type of galvanometers with the difference, that a low resistance shunt is put across the coil of the galvanometer. They are used to measure current. The shunt serves two purposes.

(i) It diverts most of the current from the galvanometer and so only a *fraction of the main current passes through the latter* (ii) It reduces the equivalent resistance of the system formed by the galvanometer and the shunt. *Thus, an ammeter has got an extremely low resistance* The circular scale is calibrated in amp or milli amp. by some standard current measuring instruments The pointer rests on zero, marked on the left hand side of the scale. It moves toward right when current is passed through it. Actually if one amp. current is flowing in the circuit a small fraction of it

enters the coil which gives the deflection. This deflection is called one amp and so on. Thus it is calibrated.

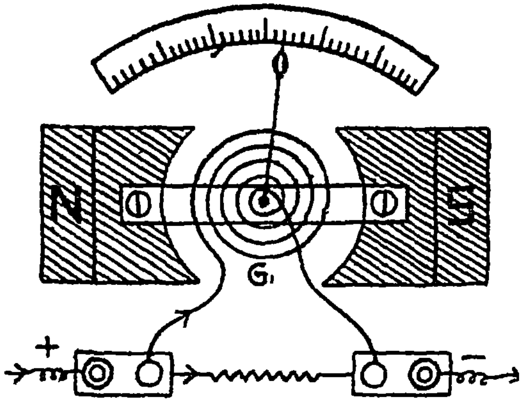


Fig 49 (*a*)

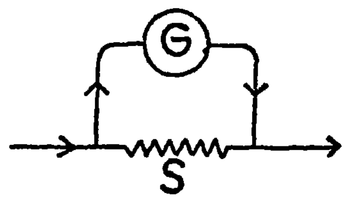


Fig 49 (*b*)

As the pointer moves only in one direction it is essential that *the current should enter only from one terminal* Hence + and — are marked on the terminals The current should always enter from the *positive terminal. The ammeters are always put in series in the circuit.* As the resistance of the instrument is extremely reduced, they do not change the value of the current when placed in a circuit They simply measure the current without changing it Their range can be changed by employing suitable shunt

§ 11. **Voltmeters** :—This is also a moving coil pivoted type of galvanometer In this case instead of a low value shunt *high resistance is put in series with the coil* This enormously increases the resistance of the instrument. As it is used to measure potential difference between two points in a circuit, it is put in parallel to those points between which *p d* is to be measured As it has got a very high resistance it takes up to very little current from the main circuit when connected in parallel Thus, the current strength in the main circuit remains undisturbed. *In this case also only a fraction of the* main current enters the instrument It is calibrated by the help of some standard instrument As in ammeters the zero of the scale starts from left hand, and the pointer moves towards right For similar reasons current should always enter from the *same termi-*

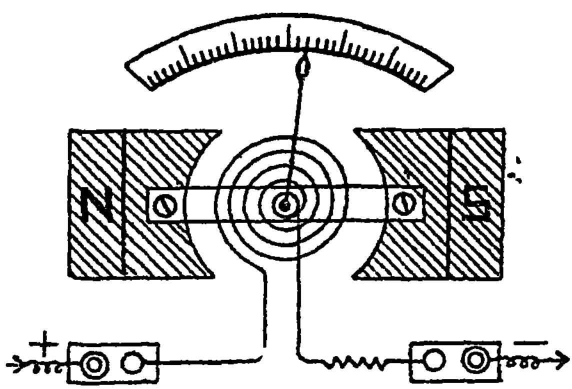


Fig 50 (*a*).

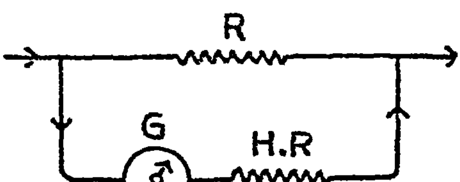


Fig. 50 (*b*)

nal, i.e., + marked at the top. The range of a voltmeter can also be changed by putting a suitable resistance in the series with the galvanometer coil.

§ 12. Resistance boxes :—Many times for the regulation of current we require standard values of resistances. For this purpose resistance boxes are made. A resistance box is a box of wood or ebonite on the top of it brass blocks are laid in lines with gaps between them. Plugs can be fitted in these gaps. Below the gaps resistance coils of fixed resistances are placed. To construct a proper resistance coil, desired length of resistance wire generally either of Constantan or *Eureka* is taken. It is doubled and wound upon a non-conducting cylinder as shown in fig. 31 (b). It is placed below two gaps. One end of the coil is connected to one block of brass, and the other end to the next block of brass. In this way between two blocks one coil is placed. Number of such coils of different resistances are placed in series. The value of the resistance of the coil is marked between the two blocks. Wires of Constantan or Eureka are used because they possess high specific resistance and low temperature coefficient. The wire is doubled to remove self-induction effect.

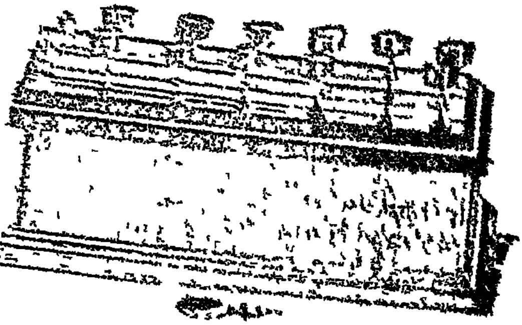


Fig. 31 (a).

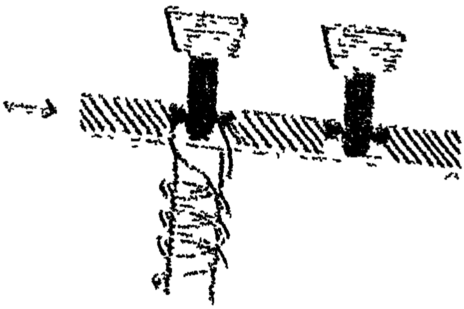
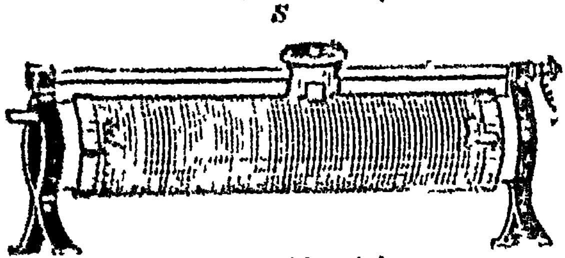


Fig. 31 (b).

the similar reasons is either of Constantan, or Manganin. The wire is wound on a non-conducting cylinder generally of china clay. Each turn is insulated from the other. A sliding contact S slides over the cylinder and makes contact with the wire. C is a terminal connected to the thick rod along which S moves If A and C are connected in the circuit, and if the current enters the rheostat at A, it passes through the wire between A and S and leaves at C. It does not flow between S and B. If S is moved towards A, the current passes through lesser and lesser number of turns. Consequently the resistance decreases and current increases Instead of this, if S is moved towards B, more resistance is introduced in the circuit and the current decreases. Similarly, if B and S are connected in the circuit, current will pass through BS. By moving S towards B the resistance in the circuit will decrease and *vice versa*. If A and B are connected in the circuit, it will behave as a fixed resistance. On the top of the instrument is usually something written like 22 ohms. 2 5 amp It means that the maximum current which can be passed through the rheostat is 2·5 amp without damaging it. If current exceeds 2 5 amp, coil will be burnt off due to excessive heating 22 ohms. denotes the maximum resistance it can offer when connected between A and B.



Rheostat
Fig 52 (a).

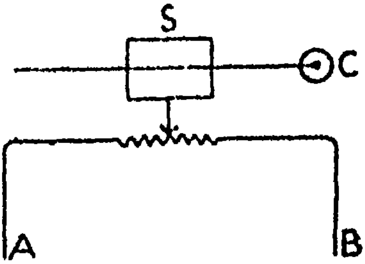


Fig 52 (b).

§ 14. **Verification of ohm's law:**—(for details see a text Book of Practical Physics by the authors :—The law can be easily verified by a simple experiment in the following way.

(i) Connect a battery B, a key K, a variable resistance S, an

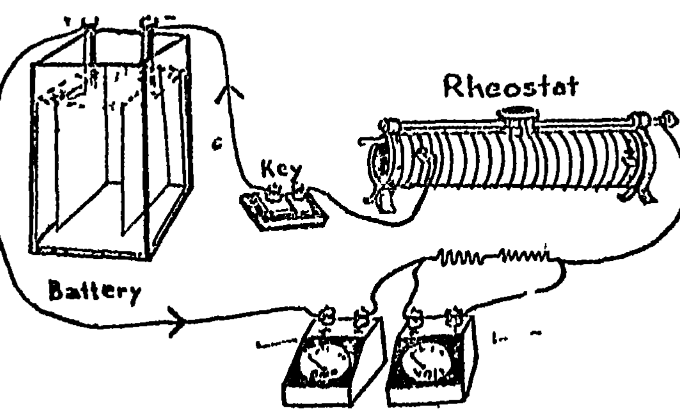


Fig 53 (a)

ammeter A and a fixed resistance coil R in series as shown in

fig. 53 (b) A voltmeter is put in parallel to the resistance coil R. Care should be taken to see that in ammeter and voltmeter current should always enter from the positive terminal.

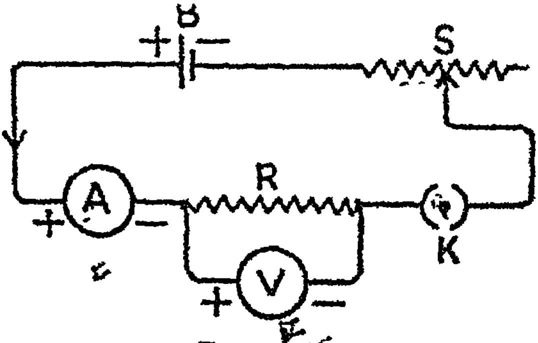


Fig 53 (b)

(ii) Pass current through R by inserting the plug in key K. Note down the value of p. d. across R by the help of the voltmeter. The ammeter will give you the value of the current passing through R. This will be one set. Let V and i be respectively the p d and the current. Then find the ratio between V and i.

(iii) Now change the resistance in the circuit by moving the sliding contact of the rheostat. It changes the current in circuit; consequently p d. across R will also change. Note down the new values of current and p d This will be another set In this way change the current in the circuit for a number of times, each time noting the value of V and i.

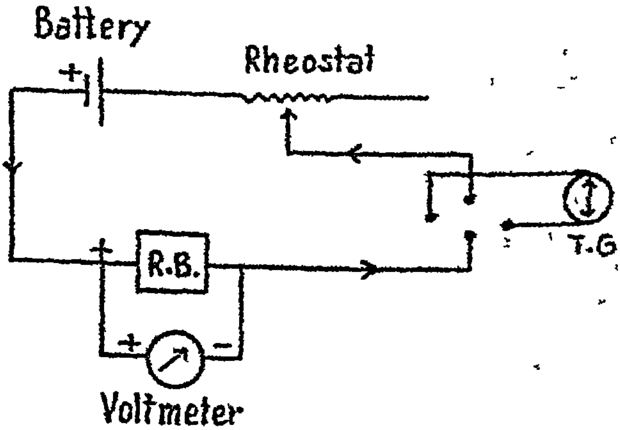


Fig. 53 (c).

(iv) *Find the ratio* $\frac{V}{i}$ *for each set. You will see that it comes out to be constant for each set which is equal to* $R\left(\frac{V}{i}=R\right)$. *It proves Ohm's law.*

The temperature of the resistance should not change throughout the experiment.

The law can be verified by a tangent galvanometer instead of an ammeter The procedure is exactly the same except that the ammeter is replaced by the tangent galvanometer in the circuit. Now the current is $i=K\tan\theta$, where K is the reduction factor of the tangent galvanometer and θ is the deflection. In this case $\frac{V}{K\tan\theta}$ will come out to be constant. [See fig. 53 (c)]

§ 15. Ohm's law applied to a circuit :—Internal resistance of a cell :—When a current passes through a cell it has to pass through the electrolyte constituting that cell The electrolyte offers certain amount of resistance to the passage of the current through the cell. *This resistance is called the internal resistance of*

the cell It varies from cell to cell *The primary cells have more internal resistance than secondary cells* which will be described in the next chapter.

The internal resistance of a cell depends upon the following factors

(1) **The Electrolyte :**—Different electrolytes will offer different resistances.

(2) **Size of the Plates :**—The larger is the size of the plates the lesser will be the resistance offered and *vice verse.*

(3) **The nearness of the Plates :**—The nearer are the two plates in the cells lesser will be the resistance offered and *vice versa*

If Ohm's law is applied to a circuit it will be found that a certain fraction of the e m f is spent in driving the current through the cell.

Connect a cell across a resistance R as shown in the fig. 54. Let E be the e m f. of the cell, B its internal resistance, R the external resistance and i the current flowing through the circuit Then by Ohm's law,

$$\text{Current} = \frac{\text{Total e m.f. in the circuit}}{\text{Total resistance in the circuit}}$$

$\therefore\ i = E/(R+B)$

or $E = iR + iB$

If V is the p d. across R in the closed circuit

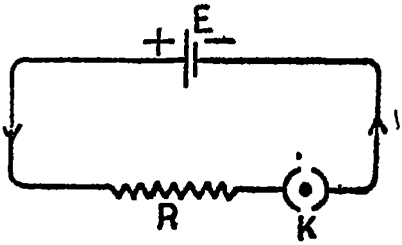


Fig. 54.

$$V = iR$$

or $$E = V + iB$$

Hence, in the closed circuit the p d across the plates which is available for outside circuit is less than the e m f by the amount iB. *Thus, the amount of p d $=iB$ is unnecessarily wasted. Thus, they are called lost volts* The amount of energy is spent in driving the current within the cell From above equation.

$B = \frac{E-V}{i}$, from this equation the internal resistance of a cell can be determined.

It has already been discussed that the e.m.f. comes into existance on account of the *chemical reactions taking place in the cell.* Thus, it remains the same so long as the plates and the electrolyte is the same on the other hand, *the potential difference is built up across the two plates on account of e m f. acting in the cell.* In the open circuit they are equal and act in the opposite direction In the closed circuit a part of the e.m f is consumed in driving the current within the cell, *hence, potential difference in the closed circuit is always less than the e m f.*

§16. Grouping of Cells :—

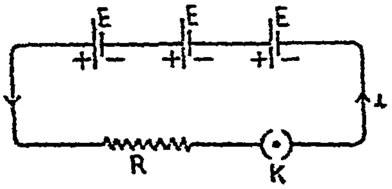


Fig 55 (*a*).

(1) Cells in Series :—When a number of cells are so arranged that the positive *pole of one is joined to the negative pole of the other, they are said to be arranged in series as shown* in fig. 55 (*a*) and 55 (*b*)

Let n be the number of cells joined in series, each possessing an internal resistance $= B$ ohms, and e m f. $= E$ volts. Let them be connected to an external resistance R ohms. Let i be the current through R. Then by Ohm's law

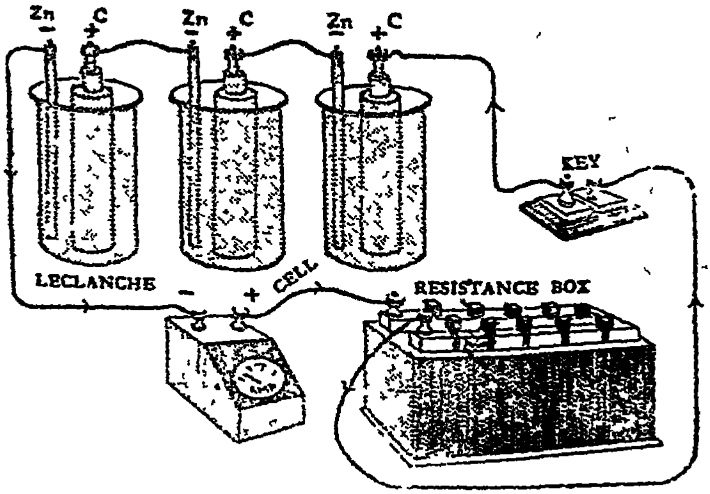


Fig. 55 (*b*).

$$\text{Current} = \frac{\text{Total e.m f in the circuit}}{\text{Total resistance in the circuit}}$$

Total e m.f. in the circuit $= nE$, as they are arranged in series, e.m f. will increase n times

Total internal resistance $= nB$ ohms.

External resistance $= R$ ohm.

$$\therefore \qquad i = \frac{nE}{nB+R}$$

Now two things are possible !

(i) *When R is small in comparison to nB, $i = E/B$ i.e., the current is the same as in the case of single cell and there is no advantage.*

(ii) *When R is quite large compared to nB, $i = \frac{nE}{R}$ i. e., the current increases n times in the circuit compared to a single cell. Hence if the internal resistance is smaller, to get strong current the cells should be arrrnged in series.*

—(2) **Cells in parallel** :—When a number of cells are so arranged that *all the positive poles are connected to the same point A and all the negative poles to the other point B, they are said to be arranged in parallel.* The points A and B respectively form the positive and negative pole of the resultant cell.

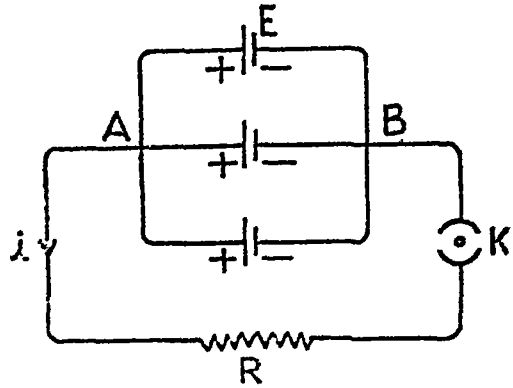


Fig 56 (*a*).

Let the number of cells be n, each possessing e m.f. $=E$ volts, and internal resistance $=B$ ohm. Let i be the current flowing in the external resistance R due to this arrangement See fig 56 (a) & (b)

As the cells are arranged in parallel their internal resistances are also arranged in parallel, hence the total internal resistance$=B/n$ ohm.

$\therefore$ Total resistance in the circuit $= B/n + R$

and total e.m f in the circuit $= E$

Therefore, by Ohm's law,

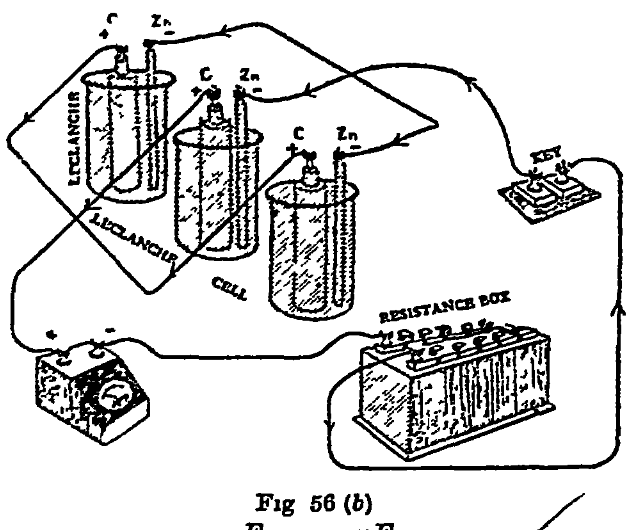


Fig 56 (*b*)

$$i = \frac{E}{B/n+R} = \frac{nE}{B+nR}$$

In this case also two things are possible

(*i*) *When* $R >> B$

$$i = \frac{nE}{nR} = E/R,$$ *i e., the current is the same as due to a single cell*

(*ii*) *When* $nR << B$

$i = nE/B$ *i e, the current increases n times Thus, if the cells possess high internal resistance, to get maximum current in the external resistance the cell should be arranged in parallel*

(3) **Mixed grouping** :—When the cells are so arranged that a few of them are in series while a few are in parallel, they are said to be arranged in mixed grouping. As shown in the diagram there are 12 cells arranged in mixed grouping They are connected in three rows, each row containing 4 cells

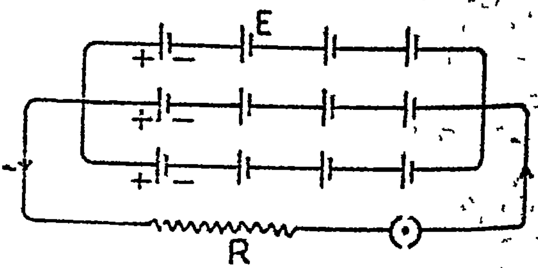


Fig 57.

For general case let there be m rows each containing n cells Let the e m f and internal resistane of each cell be respectively E and B Let R be the external resistance through which the current i is flowing.

As there are n cells in series, the e m f. acting in each row$=nE$. As all the rows are connected in parallel the resulting e m.f. acting in the circuit $= nE$.

The internal resistance acting in each row $= nB$

$\therefore$ Total internal resistance in the circuit due to m rows in parallel $= \dfrac{nB}{m}$.

$\therefore$ Total resistance in the circuit $= R+\dfrac{nB}{m}$

$$\therefore \qquad i=\frac{nE}{R+nB/m}=\frac{mnE}{mR+nB}.$$

Condition for Maximum Current —The current in the mixed grouping will be maximum only when certain conditions are satisfied. The above relation, $i=\dfrac{mnE}{mR+nB}$ can be put as,

$$i=\frac{mnE}{(\sqrt{mR}-\sqrt{nB})^2+2\sqrt{mRnB})}$$

It is very clear that i can be maximum only when the denominator on the right hand side is minimum Now in the denominator $2\sqrt{mRnB}$ is always positive and a constant term. Hence it will be minimum only when the other variable term $(\sqrt{mR}-\sqrt{nB})^2$ is minimum.

$$\therefore \qquad (\sqrt{mR}-\sqrt{nB})=0$$

or

$$mR=nB$$

or

$$R=\frac{nB}{m}.$$

The term on the right hand side is nothing *but equal to total internal resistance of the cells.* If the cells are arranged in such a way that they fulfill the above condition the current in the circit will be maximum

Thus, in a mixed grouping current in the external resistance will be maximum when the cells are so arranged that the external resistance (R) is equal to the total internal resistance $\left(\frac{nB}{m}\right)$

§17. **Solved problems:—1.** *A lamp of resistance 80 ohms takes a current of 0·75 amp. What voltage is required to work it?*

By Ohm's law $i=\frac{E}{R}$

or $$E=iR=0{\cdot}75\times80=60 \text{ Volts.}$$

2. *What length of wire of diameter 0·024 cm. and of specific resistance 48 micro ohms per cm. cube would be required to construct a coil having resistance = 1 ohm?*

From §2 we have

$$R=\sigma\frac{l}{s}.$$

In this problem $\sigma=48\times10^{-6}$ ohm cm.

$$S=\pi(.012)^2 \text{ sq. cm}$$

$$R=1 \text{ ohm}$$

$$\therefore \qquad l=\frac{3{\cdot}14\times(.012)^2}{48\times10^{-6}}\times1.$$

$$=9{\cdot}42 \text{ cm.}$$

3. *Two coils in series have a resistance of 18 ohm and in parallel a resistance of 4 ohm. Find their respective resistances.*

Let their individual resistances be respectively R_1 and R_2

$$\text{Then, } R_1+R_2=18$$

$$\text{and } \frac{1}{R_1}+\frac{1}{R_2}=4$$

Solving these two equations we get

$$R_1=12 \text{ ohm.}$$
$$R_2=6 \text{ ohm}$$

4. *A galvanometer of resistance 100 ohm can safely take a current of 1 milli amp. Calculate the resistance of the shunt if a current of 1 amp. is to be measured by it.*

The current (i) flowing in the main circuit is 1 amp., but only 1 milli amp. is to flow in the galvanometer. Rest of it should pass through the shunt. With usual notations we have

$$IG=i\frac{S}{S+G}$$

In this problem, $IG=1$ milli amp $=\frac{1}{1000}$ amp.

$$i=1 \text{ amp}$$
$$G=100 \text{ ohm}$$

$$\therefore \qquad \frac{1}{1000}=1\,\frac{S}{S+100}$$

or $$S=\tfrac{100}{999} \text{ ohm.}$$

5. *A milli. ammeter reading up to 5 milli. amp. l tance of 5 ohm. How would you convert it to measure (i milli. amp. (b) up to 100 volts*

(*a*) In this case let it be shunted by a resistance of S, then

$$IG = i\frac{S}{S+G}$$

or

$$\cdot 005 = 025\frac{S}{S+R}$$

or

$$S = \tfrac{5}{4} = 1{\cdot}25 \text{ ohms.}$$

Hence a resistance of 1 25 ohms. should be placed to its coil

(*b*) In this case let a high resistance of the value in series with its coil.

The current in the galvanometer=0 005 amp.

The total resistance in the circuit $= G+R = 5+R$.

The p d to be measured =100 volts.

$$\therefore \quad 005 = \frac{100}{R+5}$$

or

$$R = 19995 \text{ ohm.} \quad \textit{Ans.}$$

6 *An electric circuit contains a cell of e m f. internal resistance 0·5 ohm connected with three con resistances 1, 2 and 3 ohm. respectively all in series. difference of potential existing between the ends of the middle*

The external resistance in the circuit=1+2+3=6 o

Internal resistance =0 5

Total resistance in the circuit =6 5

Let i be the current in the circuit

$$\therefore \quad i = \frac{E}{R} = \frac{2}{6\,5} \text{ amp.}$$

As the current is the same in all the three coils, p the middle wire (V)$=iR$.

$$\therefore \quad V = \frac{2}{6\,5} \times 2 = 615 \text{ volt.}$$

7. *Find the resistance of a battery which in open c an e. m f. of 6 volts, and which when producing a current has a p. d. of 4 volts between its poles.*

Let the internal resistance of the cell be B; the $B = \dfrac{E-V}{i}$, where E is the e.m f in the open circuit, V its p d., and i is the current.

In this problem $E = 6$ volts.

$V = 4$ volts

$i = 2$ amp.

$$\therefore \quad B = \frac{6-4}{2} = 1 \text{ ohm.}$$

8 *A battery of 24 cells each of internal resistance 2 ohms., and e.m f. 1 4 volts is to be connected so as to send a maximum current through a wire of 12 ohms. Show how you will connect them, and find the current in each of the cells , also find the potential difference at the ends of the external resistance.*

Let the cells be arranged in mixed grouping having m rows each row containing n cells. Then by §16(3) the condition for maximum current is

$$mR=nB$$

In this problem, $R=12$ ohm and $B=2$ ohm.

$$\therefore \qquad 12m=2n$$

also $\qquad m\times n=24 \quad$ [As the total number of cells is 24]

Solving these two equations, we get

$$n=12, \qquad \text{and} \qquad m=2$$

Thus the cells should be arranged in two rows, each row containing 12 cells arranged in reries

From §16 (3), the current in the resistance R is

$$i=\frac{mnE}{mR+nB}=\frac{12\times 2\times 1{\cdot}4}{24+24}=70 \text{ amp.}$$

This divides into two branches , therefore in each branch the current will be $=\dfrac{\cdot 70}{2}=35$ amp.

Hence, in each cell the current flowing$=0$ 35 amp

Potential difference across the resistance$=i\times R$

$$=70\times 12=8\ 4 \text{ volts.}$$

QUESTIONS

1. State and explain Ohm's law. How can it verified in the laboratory ? (See § 1 & 14)

2 What do you understand by resistance and specific resistance of a wire ? A thick wire possesses more resistance or a thin wire of the same length and material ? Explain (See § 2 & 3)

3 Define absolute units of current, potential difference and resistance and obtain practical units from them. Name the instruments by which each is measured. (See § 6)

4. Define a volt and an ampere (See § 6)

5 Enunciate and prove the laws of resistances in series and parallel In which combination the resultant resistance will be more and why ? (See § 7 & 8)

6. Explain the uses of shunts What is the resistance of a shunt which when joined to a galvanometer of resistance g will cause $1/n$th of the total current to flow through the galvanometer (See § 9)

7 Give the full theory of shunts How can you make use of the theory in altering the range of a given ammeter (See § 9 & 10)

8 What is an ammeter ? How is it used ? How does it differ from a voltmeter ? (See § 10 & 11)

9 Describe how you could convert a galvanometer into ammeter or voltmeter of any desired range ? (See § 10 & 11)

10 What do you understand by the internal resistance of a cell? How can you find it experimentally? (See § 15)

11. What do you understand by mixed grouping of cells? How will you arrange a number of cells given to you in mixed circuit to get maximum current in an external resistance. Derive the condition required. (See § 16)

NUMERICAL PROBLEMS

1. A coil of resistance 20 ohms is to be constructed of a wire of diameter 0·146 mm. and specific resistance 5×10^{-6} ohm per cm.^{3}. What length of the wire will be required? (*Ans.* 664 4 cm.)

2. One gm. of copper wire is to be drawn into a wire (*a*) 0·5 cm. radius, (*b*) 1 cm. radius Compare their resistances. (*Ans* 16 : 1)

3 You are given two resistance boxes each containing resistance coils of 1, 2 and 5 ohm Show how will you arrange them to introduce a resistance of 2 1 ohms in the circuit.

[7 ohm. from the first box and 3 ohm from the second box, and then the two boxes must be put in parallel *Ans.*]

4 The same current is made to pass through a metre long wire of 0 05 cm radius, and another wire of copper 2 metres long. The potential difference between the ends of the first wire is 1 volt, and that between the ends of the second wire is 20 volts. Find the diameter of the second wire

(*Ans.* $0\ 1/\sqrt{10}$ cm.)

5. A moving coil ammeter has a resistance of 50 ohm and gives a full scale deflection for a current of 0 5 milli amp. How can it be converted to measure (*a*) up to 200 volts (*b*) up to 2 amps.

[(*a*) A resistance of 39050 ohm. should be placed in series with coil. *Ans.*

(*b*) A shunt of the value of 0·025 ohm. Should be placed in parallel to the coil. *Ans*]

6. A voltmeter has a resistance of 1000 volts and range of 15 volts. How can it be converted to read up to 150 volts.

(*Ans* 9000 ohm resistance should be placed in series)

7 A cell in the open circuit has e.m.f. of 1·5 volts. In closed circuit when the current is 0 05 amp., the p d. between the poles of the cell is 1 2 volts. Calculate the internal resistance of the cell (*Ans.* 6 ohms)

8 There are 48 cells, each of e m.f 1 8 volts and internal resistance 3 ohms How would you arrange the cells so that they may send maximum current through an external resistance of 50 ohms. What will be the current in the resistance. (*Ans* 2 rows of 24 cells each ; 5 02 amp)

CHAPTER VIII

CHEMICAL EFFECTS OF CURRENT

§1 **Electrolysis :—Definitions**—When an electric current passes through solids no chemical changes take place in them. But when it is passed through conducting solutions (*e g*, solutions of salts, acids and bases) they get decomposed into their chemical constituents The two parts into which the solution dissociates travel in the opposite direction The liquid which gets decomposed is called the *electrolyte* The phenomenon of decomposition of a solution due to the flow of current through it is known as *electrolysis*. The vessel containing the electrolyte is called the *voltameter or an electrolytic cell*. The two conductors through which the current enters and leaves the voltameter are known as *electrodes*. The electrode through which the current enters is called *anode*, while that through which it leaves is termed *cathode* The constituent parts into which the electrolyte gets decomposed are known as *ions* The ions travelling towards anode (positive plate) and cathode (negative plate) are respectively called *anions and kations*.

§2. **Theory of electrolytic dissociation :**—Arrhenius thought that even by the mere act of solution an electrolyte is broken up into two parts known as ions Ions are nothing but *atoms or cluster of atoms carrying an electric charge. Thus they are not neutral.* The more is the solution diluted the more will be the number of ions present in the solution. When a neutral atom breaks up it gives rise to two types of ions carrying opposite charges The two types of ions carrying opposite charges remain moving in the solution. When a potential difference is applied to the solution through the electrodes, attraction takes place between the electrodes and the ions The positively charged ions are attracted by cathode and moves towards it For similar reasons the negatively charged ions go towards the anode. Hence they are called kations and anions. As the ions carry charge their *movement in the cell constitutes current*

Suppose we take a dilute solution of $CuSO_4$. It will break up into positive ions of copper and negative ions of SO_4. When p d is set up between the two electrodes Cu ions will travel towards cathode and SO_4 ions will travel towards anode—constituting a current in the cell

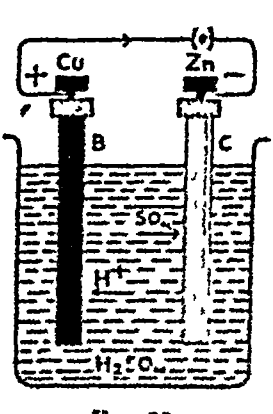


Fig 58

§3 **Faradey's law of electrolysis** — Michael Faradey studied the phenonmenon of of electrolysis in detail. From his numerous observations he arrived at certain conclusions known as Faraday's laws of electrolysis.

1st law :—*The mass of ions liberated by a current in a given time is proportional to the total quantity of electricity passing through it in that time*, *i e.*, it is proportional to the

product of the current and time for which current is passed. Let m be the mass of the ion deposited by passing current i for t seconds, then according to this law we have

$$m \propto Q \text{ (if } Q \text{ is the total quantity of electricity passed)}$$
$$\propto i\, t \; [Q = i.\, t]$$

or $$m = e\, i\, t \quad \text{... (1)}$$

where e is a constant called the **electro chemical equivalent of the element**

2nd law —*If the same strength of current is passed through a number of voltameters containing different electrolytes for the same time, the mass of different ions liberated will be proportional to their chemical equivalents, (i.e., chemical weights).*

If the same current passing for the same time through various electrolytes liberates m_1, m_2 m_3, ... gm of different elements having w_1, w_2, w_3...... equivalent weights, then according to this law,

$$m_1 \quad m_2 \quad m_3 \quad \ldots\ldots\ldots \quad w_1 : w_2 : w_3 \quad \ldots\ldots$$

§4 **Voltameters** —

(1) **The Voltameter** —**Description** It consists of a glass vessel nearly filled up with 15 to 20% acidulated solution of copper sulphate. Three plates of copper are dipped in the solution as shown in fig. 59. The two outer plates AA are joined together and have a common binding screw. They constitute the *anode* through which the current enters. The middle plate also carrying a binding screw forms the *cathode* *The cathode is well insulated from the anode plates.* It is movable and so can be taken out, dried and cleaned (see fig. 59).

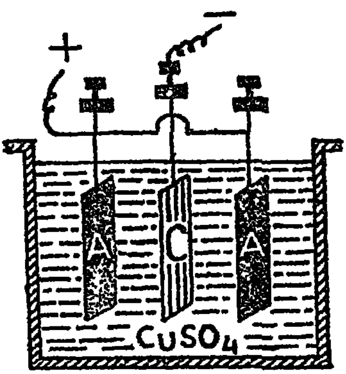


Fig 59

Working. By the very act of solution the solution of $CuSO_4$ breaks up into ions according to the following equation

$$CuSO_4 = Cu^{++} + SO_4^{--}$$

When current is passed through the voltameter, the positive ions of copper move towards cathode They give up their charge to the cathode and are deposited there Thus as the time passes the weight of the cathode increases. On the other hand the negative ions of SO_4 move towards anode. They react with the anode plate forming $CuSO_4$

$$SO_4^{--} + H_2O = H_2SO_4 + O^{--}$$

The liberated negative ion of oxygen reacts with copper forming CuO

$$Cu^{++} + O^{--} = CuO$$

CuO reacts with H_2SO_4 forming water and $CuSO_4$

Thus, as copper dissolves in solution from anode, its weight goes on decreasing. Any way, the strength of the solution remains the same, because the amount of copper liberated from the anode is equal to the amount of copper deposited on the cathode.

This type of voltameter is often used as a current measuring instrument.

(2) **The silver voltameter :—Description** It consists of a platinum or a silver cup *A* containing solution of silver nitrate. A rod of silver (*c*) is suspended inside the cup as shown in fig 60. The rod *c* forms the *anode* while the cup constitutes the *cathode*

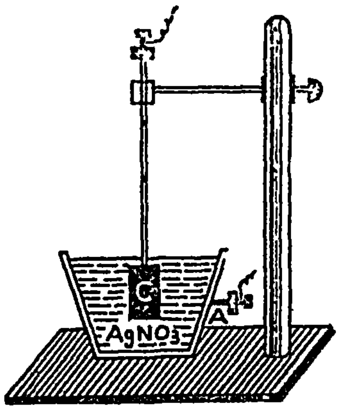


Fig. 60

Working. The solution breaks up as

$$AgNO_3 = (Ag)^+ + (NO_3)^-$$

When a current is passed, the positive ions give up their charges to the cup and are deposited there. Hence, the weight of the cup, *i.e.*, cathode increases. On the other hand, the negative ions react with anode forming silver nitrate which passes into the solution

$$(Ag)^+ + (NO_3) = Ag\,NO_3$$

Thus the weight of anode goes on decreasing and the strength of the current remains the same.

3 **The water voltameter. Description :—**It consists of a glass vessel *M* (see fig 61). Two platinum electrodes *C* and *D* are sealed at its bottom. The vessel is filled up with water. As ordinary water does not conduct electricity, a few drops of H_2SO_4 are added to it to make it more conducting. Two test tubes filled with water are inverted over the two electrodes. They are used to collect gases during the electrolysis

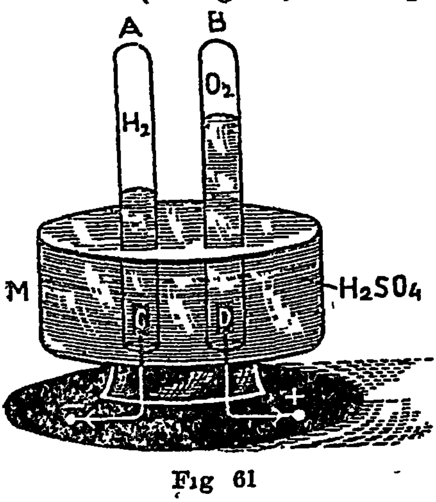


Fig 61

Working :—The acidulated water breaks up as

$$H_2SO_4 = H_2^{++} + SO_4^{--}$$

When a current is passed, the positive ions of hydrogen travel towards cathode. They give up their charges to the cathode, and hydrogen gas is evolved which is collected in the tube *A*. The negative ions of SO_4 go towards anode liberating ions of oxygen

$$SO_4^{--} + H_2O = H_2SO_4 + O^{--}$$

This is known as the secondry reaction at the anode. Oxygen ions deliver their charges to the anode and escape

in the form of gas. The gas is collected in the tube B. From the above relation it is quite clear that H_2SO_4 again comes back in the solution, and thus it is only water which undergoes chemical decomposition forming H_2 and O_2 according to

$$2H_2O = 2H_2 + O_2$$

The volume of the gas collected in the tube A will be found to be double than that collected in the tube B *i e.*, the volume of H_2 will be double than that of O_2 This result tallies with theoretical considerations. If the gases in the tubes A and B are put to test, they will be found to be hydrogen and oxygen. Hence, it is actually the hydrolysis of water

§5 Verification of the laws of electrolysis :—

Verification of the first law :—Take a copper voltameter, clean its cathode plate and weigh. Pass current through the voltameter for t_1 seconds. Take out the cathode and again weigh. The difference between the two weights will be the mass of the copper deposited Let it be m_1 gm. Now repeat the same procedure and pass the same current for t_2 seconds. Let the mass of the metal deposited in this case be m_2 gm Then it will be seen that

$$\frac{m_1}{m_2} = \frac{t_1}{t_2} \text{ when current is constant}$$

$$\therefore \quad m \propto t \text{ when } i \text{ is constant}$$

Now pass different currents i_1 and i_2 for the same time t second and determine the masses deposited Let them be m_3 and m_4 gm respectively. Then it will be seen that

$$\frac{m_3}{m_4} = \frac{i_1}{i_2} \text{ where } t \text{ is constant}$$

$$\therefore \quad m \propto i$$

From the above two equations we get

$$m \propto i\,t, \text{ which proves the law.}$$

Verification of the second law :—Put three voltameters of copper, silver and water in series as shown in fig. 62 Pass the same amount of current for the same time through them. Find out the masses of different ions liberated Let the masses of oxygen, hydrogen, copper and silver liberated be respectively m_1, m_2, m_3, and m_4. If w_1, w_2, w_3 and w_4 are respectively their chemical equivalents, it will be found that $m_1 : m_2 : m_3 : m_4 \cdots = w_1 : w_2 : w_3 : w_4$

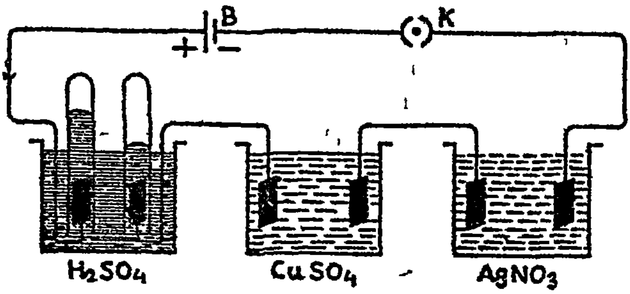


Fig. 62.

$$\text{or} \quad \frac{m_1}{m_2} = \frac{w_1}{w_2} \text{ and so on, which proves the law.}$$

§6. **Electro chemical equivalent** :—From Faraday's first law we have

$$m = e\,i\,t$$

with usual notations. As explained e is a constant called the electro chemical equivalent of that substance. If we put $i=1$ amp.. $t=1$ second

$$e = m$$

Thus, the electro chemical equivalent of an element can be defined as the mass of the element deposited when one amp current flows through the solution for one second It is denoted as e c e. The electro chemical equivalents of a few elements are given below —

Copper	000329	gm. per coulomb.
Silver	·001118	,, ,, ,,
Zinc	·000338	,, ,, ,,
Hydrogen	000010	,, ,, ,,

From Faraday's second law if m_1 and m_2 are the masses of the two substances deposited due to the flow of current i for the same time

$$\frac{m_1}{m_2} = \frac{w_1}{w_2},$$ where w_1 and w_2 are their equivalent weights. But from first law

$$m_1 = e_1 i\,t$$
$$m_2 = e_2 i\,t$$

where e_1 and e_2 are the electro chemical equivalents of the substances.

$$\therefore \qquad \frac{e_1\,i\,t}{e_2\,i\,t} = \frac{w_1}{w_2}$$

$$\text{or} \qquad \frac{e_1}{e_2} = \frac{w_1}{w_2} \qquad . \quad (1)$$

from this equation if w_1, w_2 and e_1 is known, e_2 can be determined and *vice versa*.

§7. **Experimental determination of e c.e. of copper** :— Connect a copper voltameter V, a commutator key K, a tangent galvanometer $T\,G.$, a rheostat S and a battery B in series as shown in fig. 63 First of all, by the help of the rheostat adjust the current in the circuit in such a way that the galvanometer gives a deflection of near about 45°. Then stop the current and remove the cathode plate from the voltameter. Thoroughly clean the plate, dry and weigh. Put the plate again in the voltameter and by the help of the key allow the current to pass through the circuit Care should be taken to see that the current remains uniform i e, the deflection should not change. It can be realised by the

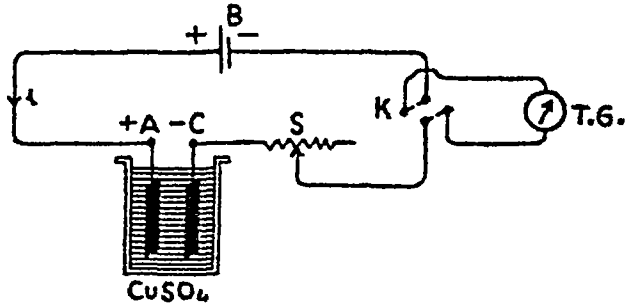


Fig. 63.

adjustment of S. Note the deflection in $T.G.$ Let this be θ_1 and θ_2 After nearly 20 minutes reverse the current in the tangent galvanometer and allow the current to flow for the next 20 minutes. Let the deflection in galvanometer in this case be θ_3 and θ_4. The mean of these four readings will give you the mean deflection θ. Now remove the cathode plate, Again clean, dry and weigh. The difference between the two weights will give you the mass of the copper deposited. Let it be m gm Let i be the current flowing through the galvanometer for an interval t seconds, then by Faraday's first law

$$m \propto i\,t$$
$$= e\ i.\ t. \text{ where } e \text{ is the e c e of copper}$$

But
$$i = \frac{10RH}{2\pi n} \tan\theta,$$

where R is the radius and n the number of turns in the coil of the tangent galvanometer. H is the earth's horizontal component

or $i = K \tan\theta$, where K is the reduction factor of the galvanometer

$$\therefore \quad m = e\,K \tan\theta . t$$

or
$$e = \frac{m}{K \tan\theta\ t}$$

Thus if K is known e can be determined. In the same way if e is known the reduction factor of the galvanometer can be found out. Knowing e c e of copper, e c e of any other substance can be determined by equation. (1) of the last article

§8. Voltameters as current measuring instruments:— Voltameters are regarded to be as extremely standard current measuring instruments. According to Faraday's first law if e.c.e. of a substance is known, the current in the circuit can be calculated. As the mass of the metal deposited, and time for the flow of the current can be determined with fairly high accuracy, the current determined by voltameters is also fairly accurate. For the same strength of current the amount of silver deposited is much more than that of copper. Silver voltameters are preferred where accurate measurement of the current is the object. Ammeters are calibrated with their help Rather even an ampere is defined in terms of the silver deposited *An ampere is defined as the amount of current which will deposit 0·00118 gm of silver in one second.*

§9. Faraday's constant:—Faraday proved that a constant amount of charge is always required to liberate gm equivalent of any element For example e c.e of copper is 0011183 gm. per coulomb, and its equivalent-weight is 107·88 gm (because it is monovalent)

$\therefore$ 0·0011183 gm of silver is liberated by 1 coulomb

$\therefore$ 1 gm equivalent (i e 107 gms.) of silver will be liberated by

$$\frac{107}{0011184} = 96470 \text{ Coulombs}$$

As the masses liberated are proportional to equivalent weights, 96470 coulombs of charge will always be required to liberate gm.

equivalent of any ion. *Thus, it is a constant quantity and is called Faraday's constant.*

To liberate 1 gm atom of a monovalent substance 96470 coulombs of charge are needed If the substance is divalent, double the charge will be needed for a gm. atom, and so on. If there are n atoms in a gm. atom of any mono-valent substance each carrying a charge e,

$$ne=96470$$

But the accepted value of n is equal to $6\ 16\times10^{23}$

$$\therefore \qquad e=\frac{96470}{6\ 16\times10^{23}}=4{\cdot}77\times10^{-10} \text{ e s u.}$$

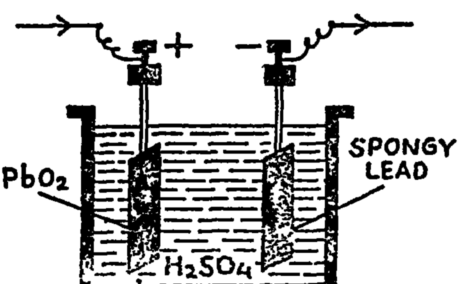
This is the smallest amount of charge carried by an ion.

§ 10. **Secondary cells :**—The primary cells have already been described in chapter I. The e m f developed in them is due to the chemical reactions taking place in their electrolytes These chemical reactions are *irreversible.* After the production of the current, the products of the reaction are wasted and cannot be transformed into original substances But there are other types of cells also in which the reactions are reversible and the products are not wasted They are called *secondary cells or accumulators.*

When current is made to pass through them electrolysis takes place. Electrical energy is converted into chemical energy and is stored up in the cell. When the cell is connected to an external circuit, i e, when current is drawn from it, the chemical energy is converted back into electrical energy. *The original substances are again obtained Thus, the difference between a primary cell and a secondary cell is that in the primary cell chemical energy is converted into electrical energy, while in secondary cells electrical energy is first stored up in the cell as chemical energy, and this chemical energy is then converted into electrical energy* As the current is obtained by secondary reaction they are called secondary cells The process of converting electrical energy into chemical energy is known as *charging the cell,* while the process of getting back the electrical energy from chemical energy is *known as discharging it.*

There are two types of secondary cells (1) **Acid accumulators** (2) **Alkali accumulators.** In this book we shall consider only the acid accumulators

§ 11. **Acid accumulators :**—It consists of a glass vessel containing dilute sulphuric acid. Two lead plates as shown in fig 64 are dipped in the solution The plates are constructed in the *form of grids or net works as shown in fig 65. in the interstices of which is filled litharage (PbO)* PbO acts with H_2So_4 to form $PbSO_4$ Thus, to start with both the plates contain a mixture of *PbO* and *$PbSO_4$* The density of the acid is 1 17 to 1 19



Fig, 64.

Changing :—Current is passed in the cell from an external source, say, D.C mains Due to this, hydrolysis of water takes place. Hydrogen is evolved at the cathode while oxygen travels towards anode. The reactions are as follows :—

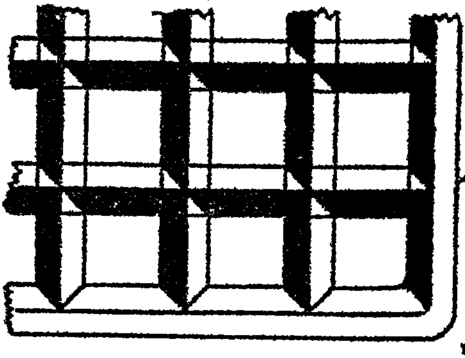


Fig. 65.

At the positive plate :—

$$PbO + O = PbO_2 \qquad ...(1)$$

$$PbSO_4 + O + H_2O = PbO_2 + H_2SO_4 \qquad ..(2)$$

At the negative plate :

$$PbO + H_2 = Pb + H_2O \qquad ...(3)$$

$$PbSO_4 + H_2 = Pb + H_2SO_4 \qquad ...(4)$$

By charging positive plate is converted into dark brown peroxide of lead, while the negotive plate becomes spongy lead. According to reactions (3) and (4) the amount of sulphuric acid increases in the solution ; hence its density goes up When it is completely charged, the density becomes near about 1 25 to 1·28. Its e m.f becomes near about 2·2 volts.

Discharging. Currents can be obtained from such a cell, when it has been charged. When currents are drawn in the external circuit, it flows from anode to cathode in the outer circuit, and just in the reverse direction in the cell. Again the hydrolysis takes place. But now as the direction of the current has become reversed hydrogen is evolved at the anode, and oxygen at the cathode according to the following equations :

At the positive plate ·—

$$PbO_2 + H_2 = PbO + H_2O \qquad ...(5)$$

$$PbO + HSO_4 = PbSO_4 + H_2O \qquad ...(6)$$

At the negative plate :—

$$Pb + O = PbO \qquad ...(7)$$

$$PbO + H_2SO_4 = PbSO_4 + H_2O \qquad ...(8)$$

The reactions are self-explanatory When the cell is discharged, the original products PbO and $PbSO_4$ are again obtained. Water is formed during discharge lowering the density of sulphuric acid. It again reaches the initial value of 1·17 to 1·19. Thus, the cell regains its initial conditions, and can be recharged. To know about its electrical condition the specific gravity of the acid should be occasionally measured by a hydrometer.

Notes :—(1) It should not be charged by passing a *heavy current* ; otherwise the plates will get damaged and the material will *heal off falling upon the base.*

(2) When it discharges, its e.m.f. becomes near about 1·8 volts and specific gravity equal to 1·17. No current should be taken from this cell after this stage , otherwise *insoluble lead sulphates will be formed greatly affecting the efficiency of the cell.*

(3) The capacity of the cell is expressed in *ampere-hours* which shows the total amount of current which can be drawn from such a cell when it is fully charged. If the capacity is 40 ampere hours, it can give a current of 40 amperes for one hour, and a current of 10 amperes for four hours etc.

(4) Its terminals should never be short circuited, otherwise huge current will pass through it damaging the plates. This is due to its internal resistance being very low.

Generally, to increase its capacity instead of two plates, a number of plates are taken The plates are arranged in parallel alternately connected to the two electrodes.

§ 12. Advantages :—The accumulator possesses the following advantages over that of primary cells :—

(1) It has got a number of plates which possess large areas, and are placed very close to each other This arrangement extremely reduces the internal resistance of the cell which is of the order of 0 0001 to 0 001 ohm. As it has got a high e m f and low internal resistance, heavy and steady currents can be obtained from such a cell This is not possible in the case of primary cells which have got very high internal resistance

(2) As the reactions are reversible, it can be recharged.

(3) It can be used for lighting buildings, operating cars, etc. where strong currents are needed.

Despite all these advantages, it has got a few disadvantages also. It is very heavy and therefore, cannot be transported easily. Its cost is quite appreciable in comparison to primary cells Apart from this, it requires very careful handling. If it is not properly charged at the proper time it will be rendered useless.

§ 13. Practical applications of electrolysis :—Electrolysis finds a number of useful applications in industry The following are a few of them —

(1) Electroplating, (2) electrotyping, (3) refining of metals, (4) manufacture of chemicals (5) medical applications etc.

Out of these electroplating is the most important which we shall consider in detail

Electroplating :—It is a process by which the baser metals, *e g, copper, iron, etc* are coated with a *fine layer of more attractive or durable metals like gold, silver, nickel etc* It is done to impart lustrous appearance to any article or to protect it from oxidation. It is called gold plating, silver plating, or nickel plating depending upon the nature of the coating desired For example, if, silver is to be placed over a brass piece, take a glass vessel and fill it with with solution of silver nitrate Make the brass piece as cathode and a pure silver rod as anode When a constant current is passed through the solution, a fine layer of silver will be deposited on the brass piece. For layers to be smooth and uniform, current employed should be smaller in strength, otherwise the layers will be chopy and peel off soon Thus, the articles to be plated should be made of cathode, while the metals to be deposited should be made anode

§ 14. **Solved problems :—1.** *The mass of copper deposited in a copper voltameter is 0·75 gm. in 10 minutes. Find the current flowing through the voltameter [e.c.e. of copper=·000328 gm per coulomb]*

From the first law of electrolysis $m=e\,i\,t$

Substituting the values in this equation we get

$$0{\cdot}75 = {\cdot}000328\, i \times 10 \times 60$$

$$\therefore \quad i = 381 \text{ amp.}$$

2. *Three voltameters containing solutions of copper sulphate, silver nitrate and sulphuric acid are connected in series. A current of 10 amp. is passed for 5 hours through them Calculate the masses of silver, copper and hydrogen liberated [e c e. of silver =0·00118 gm /coulomb, at. wt. =107·88, at. wt. of copper=63 57 ; and at weight hydrogen=1·008]*

From the first law of electrolysis if m is mass of the silver deposited

$$m = e\,i\,t, \text{ substituting the values}$$

$$m = 0{\cdot}00118 \times 10 \times 5 \times 60 \times 60$$

$$= 201{\cdot}24 \text{ gms. of silver.}$$

If m_1 and m_2 are the masses of copper and hydrogen liberated, according to Faraday's second law they are proportional to their chemical equivalents. As equivalent weights = atomic weight devided by valency

$\therefore$ Chemical equivalent of silver $= \dfrac{107\,88}{1} = 107{\cdot}88$

Chemical equivalent of Copper $= \dfrac{63\,57}{2} = 31{\cdot}78$

Chemical equivalent of hydrogen $= \dfrac{1\,008}{1} = 1{\cdot}008$

$$\therefore \quad \frac{m}{m_1} = \frac{201\,24}{m_1} = \frac{107\,88}{31\,78}$$

or $\quad m_1 = 59\,38$ gm

Similarly $\quad \dfrac{m}{m_2} = \dfrac{201\,24}{m_2} = \dfrac{107\,88}{1\,008}$

or $\quad m_2 = 1{\cdot}88$ gms.

Thus, the weight of silver deposited $= 201\,24$ gm.
the weight of copper deposited $= 53{\cdot}38$ gm.
the weight of hydrogen deposited $= 1{\cdot}88$ gm.

3 *A copper voltameter is connected in series with a tangent galvanometer and a battery. In half an hour the mass of copper deposited is 0·2988 gm The deflection in the galvanometer is 45°. Find the reduction factor of the tangent galvanometer [e c.c. of copper =·00033 gm /coulomb]*

From the formula arrived at in the above article, we have

$$\_\frac{K \tan\theta}{}$$

$$\therefore \qquad K=\frac{e\,m}{\tan\theta.\,i}.$$

Substituing the values, $K=\dfrac{0003\times0\ 2988}{\tan 45^\circ\times30\times60}$

or $K=\dfrac{00033\times0\ 2988}{1\times30\times60}$ (for $\tan 45^\circ=1$.)

$=0\ 50$ amp

QUESTIONS

1 What do you understand by electrolysis ? (See §1)

2 State and explain Faraday's laws of electrolysis How can they be verified in the laboratory ? (See §2 and 4)

3 What is the principle of a voltameter ? Describe Copper, Silver and water voltameter. Give the necessary theory. (See §3)

4 Define electro-chemical equivalent of an element How can it be determined in the laboratory by the help of a tangent galvanometer ? (See §5 and 6)

5. Describe how would you determine the ratio between the electro-chemical equivalent of two substances ? (See §5 and 6)

6 How can you measure current by voltameters ? Are they better than ammeter ? (See § 7)

7 Define Faraday's constant. (See § 8)

8. What is the difference between a primary cell and a secondary cell (See § 9)

9 Describe in details giving diagrams the action and working of a lead accumulators, what are its merits and demerits ? (See §9 and 10)

10 Briefly mention the different industrial applications of electrolysis, and describe electro plating in details. (See § 11)

NUMERICAL QUESTIONS

1. How long should you pass an ampre of current in a silver voltameter to get a deposit of 0 11183 gm of silver; e c e of silver being 0 0011183 gm. per coulomb (*Ans* 1 min 40 sec)

2. Three copper voltameters in parallel are connected to a battery with resistance If after 30 minutes the deposits are 0 763, 0 742, and 0 785 gm respectively, find the strength of the current drawn from the battery. (e c e of copper= 000329 gm per coulomb) (*Ans* 3 86 Amp.)

3 Calculate the value of the current required to deposit 0 972 gm of chromium in 3 hours, if e c e of chromium is 0 00018 gm. per coulomb (*Ans.* 0 5 Amp.)

4 A copper voltameter is put in series with a tangent galvanometer having 10 turns and radius 5 cm. If the deflection is 60°, calculate the mass of copper deposited in 30 minutes [e c e of hydrogen=·C0001045, H=0 36; and atomic weight of copper (divalent)=63 57] (*Ans* 0 296 gm)

CHAPTER IX

ELEMENTARY IDEAS ABOUT ELECTRO-MAGNETIC INDUCTION

§1. Electro-magnetic induction :— Induced currents :— Whenever a current is passed through a conductor magnetic field is produced. This was discovered by Oersted and has been treated in details in Chapter II Faraday argued that if it was true, why its converse should not also be true, *i e.*, why a magnetic field should not also give rise to an electric current? Thinking in this direction he was able to demonstrate in the year 1831 that *whenever, the number of lines of magnetic force passing through a circuit changes, induced current is generated in the circuit The current lasts only while the number of lines of force passing through the circuit varies* This can be easily achieved by either *moving a magnet near the circuit or moving a circuit near the magnet* The electro motive force so developed in the circuit *is called induced e m. f* Whereas this phenomenon is known as *electro-magnetic induction.* The number of lines of force threading the circuit is known as *magnetic flux*

§2. Production of induced currents :—

(*i*) **By the motion of a magnet near a coil :**—Connect the two ends of a coil *C* of insulated wires to the terminals of a sensitive galvanometer *G* Let the pointer in the galvanometer give deflection towards right when the direction of the current in the coil is anti-clockwise, and towards left when the direction is clockwise, (It can be ascertained by connecting a cell in series with the coil and noting the deflection in the galvanometer before doing experiment for obtaining induced currents). Then move *a strong bar magnet NS towards the coil inserting the north pole of the former into the latter* as shown in the figure The motion of the magnet varies the magnetic flux linked with the coil *Consequently induced current is developed in the coil* and the galvanometer needle will be deflected The pointer moves towards right showing that the direction of the induced current produced in the coil *is anti clockwise* if you stop the motion of the magnet, variation in the magnetic flux also stops - Consequently the induced current ceases to exist and the galvanometer shows no deflection. This clearly demonstrates the fact that current lasts only for the time when the *flux is changing* and is therefore, *temporary.* The *faster is the motion* of the magnet, rapid will be the change in the magnetic flux, and more is the induced e m. f. produced in the coil. This will be indicated by the pointer of the galvanometer.

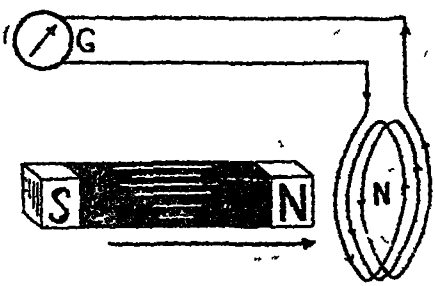


Fig. 66

Now withdraw the magnet swiftly from the coil as shown in fig 67. Again the flux changes and induced current is produced in the coil. But in this case the deflection of the pointer will be *towards left*, showing that now the current in the coil is flowing in the *clockwise direction*.

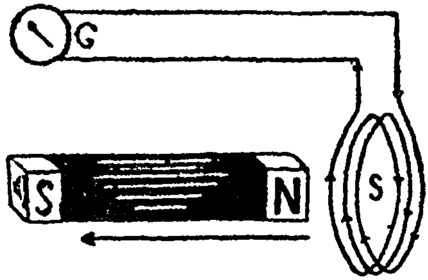


Fig. 67.

From Chapter V §5, we know, that when current in a coil is flowing in the anticlock wise direction the face of the coil from the observer's side will behave as a north pole, whereas if it is flowing in the clock wise direction the face will behave as a south pole Thus, when the north pole is moved towords the coil, the face of the coil facing the pole behaves as a north pole On the other hand when the same pole is withdrawn, the face behaves as a south pole If instead of the north pole, south pole is inserted in the coil deflections in the galvanometer will be reversed The above mentioned results can be tabulated as follows .—

	Motion of the magnet	Direction of the induced current	Polarity of the near face of the coil
1	*N* pole inserted	Anticlockwise	North
2	*N* pole withdrawn	Clockwise	South
3	*S* pole inserted	Clockwise	South
4	*S* pole withdrawn	Anti clock wise	North

(ii) **By Starting or stopping current in a coil** :—Connect a coil *P* of insulated wires in series with a battery *B*, a variable

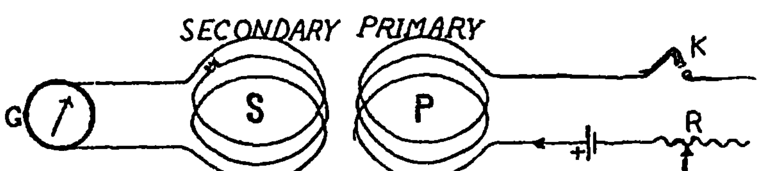


Fig 68.

resistance *R* and a press key *K* Another coil *S* also of insulated wires connected to a sensitive galvanometer *G* is placed near the former The former coil *P* is called the primary and the latter coil *S* is called the secondary Now press *K* so that the current passes through the primary As soon as the key is pressed the galvanometer will give a sudden deflection, indicating the production of the induced current in the secondary The current so produced is temporary and last for a very short duration It is reduced to zero when the current in the primary attains a steady value The direction of the induced current *C* in the secondary) so produced at the make of the circuit, *is opposite to the direction of the current flowing*

in the primary. Hence, this current is called *inverse current.* Similarly when the circuit is broken by releasing K, again the galvanometer will register a deflection But now the deflection will be *opposite to that of the former, i e*, in this case the direction of the induced current produced in the secondary will be the *same as* that of the current flowing in the primary. Thus, currents are induced in the secondary coil, at the make and break of the direct current in the primary coil This can be explained very easily.

When current is allowed to flow in a circuit by pressing the key, current does not attain its steady value instaneously It takes some time in rising from zero value to some steady value. In that interval of time *current varies* in the primary giving rise to *varying magnetic field* This changes the magnetic flux linked with the secondary, and induced currents are produced there. When the current attains the steady value, there is no change in the magnetic flux, and hence there are no induced currents. Similarly at break current takes some time to vanish from some steady value to zero value. This give rise to *changing magnetic flux* producing induced currents in the secondary. Hence at the make inverse currents are produced while at the break direct currents are produced.

§3 **Faradays' laws of electromagnetic induction :**—Faraday performed a number of experiments as described above and arrived at certain conclusions regarding the production of induced currents. They are as follows :—

(1) *Whenever the magnetic flux linked with a circuit changes induced currents are produced*

(2) *The value of the induced e.m f. so generated is directly proportional to the rate of change of magnetic flux. If the change in flux is rapid more will be the magnitude of the inducred e.m f produced, and vice versa*

If the number of magnetic lines of force threading a circuit changes from N_1 to N_2 $(N_2 > N_1)$ in t seconds, the induced e m f. e in electro-magnetic units is given by,

$$e \propto -\frac{N_2-N_1}{t}$$

or $$e = -K\frac{N_2-N_1}{t}, \text{ in electro-magnetic system if } K=1,$$

$$e = -\frac{N_2-N_1}{t}.$$

The minus sign is put simply to indicate the e.m f. so produced opposes the change.

(3) *The current so produced is temporary and lasts only while the magnetic flux is changing.*

§4. **Lenz's Law** .—According to §2 (*i*), when north pole of a magnet approaches a coil, induced current is produced in the latter in the *anti-clock-wise direction*, *i.e.* its near face opposite to that of the pole acquires *north polarity*. Therefore, there will develop a force of repulsion between the two. Hence the induced current

produced in the coil will *oppose the motion of the magnet* Similarly when the magnet is withdrawn the nearer face developes *south polarity.* Thus, now the induced current tries to attract the pole when it is moving always. So again the *motion of the magnet is opposed* Therefore, there always exists a *mutual opposition* between the two From such experiments *Lenz* came to a certain conclusion regarding the direction of the induced current, which can be stated in the form of a law, as follows —

The direction of the induced current produced is such that it always tends to oppose the cause which has produced it.

§5. **Mutual Induction** —In §2 (*ii*) we have seen that when the current is made or broken in the primary coil, current is induced in the secondary coil. The production of the induced current in the secondary coil due to the variation of current in the primary is known as **mutual induction.** The e.m.f generated due to mutual induction can be increased, (*i*) by winding the coil on *soft iron cores* which increases the number of magnetic lines of force in the coils, (*ii*) *by increasing the number of turns* of the coil in the secondary, (*iii*) by increasing the *rate of change of magnetic flux, i.e.,* by increasing the frequency of make and break of the circuit.

§6. **Self-Induction** —When the current is changed in a circuit induced currents are produced in the circuit itself, opposing the current flowing in the circuit. This is known as **self-induction.** As for example if you pass current through a coil of many turns, the current takes some time to attain some steady value rising from zero value. In this interval, the magnetic flux linked with the coil changes giving rise to induced current The current so produced flows in a direction opposite to that of the rising current But it lasts only for a small time and dies when the original current attains steady value The same phenomenon is experienced when the current is broken in the circuit Current takes time to die out, and hence the magnetic flux changes giving rise to induced current flowing in the same direction as that of the original current Thus it prolongs the stay of the current and sparking is noticed at the tap key. As described in the construction of resistance boxes, the coils are *doubled and then wound.* This is to eliminate the effects due to *self-induction.* As the coils are wound in the *opposite directions,* the self induced currents produced on one side of the coil will be neutralised by those produced on the other side

§7. **Practical Applications** .—The phenomenon of electro-magnetic induction finds quite a good number of practical applications. The following are a few appliances based upon this principle —

(1) **Induction Coil** —It is based upon the principle of mutual induction It is employed to convert strong currents at low potentials into weak currents at high potentials

(2) **Dynamo** —When a coil moves in a magnetic field current are induced in it Dynamo is based upon this principle. It is employed to generate electric current which is so very important for human necessities and industrial advancements

(3) **Electric motor** .—It is just the reverse of a dynamo, by which electrical energy is converted back into mechanical energy. Every body is familiar with an electric fan so very necessary in a tropical country like India. It employs an electric motor.

(4) **Transformer** .—It is a device by which low voltage can be transformed into a high voltage and *vice versa* in alternating currents.

QUESTIONS

1. What do you understand by the phenomenon of electro-magnetic induction ? (See § 1)

2 What are induced currents and how can they be produced in the laboratory ? (See § 2)

3 State faraday's laws of electro-magnetic induction (See § 3)

4 What is Lenz's laws and how can it be verified ? (See § 2 and 4)

5. Give in brief what do you understand by mutual induction and self-induction (See § 5 and 6)

6 Name the various practical applications in which the phenomenon of electro-magnetic induction has been applied. (See § 7)

Section VI

SOUND

Let us again consider the case of a long wooden scale fixed at one end while the rest of it is quite free as shown in Fig 1. The end A is fixed while the length AB is horizontal. If you put some weight at B, it will be depressed. More and more is the weight you put, more and more is the depression i e, the displacement from its position of rest If now the weight is removed, instantaneously we find that the end B wants to return back to its initial position. Obviously, due to its displacement some force internally must have been developed which must be resisting this change of position. This force developed is called force of restitution and in this particular case it is developed on account of the elastic property of the scale. In the case of pendulum, as you have already studied, the force of restitution is developed on account of the gravitational force Due to this force of restitution, the scale tries to return to its original position. As it comes nearer and nearer, the force of restitution becomes smaller and smaller, but the momentum goes on increasing. Ultimately, when the scale is in its equilibrium position, the force of restitution acting is zero but the momentum is maximum. As a result of this momentum, the scale goes past its original position to the other side and in that process the momentum is destroyed At the same time, the force of restitution goes on developing When the momentum becomes zero, the force of restitution brings it back and the process is repeated To make this motion a simple harmonic motion, the following points must be satisfied.

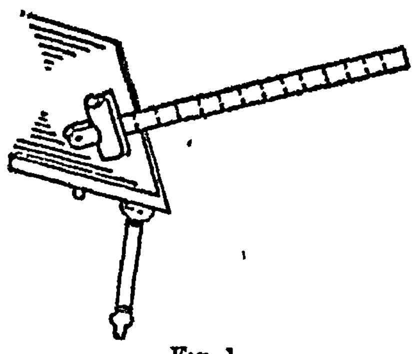


Fig 1

1. *It should be strictly to and fro motion.* There should not be any rotatory or spinning motion

2. *The motion should take place along a straight line. For this it is necessary that the displacement from the equilibrium position should be small.*

3. *The acceleration (i e, the force of restitution) should always direct the particle towards the position of rest.*

4. *The acceleration should be directly proportional to the displacement.*

To very great extent the examples quoted above satisfy the above four specifications and more or less they are the examples of simple harmonic motion.

§ 2. **Deduction of periodic time :**—Let us consider an imaginary case of a heavy particle P of mass m moving round the circumference of a circle of radius a.

Let the linear speed of its motion be v. Let the angular velocity of P be ω, *i e.*, let it describe an angle ω in one second.

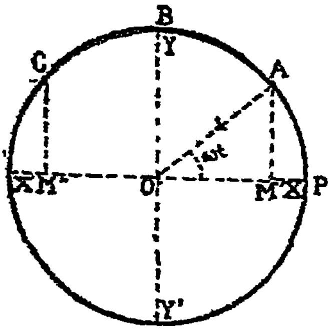


Fig 2.

We want to prove that as P circles round and round, its projection along the axis XX' or YY' performs a simple harmonic motion.

We shall consider projection on the axis of X. To start with when P is just on the axis, its projection M is displaced to a distance a from the centre O. After time t, when the particle P is at A, subtending an angle ωt, its projection is at M', when P is at B, its projection is at O, when it is at C, its projection is at M'', when at X', it is at X' and so on. Thus we find that as P moves round the circumference of the circle along $PABCX'Y'P$, its projection moves along XX' from P to O and then to P Thus, the time in which P completes one rotation along the circumference of the circle, its projection M completes one to and fro motion along the axis XX'.

As P is moving round and round, it can do so only under a centrifugal force Thus we know is mv^2/a and is directed towards the centre O. Therefore, its projection M will also have the component of this force which would be directed towards O always.

Thus, if $OA=a$ represents the force mv^2/a in magnitude, its component force acting towards O would be represented by its projection $OM'=x$ at the instant t. Hence, the component force dragging the projection point towards O would be $\frac{mv^2}{a}\times\frac{x}{a}=\frac{mv^2x}{a^2}$. This, in other words is the force of restitution.

Hence, the acceleration of the projection at any instant t when its displacement is x form O would be

$$\text{acceleration}=\frac{\text{force}}{\text{mass}}=\frac{\frac{mv^2}{a^2}\cdot x}{m}=\frac{v^2}{a^2}x$$

Thus, acceleration $=\dfrac{v^2}{a^2}\times$displacement .. (1)

We know that the circumference of the circle is $2\pi a$, and it is described with a linear speed v. Hence, if T is called the period of completing one round,

$$T=2\pi a/v \quad .. (2)$$

Similarly, with angular velocity ω, it completes one round *i.e.*, completes angle 2π. Hence,

$$T=2\pi/\omega \quad . (3)$$

Comparing eqns. (2) and (3), we get

$$\frac{2\pi a}{v}=\frac{2\pi}{\omega}$$

or

$$\frac{a}{v}=\frac{1}{\omega}$$

or

$$\omega=v/a \quad . (4)$$

Substituting this value from eq (4) in eq (1), we get

$$\text{acceleration}=\omega^2\times\text{displacement}$$

As ω^2 is a constant,

$$\text{acceleration } \propto \text{ displacement.}$$

Thus, we find that the motion of the projection on the axis satisfies all the requirements of a simple harmonic motion.

Now the period of one complete rotation=the period of one complete to and fro motion T=$\dfrac{2\pi}{\omega}=\dfrac{2\pi}{\sqrt{\omega^2}}$

$=2\pi/\sqrt{\text{constant of proportionality between acceleration and displacement}}$

So, this is regarded as a general expression for the period of a simple harmonic motion

§ 3. **A few definitions :**—In the above imaginary example, projection M has been shown to perform simple harmonic motion (S H M) round about the equilibrium point O.

When M moves from its extreme position P to O, then to the left extreme position X', back to O and then to its initial position P, it is said to execute one oscillation, rotation or vibration

The period or the time which it takes to execute one vibration is called the time of oscillation, or time of vibration or simply periodic time and is usually denoted by T

The number of vibrations it performs in one second is called its frequency and is generally denoted by n.

Hence, the time of one vibration $=\dfrac{1}{n}$ seconds.

Therefore, $T=\dfrac{1}{n}$

Or Periodic time=inverse of frequency.

The maximum displacement of M from its mean position O is called the amplitude of vibration and is denoted by a

§4. Graphical and mathematical representation of S.H.M. :—

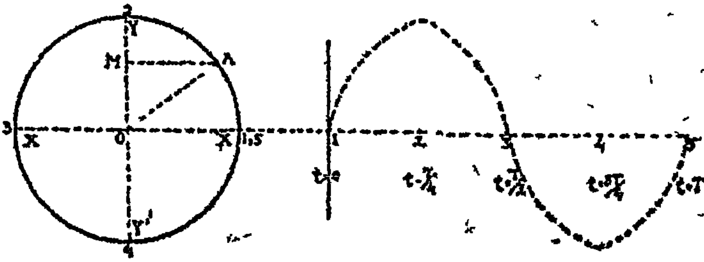


Fig 3.

As in the previous example, let the tracing point be P and consider its projection M on Y-axis When the tracing point initially at $t=0$ is at 1, its projection is at O, and the displacement from O along Y axis is $y=O$. After time t, such that $\omega t=\frac{\pi}{2}$ or putting $\omega=\frac{2\pi}{T}$ we get $\frac{2\pi}{T}\cdot t=\frac{\pi}{2}$ or $t=\frac{\pi}{2}\times\frac{T}{2\pi}$ or $t=\frac{T}{4}$ i.e after one fourth of periodic time when P is in position 2, the projection is at Y and the displacement $y=a$ is maximum. Again after $t=\frac{T}{2}$ when P is at 3, the projection is at O and $y=0$. After $t=\frac{3T}{4}$, when P is at Y', the displacement $y=-a$ i e., maximum on the negative side After $t=T$, when P returns to its initial position, the displacement is again $y=0$.

All these positions have been shown graphically. Along Y axis is represented the displacement and along X axis is represented the times The continuous wavy curve which we get represents $S.\ H$ motion graphically.

Mathematical Let at any instant t, the position of the tracing point P be at A such that the $\angle AOP=\omega t$. Then the $\angle AOM=(\pi/2-\omega t)$ where M is the projection of P on the Y axis Let OM, the displacement at any instant be called y.

Now according to trignometry, $\cos AOM=\frac{OM}{OA}$. Substituting the values $\angle AOM=(\pi/2-wt.)$, $OM=y$ and $OA=a$ we get,

$\cos\ (\pi/2-\omega t)=y/a,$

Or, $\sin \omega t=y/a$

Or, $y=a \sin \omega t,$

$$=a \sin\frac{2\pi}{T}\ldots(1) \text{ because } \omega=\frac{2\pi}{T}$$

This eq. (1) is called the equation of a $S.\ H.\ M.$

By giving t, the various values of $t=0$, $T/4$, $T/2$, $3T/4$, T, in eq. (1), we get the displacements at various instants

§5. **Phase :**—Let us now consider two tracing-points P and Q, having the same angular velocity w and tracing the same circle of

radius a. Then their projections say M and N on Y axis will perform separately two $S. H. Ms.$ which will be identical in all respects

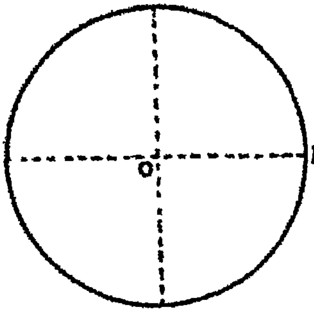


Fig. 5 a

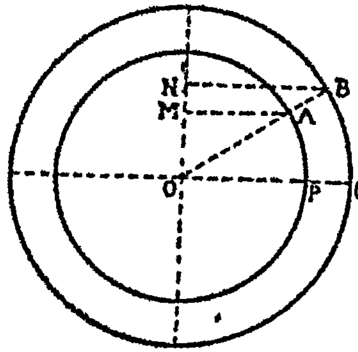


Fig. 5 b

—*i.e.* same period and same amplitude.

If P and Q trace different circles of radii a and b respectively, then also the projections describe $S.H.M$ but of different amplitudes. Mathematically we can write

$$y=a \sin \omega t \quad \ldots \quad \ldots \quad (1)$$

and

$$y=b \sin \omega t \quad \ldots \quad (2)$$

In the examples quoted above we say that both the $S.H\ Ms.$ are in the same phase. By phase we mean that in both the cases they would reach their zero displacement, maximum $+$ve or $-$ve displacements simultaneously together.

To quote a practical example we might take two simple pendulums exactly of the same length, displace them through different distances and leave them These two pendulums will perform $S\ H.\ Ms$ which are in the same phase.

In Fig 6*a* are shown two tracing points P and Q occupying different positions to start with. The $\angle POQ=\theta$. Their angular

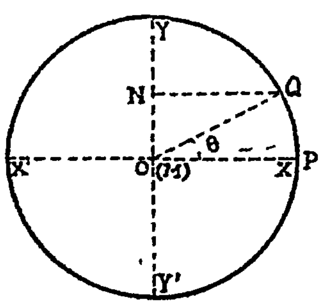


Fig 6*a*

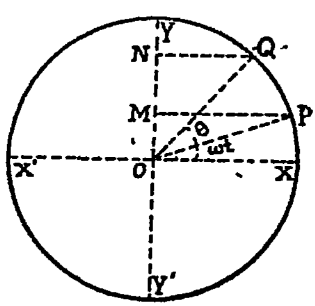


Fig. 6*b*

velocity is the same and they describe the same circle. We here find that their projections M and N perform $S.\ H.\ Ms.$ of the same period but differently At $t=0$, when displacement for M is zero for N it is equal to ON.

After time t when P subtends an $\angle \omega t$, Q subtends an $\angle \omega t-\theta$ because Q is already in advance by an $\angle \theta$. Due to this advance of

θ, the displacements of M and N at no instant are similar. When N has achieved its maximum displacement, M would be still following and so on. Mathematically, the $S.\ H.\ M.$ of M is represented by

$$y = a \sin \omega t. \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

and that of Q would be represented by

$$y = a \sin (\omega t + \theta) \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

Here this $\angle\theta$ is called the phase difference between the two $S.\ H.\ Ms$ In this particular example, $S\ H.M.$ of N is said to lead the $S.\ H\ M.$ of M or that of M to lag behind that of N by an amount θ.

Thus, when $\theta=0$, the two motions are said to be in the same phase

When $\theta=\pi$, the two motions are said to be in the opposite phase In this particular case when one motion will be having maximum $+$ve displacement other would be having $-$ve maximum displacement, when one would be crossing the zero position from say right to left, the other would be crossing it from left to right.

If a particle is made to vibrate under the action of two $S.\ H.\ Ms$ in the same direction, the resultant motion performed by the particle is also simple harmonic.

If the periods of vibrations of the two $S.\ H.\ Ms.$ is the same and they are in the same phase, the resultant $S\ H.\ M$ has the same period but its amplitude is the sum of the amplitudes of the two motions If the two motions are in the opposite phase, the resultant amplitude will be the difference of two amplitudes. In case, the two amplitudes are equal, the resultant amplitude would be zero i.e , the particle will not perform any $S.\ H.\ M.$

If the two periods differ slightly, the resultant motion is complicated. Sometimes the resultant amplitude is sum of the two amplitudes and sometimes it is the difference of the two With this is connected some very important phenomenon in sound.

QUESTIONS

1 What is $S\ H\ M$ Give its characteristic properties and deduce an expression for the period of a $S\ H.M.$ [see § 1 and § 2].

2. Define the following —

(i) Oscillation, (ii) Period of oscillation, (iii) Frequency, (iv) Amplitude and (v) Phase. [see § 3 and § 5].

3 Explain how you will represent a $S.H\ M$ graphically and mathematically. [see § 4]

CHAPTER II

WAVE MOTION

§1. **Wave Motion** :—It is a matter of common experience that when we throw a stone in a quiet pond a disturbance is created. Actually the stone strikes water at one spot A disturbance is created This disturbance starts from the spot struck and spreads outwards If a piece of paper or cork is floating on the surface of water we find that it simply performs up and down motion but is not permanently displaced from its position or is not carried forward Similarly, if we take a stretched rope and give it some jerk at one end, this jerk is carried to the other end through the rope Each portion of the rope is temporarily affected *Such a kind of disturbance which is carried from one end of the medium to the other end, without actually permanently displacing the particles of the medium is called a wave motion.* In wave motion, each particle performs to and fro motion round about its equilibrium position The disturbance is carried from particle to particle without actual permanent displacement of the particle Thus, here the energy is carried from one end to the other end of the medium but there is no permanent displacement of the particles from their mean positions of equilibrium

Thus in a wave motion we note the following :—

1 *Disturbance or energy is carried from one end of the medium to the other end*

2. *For the energy propagation medium is necessary*

3. *The particles of the medium are disturbed and perform to and fro motions round about their points of equilibrium.*

4 *The particles of the medium are not permanently displaced from their equilibrium position*

§2. **Kinds of wave-motion** :—There are two kinds of waves depending on the way of their propagation and production These are

(*i*) *Transverse,*

and (*ii*) *Longitudinal*

In transverse wave, the particles of the medium *perform to and fro motions at right angles to the direction of propagation of the disturbance* while in longitudinal wave *the vibrations of the medium particles take place along the direction of propagation* The other characteristics will be discussed in § 3 and § 5.

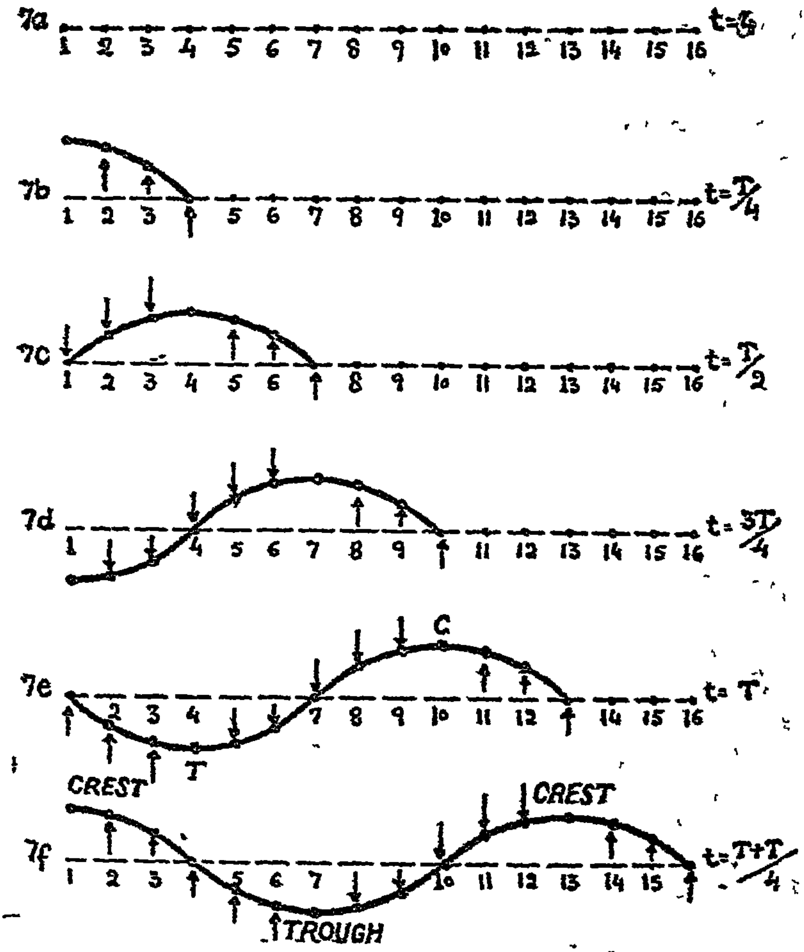
The wave motion which we are going to consider here due to its peculiar nature as will be clear afterwards is called a Progressive Wave.

§3. **Propogation of Transverse Progressive Wave** :—If we take a thin stretched wire and pluck it at one end you will find a disturbance which travels throughout the length of the wire.

Such disturbance is a transverse wave. This transverse wave can easily be seen when we produce a jerk in a long stretched rope.

In order to understand its propagation clearly we shall consider an imaginary medium having elastic properties. The medium is continuous but we shall represent it as made up of a large number of discreet particles 1, 2, 3.. etc. all lying along the same line. See Fig. 7*a*. All these particles are in rigid contact with each other.

Let us now produce a *S.H.* disturbance at the point 1. As 1 would be assumed to be in rigid contact with the *S.H.* source, it



would start performing *S.H.M.* of the same amplitude and period. Let us assume that the amplitude of vibration is a and its period is T.

Fig. 7*a*, shows the positions of the various particles of the medium when it is at rest *i.e.*, initially at $t=0$.

Fig. 7*b*, shows the positions after $t=T/4$. During this period particle 1 is displaced upwards through the maximum displacement a. Now particle 2 is in contact with 1 and therefore, it also takes up the motion of 1. But the *S.H.M.* which it performs lags in phase with respect to 1. Hence, 2 is trying to follow 1. Particle no. 3 is similarly disturbed and lags behind 2. Suppose particle 4 is

situated at such a distance that at the end of this time $t=T/4$, it is just disturbed and is trying to move upwards

Fig. 7c shows the positions after $t=T/2$. During this period no. 1 has completed its half vibration and is about to start downwards No. 2 after completing its maximum displacement is trying to follow it on its heels and so it happens onwards. Mean while no. 4 attains its -ve maximum displacement and the condition upto 7 is the same as it was previously from 1 to 4. Thus, we find that the condition of the medium from 1 to 4 has progressed to from 4 to 7

Now Fig. 7d is self explanatory. The condition of the medium from 4 to 7 has progressed to from 7 to 10.

Fig. 7e denotes the condition after time $t=T$. Now 1 has completed one vibration and is about to start on its second vibration. Meanwhile the disturbance has reached upto 13th particle which is also about to start on its vibration.

As shown in Fig. 7f, we find that no 1 and no. 13 continue to vibrate exactly identically or there is no phase difference between the *S.H.Ms.* of these two particles. Actually 1 has performed one complete oscillation more than 13 and sometimes we say that phase difference is 2π. This phase difference 2π is taken as in the same phase.

The condition at 7 is just the opposite. We say that it is in opposite phase to 1 or 13 and that its phase differs by π.

Thus as explained above the disturbance will be carried forward and forward as time progress.

We note here the following :—

1 *All the particles are performing S.H Ms of the same period and amplitude and this motion is at right angles to the direction of propagation.*

2. *Particle no* 1 *and* 13 *are vibrating in the same phase All other particles lying in between differ in phase from each other. Such two particles which vibrate in the same phase are said to be separated by a distance of one* **wavelength.** We have also seen that the disturbance produced at the point 1 at $t=0$ has travelled a distance between 1 to 13 during time $t=T$ *This distance through which the disturbance is propagated in one periodic time is called* **wavelength** *and is denoted by the symbol* λ If you consider the point 7, it lies midway between 1 and 13 and the distance 1 to 7 or 7 to 13 *i.e.,* distance between two points in the opposite phase is $\lambda/2$ *i.e*, half wavelength.

3 By a study of the figures 7*a* to 7*f* we find that the medium is distorted *i e.,* there is change of shape of the medium when such a wave is propagated. In Fig 7*f*, the upper most points 1 and 13 are called crests and at 7 etc, are called troughs *Thus, a transverse wave is propagated in the form of crests and troughs. Every point in its turn becomes a crest and a trough.*

4. The distance between two points does not change, *i.e.*, they neither come closer or drawn apart. In other words, due to such a propagation, *there is no change in the density of the medium.*

5. Due to these peculiar properties, a transverse wave can be generated only in a medium which possesses the property of rigidity.

§ 4. Relation between velocity, period or frequency and wavelength :—The rate at which the disturbance is propagated in the medium is called the velocity of the wave and is denoted by V.

Now we know that in the periodic time T the wave travels through a distance equal to wavelength λ, hence, the distance it travels per second *i e*, velocity $V=\lambda/T$.

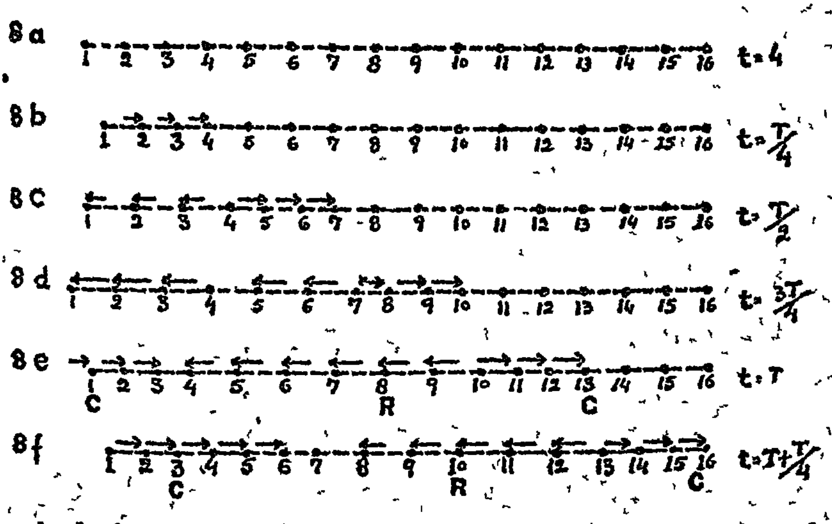
We already know that if n is the frequency of vibration, then $n=1/T$. Hence, substituting this in the above, we get

$$V=\lambda/n$$

Or, **velocity=wavelength×frequency.**

§ 5. Propagation of longitudinal progressive wave :—Examples of transverse wave propagation are easier to quote because there is distorsion of the medium which can be easily observed. Examples of longitudinal wave propagation are scarce. An approximate example may be quoted as the vibrations performed by a spring or the disturbance produced in a long railway train to which an engine is attached with a thump.

In order to understand longitudinal wave propagation we can consider the same imaginary example as in the transverse waves. Only here the vibrations of the particles will take place along the direction of propagation and not at right angles to it as in the transverse waves

See Fig. 8*b* This represents the condition at $t=T/4$. As particle no. 1 attains its maximum displacement it has pushed

no. 2 which in turn pushes no. 3 and the disturbance has just reached no. 4.

In Fig. 8c, the condition is depicted at $t=T/2$. Whatever was the condition at 1 to 4 has travelled to 4 to 7. Particle no. 1 has returned to its initial position and is about to start towards its left.

According to Fig 8d no. 1 has attained its negative maximum displacement and meanwhile the disturbance has just reached particle 10.

Fig. 8e represents the condition at $t=T$ By now particle no 1 has completed one vibration and is about to start on its second vibration Meanwhile the disturbance has reached particle no. 13 which is about to start.

If we carefully study Fig 8e, we find that the condition at points 1 and 13 is exactly identical and hence, they are exactly in the same phase and hence as in transverse wave the distance between these two points is defined as the wavelength In fact all other characteristics in this wave are similar to those of transverse wave.

However the following points where it differs may be noted .—

1 All *particles perform S H Ms. in the same direction in which the wave is propagated.*

2. The *medium is not distorted* but at some places the particles come too closer together and at others they are drawn farther apart. In other words, we say compressions and rarefactions are produced in the medium. Compressions are produced wherever the particles, come very close together and rarefaction where they are drawn maximum apart *Thus, instead of crests and troughs here we get a wave in which compressions and rarefactions are produced* These then go on progressing further and further. In transverse wave the distance between two consecutive crests or troughs is one wavelength while in longitudinal wave the distance between two consecutive maximum compressions or rarefactions is one wavelength.

3. Longitudinal waves, therefore, can be produced in *all those media which possess bulk modulus It is not necessary for the medium to possess modulus of rigidity* as there is no question of change of shape.

§ 6. Graphical representation of a longitudinal wave — The graphical representation of a longitudinal wave is exactly similar to the transverse wave , only in transverse wave, its graphical representation agrees with its actual appearance In longitudinal wave the slopes represent the compressions and rarefactions—the positive slope representing the rarefaction and the negative one the compression

That is why whenever a sound wave is represented though its nature is longitudinal its graphical appearance is exactly similar to that of tranverse wave

QUESTIONS

1. What is a transverse wave ? How is it propagated ? Discuss its various characteristics [See § 2 and § 3]

2 Define wavelength, frequency and velocity of a wave How are these inter-related ? [See § 4]

3. Distinguish a longitudinal wave from a transverse wave giving its characteristics and mode of propagation [See § 5]

SOUND AS A WAVE-MOTION

§ 1. **Sound :**—It is a common experience that when a metallic vessel is struck it produces sound. Every one is familiar with the huge college bell. If you observe it very carefully you will find that the plate struck is vibrating. If you touch it and stop it vibrating you find that the sound is instantly stopped. Similarly, if you consider a big vessel filled with water and strike it, you find that while the vessel is producing sound, it is also producing ripples on the surface of water. All these experiments go to prove that sound is produced only when a body begins to vibrate to and fro. Greater the force with which you strike a vessel, greater becomes the amplitude of its vibration and hence greater is the intensity of sound produced.

The problem before us is "How does sound produced by the vessel travel upto our ears ?"

Sound is a peculiar sensation of the ear which is interpreted by the brain as sound. Sound is obviously a kind of energy. How does this energy travel from the source upto ears ?

§ 2. **Sound as a wave motion :**—Let us suppose a toy paper boat is floating on the surface of a still pond away from the bank. We want to disturb the boat. This we can do by the following two ways :

1. By throwing a stone or some such thing directly at the boat or

2. By flapping the water near the bank by hand and thus producing ripples on the surface of water. These ripples or waves would then spread outwards and would in time disturb the paper boat.

Thus, in the first case the energy was carried direct from hand to the boat by stone while in the second case it was transmitted through the medium water. The water made to and fro motions but there was no permanent displacement.

When a source produces sound, there are two ways in which it can travel upto ear

1. The sound source might send some minute particles to ear to create the sensation of sound, or

2. The sound source might produce ripples in the intermediate medium which in turn might disturb ear.

Now we want to show that sound travels by the second way and that it is a wave motion.

§ 3. **Proof for sound as a wave motion :**—We know that a wave has the following important properties :—

(*a*) A *vibratory source* is needed for the production of a wave.

(*b*) *Medium* is required for its propagation.

(c) It takes *time to travel* from one point to the other.

(d) *Wave* is reflected.

(e) Wave has the property of refraction.

(f) Two waves can interfere.

(g) Wave possesses the property of diffraction

Now we shall show that sound propagation possesses all these properties and hence it is a wave propagation.

(a) **Vibratory source :**—As already explained, sound is only produced by the vibratory motion of a body If you take an

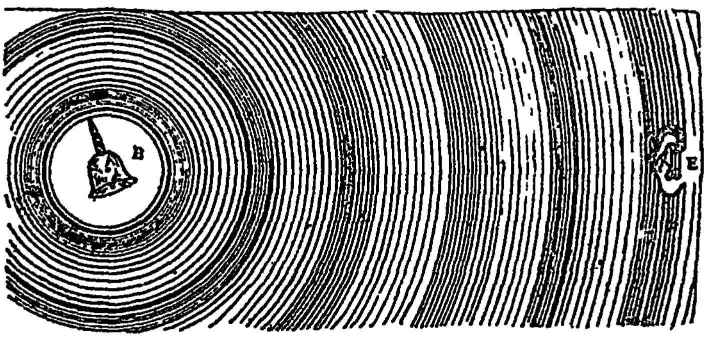


Fig. 9

inelastic body which cannot produce vibrations, we shall find that it will also not produce sound. On touching a sounding body, the sound is stopped only because its vibrations are damped See fig. 9. It shows a bell ringing and due to it longitudinal waves are created in the medium.

(b) **Medium :**—You are already familiar with the jar and bell experiment In a huge Bell jar an electrical bell is kept See fig 10. If you ring the bell, you can hear its sound. Now complete the connections with a vacuum pump and produce vacuum in the jar If you now ring the bell, you do not hear any sound. If at all you hear, it is very faint. Previously you could hear the sound because it was easily transmitted through air, but in the second case no sound could be transmitted without a medium Thus, this simple experiment proves that sound propagation needs some medium and that it cannot travel in vacuum.

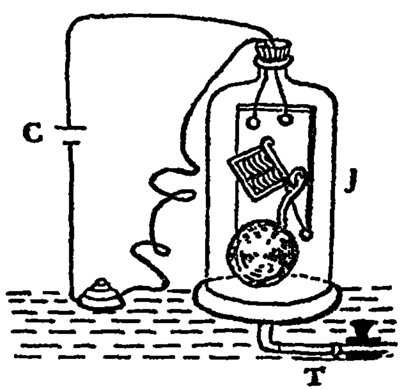


Fig 10

(c) **Time to travel :**—Those who are track athletes know that a time keeper is supposed to start his stop watch by the flash of the gun of the starter and not by its sound The reason is because sound takes time to travel. Similarly, the bullets from a pistol kill a person before he could hear the sound of the explosion We see

lightning and then alone after sometime we can hear the thunder it produces.

When the experimental determination of the velocity of sound is made, it is found that it comes out to be only 332 meters per second or 760 miles per hour approximately. In fact this velocity is so small that now-a-days we have supersonic planes which can fly much faster than this velocity of sound.

(*d*) **Reflection** :—You are already familiar with this property of reflection in light. This holds good in the case of sound also. All the laws of reflection which hold good in the case of light also hold good in the case of sound also. See fig 11. Here sound produced is made to travel along a particular direction along the tube. Now at the other end of the other tube you can hear or detect the sound distinctly if it is equally inclined with the reflecting surface.

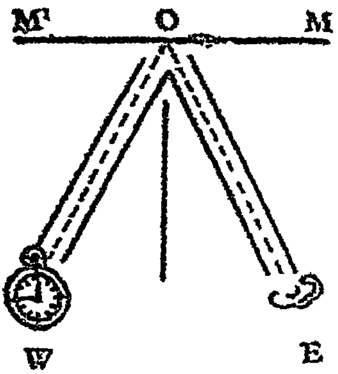


Fig 11

In practical life you are familiar with the phenomenon of echo. Whenever we shout in a very big hall or in a deep well or outside in open country surrounding which there are hills or huge buildings, we hear our sound back after sometime. This sound heard after some lapse of time is called an echo and is obviously due to reflection of sound Whenever sound travelling through one medium is incident on another medium (it may be denser or rarer than the first medium) it gets reflected.

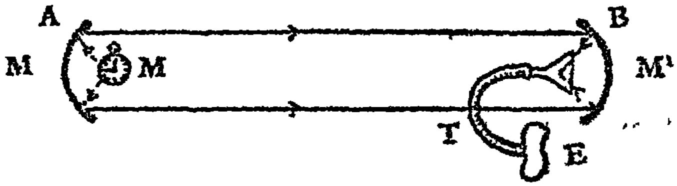


Fig. 12.

Now normally when a sound is heard by an ear, sensation of sound persists for approximately 1/10 second Therefore, when we speak in a small room, the sound reflected from the side walls etc is heard simultaneously with the direct sound However, if the dimensions of the room are very big, we get a different phenomenon. The velocity of sound is approximately 1120 ft per second. Suppose the wall of the room lies at a distance of 56 ft. or more. So the sound produced will have to travel 56 ft one way to reach the wall and return through 56 ft again to reach the ear of the speaker. Thus, it would cover these 56+56=112 ft or more in 1/10 second or more. During this period the direct sound heard by the ear would have subsided and hence it would be heard as a distinct sound. Therefore, an echo would occur only if the reflecting medium is distant 56 ft or more.

A musician finds it easier to sing in a closed room. He does

not like to sing in open air. The reason is obvious. The reflected sound from the walls augments his voice.

Actually while planning an auditorium reflection from walls etc is a very important factor which has to be borne in mind.

(e) **Refraction** :—Like light, sound is also refracted in passing from one medium to the other. As in light, there are arrangements by which sound can be made to focus at a point or may be made to spread along parallel lines.

However, we would be considering only the refraction of sound by wind

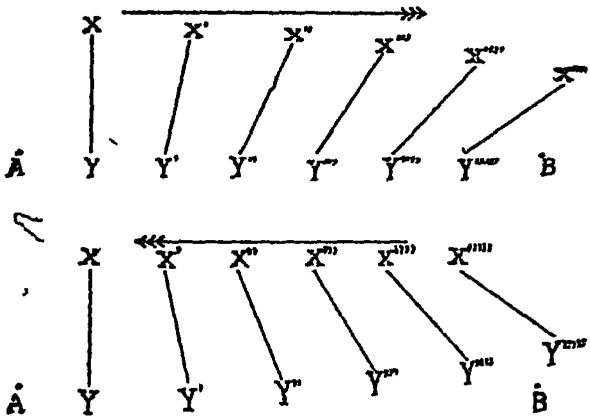


Fig 13

Suppose A is the source of sound and B is the observer Wind is blowing from A to B.

Now when sound is produced, let us suppose that its energy is spread along XY. If wind is not blowing (i e., there is no bodily movement of air) then with time the energy would travel forward and $X'Y'$, $X''Y''$ etc. would be parallel to XY and B would hear sound after sometime. If, however, wind is blowing, the position of the air near ground level would travel slower than the portion of air higher Hence, as XY region travels forwards, its next position would be inclined i e, along $X'Y'$, $X''Y''$ as shown in fig 13*a* Thus, the whole of sound energy would pass over B and sound would be heard distinctly. If the wind velocity is in the opposite direction, reverse would happen (as shown in fig 13*b*) and most of the sound energy would not reach the observer. Hence, whenever we try to hear along the direction of wind we hear distinctly and opposite to wind direction indistinctly.

(f) **Interference** :—This phenomenon of interference is a very important and distinct property of wave phenomenon. Two exactly identical waves, identical in all respects, produce this phenomenon When such two waves reach a particular spot simultaneously in the same direction and in the same phase, they help and augment each others effects On the other hand, if the two waves reach there in opposite phase they cancel each others effect

Let A and B be two identical sources of waves. If O is a point situated at equi-distance from both A and B, the two waves would reach the spot O in the same phase and the point O would begin to vibrate with double the amplitude. On the other hand if M is such a point that the distance BM is greater than the distance AM by $\lambda/2$ (where λ is the wavelength), the two waves would reach there in opposite phase That is if the disturbance from A wants

to move it in one direction, that from B would try to move it exactly in opposite direction. The result would be that the point M

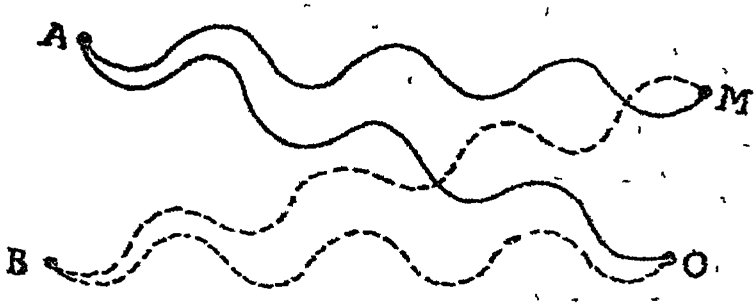


Fig. 14.

would not be disturbed at all.

Thus, we say that the two waves reinforce each other at O or such other points where they are in the same phase (as explained in § 5, chapter I, when phase is zero or 2π or 4π i e., path difference λ or 2λ etc.,) and destroy at M or such other points where they are in opposite phase (i.e , phase difference π or path difference $\lambda/2$ etc). At O, therefore, there would be maximum energy and at M there will be no energy

Now in sound if we take two identical sources of sound, identical in all respects, we find that at some places the two together produce maximum sound while at others minimum sound. This phenomenon is called interference of sound.

A practical demonstration of this can be shown by the following :—

Quincke's Tubes :—As shown in the figure ABC is a hollow brass tube of wide bore. At D and E and sliding into it there is another tube PQR. This tube can be inserted in or can be taken out. There are graduations marked at the ends P and R just to show the displacement of the tube. A and C are the two symmetrical open ends where respectively the source and an ear can be placed.

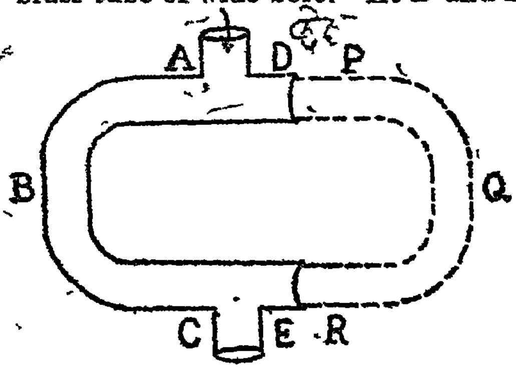


Fig 15

Now a source is sounded at A. The energy divides and one part travels along ABC and the other PQR. Thus, we get two identical waves. If the path ABC is exactly equal to the path $DPQRE$, the two waves would reach at C simultaneously in the same phase and sound heard will be maximum.

Note this position of PQR on the scale marked on it. Now gradually draw it out so that this path increases. A stage will come when at C you will not hear any sound. It means that the path $DPQRE$ has increased by $\lambda/2$, where λ is the wavelength of

the sound produced at A. By noting the position PQR, this term λ can be determined.

Thus, we not only demonstrate the phenomenon of interference but also can determine wavelength of the sound.

(*g*) **Diffraction :**—This is also a phenomenon peculiar with the wave phenomenon. When a wave comes across an obstacle, it bends round the corners of the obstacle and enters the region of geometrical shadow.

We know that even when our friend is standing behind a wall, we can hear him. The reason is obvious. The sound energy starting from him can bend at corners and reach us. Had sound consisted of some shooting particles this would not have been possible.

Thus, from the above we find that sound possesses all the properties of a wave. And, hence, sound must travel from one place to the other in the form of a wave.

§ 4. **Sound is a Longitudinal Wave :**—Now the question is whether it is a transverse wave or a longitudinal. The answer to this question would be indirect.

We know that sound needs a medium and that medium is air which is a gas Gases do not have shape of their own and hence do not possess the property of rigidity. For the production of transverse waves, we have already studied that the medium must possess rigidity *i e*, it should be capable of regaining its shape once distorted Hence, transverse waves cannot be generated in gases. Therefore sound cannot be a transverse wave motion.

The only other alternative is that sound must be a longitudinal wave motion

When the strings of a violin are struck, they begin to perform transverse oscillations Due to these longitudinal waves are generated in air. These waves then reach our ears and create the sensation of sound Actually after striking the membrane of the ear, the membrane performs transverse vibrations

QUESTIONS

1 Prove that sound is a longitudinal wave propagation [See § 3 and § 4]

2. How do you explain refraction of sound by wind [See § 3e]

3. What is echo ? Explain why only a deep well can give an echo ? [See 3*d*]

4. What do you understand by interference ? Explain Quincke's tube arrangement to determine wavelength of sound. [See § 3*f*]

VELOCITY OF SOUND

§ 1. **Newton's Formula** :—In the previous chapter we have seen that sound is propagated in the form of longitudinal waves in air When a longitudinal wave progresses it does so in the form of compressions and rarefactions. These compressions and rarefactions take place in quick succession. Newton assumed that while these compressions and rarefactions are produced there is no change of temperature involved Hence, the bulk modulus of the medium which we should consider should be the isothermal bulk modulus of the medium

Now, we have to assume that the velocity V of a longitudinal wave in a medium possessing density d and bulk modulus E is given by

$$V=\sqrt{E/d} \qquad ...(1)$$

From the above consideration we know that E represents the isothermal bulk elasticity of the medium.

Now, we know that for a perfect gas, the isothermal bulk elasticity is equal to its pressure P (This has to be taken for granted). Hence, the above formula can be according to Newton rewritten as

$$\mathbf{V}=\sqrt{\mathbf{P/d}} \qquad .. (2)$$

At N T P,, the atmospheric pressure $P=76\times 13\ 6\times 981$ dynes per sq cm and for air $d=0\ 00129$ gm per c c. Substituting these values in the above we get,

$$V=\sqrt{\frac{76\times 13\cdot 6\times 981}{0\ 00129}}=280 \text{ metres per sec.}$$

§ 2. **Laplaces' Correction** :—Experimentally when the value for velocity of sound is determined and all corrections (as discussed later) are applied we get a value of 332 meters per second at N.T.P. This difference of 332−280=52 meters/sec is a very large difference between the theoretical and experimental values. This cannot be explained as an experimental error. Hence, there must be something wrong with the theoretical value. In theoretical value the only mistake can be with the assumptions. *Laplaces questioned the assumption that the bulk modulus should be the isothermal bulk modulus.* According to him when a longitudinal wave progresses in a medium, the changes which the medium undergo cannot be isothermal When compression takes place, heat must be produced. Likewise with rarefaction cooling must take place These take place in such quick succession that there is no time for equalisation of temperature. *Hence, these changes take place adiabatically with change of temperature.* Therefore, the elasticity to be considered cannot be isothermal but must be adiabatic If adiabatic elasticity is represented by E_ϕ, we get

Now, we know that adiabatic elasticity of a gas is equal to $P\gamma$ where P is the pressure and $\gamma = C_p/C_v$ where C_p and C_v are respectively the specific heats of a gas at constant pressure and volume. (All this has to be taken for granted). By making this substitution in the above, we get

$$V = \sqrt{\frac{P\gamma}{d}} \quad \dots (3)$$

$$= \sqrt{\gamma} \times \sqrt{P/d}$$

Now, at N.T.P, we have already seen that

$$\sqrt{P/d} = 280 \text{ metres/sec}$$

and for a diatomic gas like air $\gamma = 1{\cdot}4$, hence

$$V = \sqrt{1{\cdot}4} \times 280$$

$$= 331 \text{ metres/sec}$$

This theoretical value of 331 metres/sec very closely agrees with the experimental value of 332 metres/sec Hence, correction as applied by Laplaces must be valid Therefore, now we regard correct expression for velocity of longitudinal wave of sound in a gas as

$$V = \sqrt{\gamma P/d}$$

§ 3. **Effect of various factors on velocity of sound :**—In order to determine correct value of the velocity of sound we must take into consideration the following factors which might affect it.

(a) *Humidity.*
(b) *Pressure*
(c) *Temperature.*
(d) *Wind*
(e) *Personal*

(a) **Humidity :**—When the humidity of air changes, assuming that other factors remain the same, the density of the medium would change As water vapour is lighter than air, therefore *as humidity would increase*, the density of the medium would decrease. In the expression for velocity $V = \sqrt{\gamma P/d}$, because d occurs in the denominator, therefore *velocity of sound would increase.*

This is the reason why on a rainy day distant sounds are readily and distinctly heard

(b) **Pressure :**—According to Boyle's law we know that other factors remaining constant, if pressure P varies, the ratio P/d remains constant Because in the expression for V, the ratio P/d occurs, therefore, even if the pressure of the medium changes, the velocity V would remain unaffected.

Therefore, *there is no effect of pressure on the velocity of sound and hence it may not be noted*

(c) **Temperature :**—Temperature has marked effect on velocity of sound. To understand it, let us first consider the gas equation We know that according to gas equation

$$\frac{Pv}{T} = \frac{P_1v_1}{T_1}$$

where the symbols have their usual meanings. If the mass of the gas be m and d, d_1 its respective densities at the temperatures t and t_1, we have $v=m/d$ and $v_1=m/d_1$. Also $T=273+t$ and $T_1=273+t_1$ where t and t_1 are expressed in centigrade scale. Making these substitutions in the above, we get

$$\frac{P.m}{d(273+t)}=\frac{P_1.m}{d_1(273+t_1)}$$

Dividing the denominator in the above by 273, we get

$$\frac{P.m.}{d\left(\frac{273+t}{273}\right)}=\frac{P_1.m}{d_1\left(\frac{273+t_1}{273}\right)}$$

Or,
$$\frac{P.m}{d\left(1+\frac{1}{273}t\right)}=\frac{P_1.m}{d_1\left(1+\frac{1}{273}t_1\right)}$$

Put $1/273=\alpha$ and cancel m from both the sides. Then we get

$$\frac{P}{d(1+\alpha t)}=\frac{P_1}{d_1(1+\alpha t_1)}$$

If P_0, d_0 represent the respective quantities at temperature 0, we get by putting O for t_1, P_0 for P_1 and d_0 for d_1,

$$\frac{P}{d(1+\alpha t)}=\frac{P_1}{d_1(1+\alpha t_1)}=\frac{P_0}{d_0(1+\alpha\times O)}=\frac{P_0}{d_0}$$

Or,
$$\frac{P}{d(1+\alpha t)}=\frac{P_0}{d_0}$$

Or,
$$\frac{P}{d}=\frac{P_0}{d_0}(1+\alpha t) \qquad \dots \quad (1)$$

This is another way in which a gas equation can be represented.

Now let the velocity of sound be V_0 and V respectively at $0°C$ and $t°C$. Then according to §2,

$$V_0=\sqrt{\gamma\frac{P_0}{d_0}} \qquad \dots \quad (2)$$

and
$$V=\sqrt{\gamma\frac{P}{d}} \qquad \dots \quad (3)$$

Substituting the value of P/d from eqn. (1) in eqn. (3), we get

$$V=\sqrt{\gamma\cdot\frac{P_0}{d_0}(1+\alpha t)}$$

$$=\sqrt{\gamma\frac{P_0}{d_0}}\times\sqrt{1+\alpha t}$$

Substituting the value of $\sqrt{\gamma\frac{P_0}{d_0}}$ from eqn. (2), we get

$$V=V_0\sqrt{1+\alpha t} \qquad \dots \quad (4)$$

$$=V_0\sqrt{1+\frac{1}{273}t}$$

$$=V_0\sqrt{\frac{273+t}{273}}=V_0\sqrt{\frac{273+t}{273+0}}$$
$$=V_0\sqrt{\frac{T}{T_0}}.$$

Here, we have converted centigrade temperatures into absolute.

$$\frac{V}{\sqrt{T}}=\frac{V_0}{\sqrt{T_0}}$$

Or, $$\frac{V}{\sqrt{T}}=K, \text{ a constant}$$

Or, $$V=K\sqrt{T}$$

Or, $$\mathbf{V}\propto\sqrt{\mathbf{T}} \quad \dots \quad (5)$$

Thus, *velocity of sound is directly proportional to the square root of absolute temperature With increase of temperature, velocity also increases*

Now equation (4) can also be written as

$$V=V_0(1+\alpha t)^{\frac{1}{2}} \quad \dots \quad (6)$$

As $\alpha=\frac{1}{273}$, αt is a small quantity. Therefore, when $(1+\alpha.t)^{\frac{1}{2}}$ is binomially expanded, we get $(1+\alpha t)^{\frac{1}{2}}=1+\frac{1}{2}\alpha t$, neglecting higher powers of αt Hence, the expression (6) becomes

$$V=V_o(1+\tfrac{1}{2}\alpha t)$$
$$=V_o+\tfrac{1}{2}.V_o\,\alpha.t$$

Substituting value of $V_o=332$ metres/sec and $\alpha=1/273$ we get

$$V=V_o+\tfrac{1}{2}\,332\,\tfrac{1}{273}\,t$$

or $$\mathbf{V}=\mathbf{V}_o+\mathbf{0{\cdot}6t} \quad \dots \quad (7)$$

The expression (7) is true when V and V_o are expressed in metres per second and t in °C.

Thus, *from above we find that approximately for every °C. rise of temperature, the velocity of sound increases by 0 6 metres per sec or 60 cm per sec.*

(*d*) **Wind :**—Let the velocity of sound be V and W be the wind velocity If wind is blowing in the same direction as that of sound, the sound velocity would become $V+W$. If it is blowing in the opposite direction, sound velocity would be $V-W$. If the wind direction is making an angle with the sound propagation, we will be required to consider the component of wind velocity along the direction of propagation of wind.

Therefore, in order to eliminate the wind effect, the determination of V is made in both the directions and the mean is taken.

(*e*) **Personal :**—This is such an error which cannot be helped. It varies from person to person. The best way is to take a large set of observations and get the mean value.

§ 4. **Practical determination of Velocity of Sound i air** :—The velocity of light is considered infinite compared to th velocity of sound Two distant stations are chosen. At ever station there is a gun and a stop watch. When the gun is fired a station *A*, the stop watch is started at the station *B* on seein the flash. When sound is heard by *B*, the stop watch is stopped The time interval thus noted by *B* gives the time in which soun has travelled the distance from *A* to *B*. The same experiment i repeated when sound is produced at *B* and is heard at *A*. From both the observations, the velocity of sound is calculated and th mean is taken.

Afterwards all corrections are applied and the standard value at *N.T.P.* is determined.

This is the general principle on which various methods are based.

QUESTIONS

1. State Newton's formula and discuss Laplaces' correction [see § 1 and § 2].

2 Discuss the effect of pressure, humidity and temperature on the velocity of sound [see § 3].

3. Prove that the velocity of sound is directly proportional to the root of absolute temperature of the medium [see § 3].

CHAPTER V

CHARACTERISTICS OF SOUND

§ 1. **Music and Noise** :—We have already seen that whenever a disturbance is produced in a medium, sound is heard. This sound may broadly be divided into (*a*) Music and (*b*) Noise. When the sound produced is due to a periodic disturbance of a certain regularity, with continuity and without abrupt changes in amplitude, it is called *Musical sound*. *When the disturbance is non periodic* and there are sudden changes of amplitude, the sound so produced is called Noise. We are here mainly concerned with musical sound.

§ 2. **Characteristics of musical sound** :—There are three main characteristics of musical sounds by which they can be distinguished from each other.

(*a*) *Loudness.*
(*b*) *Pitch.*
and (*c*) *Quality or timbre.*

(*a*) **Loudness or Intensity** :—*Loudness relates to the magnitude of the sensation produced on the organ of hearing.* It is something subjective. Actually the objective term is intensity of sound and *it is defined as the energy of a wave passing through unit area in unit time in a perpendicular direction.* It can be shown that *this energy is directly proportional to the square of the amplitude* of the wave. Under certain limits, we can say that *loudness of sound is proportional to the intensity.* Loudness actually in addition depends on the sensitiveness of the ear.

It has been observed that sensation of loudness in ear depends on the frequency of sound also. Hence, two sounds of the same intensity but of different frequencies may appear to be of different loudness.

Ordinarily, greater is the intensity, greater is the loudness of sound.

Generally greater is the vibrating area and greater is its amplitude of vibration, greater will be loudness of sound it produces. That is why in big colleges we find very huge bells.

The intensity and hence the loudness of sound varies inversely as the square of its distance from the source.

The unit in which loudness is measured is called decibel. If we call the threshold of hearing has a loudness of zero decibel, whisper has 10 to 20 and conversation has 60-65 decibel loudness. The loudness becomes painful if it increases above 130 decibels.

(*b*) **Pitch or frequency** :—*The degree of shrillness or graveness of sound is defined as its pitch.* For example, the horn of a car or a whistle of a railway engine may have same loudness, but whistle appears to be more shrill. Similarly voice of a woman is shriller than that of a man, though it might have low loudness. This sensation of pitch depends on the frequency of sound wave. *Greater is the*

frequency greater is the pitch. Often pitch and frequency are used in the same sense.

The ear is not equally sensitive to all the frequencies. It has been observed that the lower limit is approximately 30 vibrations per second while the upper limit varies with the age of a person. Generally it varies between 13,000 to 20,000 vibrations per second. It has been observed that ear is the most sensitive at about 1000 vibrations per second.

Sounds of frequencies greater than 20,000 *v.p.s* belong to Ultrasonics These sounds cannot be heard by a human ear.

That is why we have special whistles sound of which cannot be heard by human beings but by dogs. In recent days, this science of Ultrasonics has developed to a very great stage.

(c) **Quality or Timbre**.—A sound of only one frequency is called pure note Usually a sounding body produces complex note. Tuning fork as shown in Fig 16 is an instrument of peculiar shape. It is normally made of elastic metal of certain dimensions. *A* and *B* are called the prongs and *C* is the handle. When the prongs are lightly struck against a rubber pad, the prongs begin to vibrate. Usually the frequencies of which they are made are 256, 288, 320, 341·3, 384, 426·7, 480, 512. Forks are also made having frequencies double the above mentioned ones These frequencies are chosen on a particular scale which is called a musical scale.

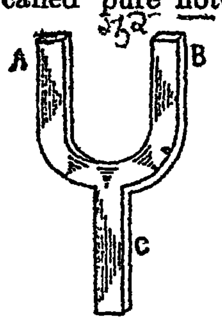


Fig 16

If a source has a frequency n and other has a frequency $2n$, $2n$ is called the harmonic of n,

Two sounds may have the same loudness and the pitch, yet we can say that they are not identical and can distinguish them from each other *That which distinguishes them from each other is called the Quality or Timbre of sound* This quality depends on the mixture of notes produced by it. Generally the note contains mixtxre of the fundamental and its harmonics. Richer a sound is greater is the admixture of harmonics it has.

Thus we distinguish a note of Tabla from that of harmonium, voice of one man from the other and so on.

QUESTIONS

1. Define and distinguish Music from Noise [See § 1]
2. What are the characteristics of sound ? Explain them giving suitable examples [See § 2]

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